PROGRESSIVE
APPLIED PHYSICS-I
For Polytechnics
(IN SI UNITS)

R.A. Banwat

EAGLE PRAKASHAN

Contents

S.No. Chapters Pages

Syllabus *(i)-(ii)*

OBJECTIVE QUESTION BANK 1 - 32

I-MECHANICS

1. Units and Dimensions UD-1-39
2. Force and Motion (FM) FM-1-62
3. Work Power and Energy (WPE) WPE-1-31
4. Rotational Motion (RM) RM-1-16
5. Simple Harmonic Motion and Vibrations (SHM) SHM-1-30
6. Surface Tension (ST) ST-1-18

II-HEAT

7. Heat and Temperature Measurement (HT) HT-1-24
8. Expansion and Solids (EOS) EOS-1-23
9. Transfer of Heat (TOH) TOH-1-31

LABORATORY WORK

Experiment Physics 1-24

Viva-Voice Physics 1-16

NEW SYLLABUS

(*Prescribed by S.B.T.E. Punjab*)

Applied Physics-I

		L T P
RATIONALE	**Hours/Weak**	**4 - 2**

Applied physics is a foundation course. Its purpose is to develop proper understanding of physical phenomenon and scientific temper in the students. The course covers basics like Mechanics, Heat, Waves, Sound, Light, Electrostatics, Electromagnetism and D.C. circuits. While teaching the subject, teachers should make maximum use of demonstrations to make the subject interesting to the students.

SYLLABUS (THEORY)

SECTION A : MECHANICS

1. UNITS AND DIMENSIONS

Fundamental and derived units is S I System. Dimensions of Physical Quantities, Principle of homogeneity, Dimensional equations, Applications of dimensional analysis. Checking the correctness of Physical equations, Derivation of simple physical relations, Limitations of Dimensional Analysis, significant figures and Error Analysis.

2. FORCE AND MOTION

Scalars and vectors, velocity and acceleration, Equations of motion, Newton's laws of motion. Force and its derivation from Newton's laws of motion, Composition and resolution of forces. Parabolic motion. Horizontal projection and projection at an angle, time of flight, Horizontal range and maximum horizontal range, simple problems : Centripetal acceleration, centripetal and centrifugal forces, Concept of friction and its applications, Applications to banking of roads.

3. WORK, POWER AND ENERGY

Work and its units, Work done on bodies moving on horizontal and inclined planes (consider frictional forces also). Concept of power and its units. Calculations of power (simple cases). Concept of kinetic energy and potential energy, Expressions for P.E. and K.E. Conservation of energy in the case of freely falling bodies. Principle of conservation of energy.

4. ROTATIONAL AND SIMPLE HARMONIC MOTIONS

Definition of moment of inertia, Moment of inertia of disc, ring and sphere. Torque and angular momentum and their inter relation. Principles of conservation (angular momentum and its applications). Kinetic energy of rolling body, S.H.M-derivation of displacement, velocity, acceleration, time period and frequency. Motion of cantilever. Free, forced and resonant vibrations (No derivation).

G.M. College of Education
Raipur, Bantalab
Jammu.
Acc. No. 3624 (D)
Dated 4/11/2015

SECTION B : HEAT

1. TEMPERATURE AND ITS MEASUREMENT

Concept of heat and temperature on the basis of K.E. of molecules. Units of heat, Basic Principles of measurement of temperature. Thermo couple, Bimetallic and resistance thermometres. Pyrometers. Ranges of these thermometers, Criteria for the selection of thermometers.

2. EXPANSION OF SOLIDS

Co-efficient of linear, surface and cubical expansions and relation amongst them. Thermal stresses (qualitative only) and their applications.

3. HEAT TRANSFER

Three modes of transfer of heat, Coefficient of thermal conductivity, its determination by Searle's method and Lee's disc method, Conduction through compound media (Series and parallel for two materials only), Heat radiation, Characteristics of heat radiations. Prevost's theory of heat exchanges, Black body radiations, Emissivity and absorbtivity, Kirchoff's law and Stefan's law of radiation.

PRACTICALS

1. To determine the density of a cylinder using vernier callipers and balance.
2. To determine the area of cross section of a wire using screw gauge.
3. To determine the thickness of glass piece using spherometer.
4. Calculation and verification of period of vibration of a cantilever (use graph).
5. To verify Parallelogram law of forces.
6. Measurement of K.E. gained by a body dropped through height *h*.
7. To find the coefficient of linear expansion of a given metal rod.
8. Calibration of Thermo-couple.

IMPORTANT FEATURES
OF
NEW EDITION

- As per New Syllabus
- Easy Language
- Each topic backed up by illustrated Figures
- A lot of simple solved and unsolved numerical problems

OBJECTIVE PHYSICS

- SHORT ANSWER TYPE QUESTIONS
- FILL IN THE BLANKS
- TRUE OR FALSE
- MULTIPLE CHOICE QUESTIONS

1. UNITS AND DIMENSIONS

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. What is measurement of physical quantities ?

Ans. The physical quantity to be measured is compared with a known fixed unit quantity of the similar nature. The known fixed quantity is called the *unit*.

Q.2. What are the desirable characteristics of a (Physical Standard) unit of a physical quantity.

Ans. Physical Standard (Unit) should ;

(i) be of a suitable size.

(ii) be easily reproducible.

(iii) be well defined.

(iv) not vary with time and temperature changes.

(v) be imperishable.

Q.3. Why wavelength of light radiation is selected as a standard for measuring length ?

Ans. The wavelength of light is not affected by time, temperature and place. Thus it is invariable and can be reproduced at any place and at any time.

Q.4. Define one second according to an atomic clock.

Ans. One second is defined as 9192631770 periods of a cesium clock. In an atomic clock, cesium atom vibrates 9192631770 times in one second.

Q.5. Define one metre in terms of wavelength of light.

Ans. The length of 1650763.73 waves of Kr^{86} is equal to one metre.

Q.6. Define international kilogram.

Ans. A kilogram was initially defined as the mass of one cubic decimetre of water at 4°C. Water was taken as standard because density of water at 4°C is maximum.

Or

International kilogram is defined as *the mass of a platinum iridium cylinder whose height and diameter are equal*. It is kept at the International Bureau of Weights and Measures at Sevres near Paris.

Q.7. What is Atomic Unit of Mass (a.m.u).

Ans. In atomic and nuclear physics, mass is measured in very small unit known as *atomic mass unit* (*a.m.u.*). One *a.m.u.* is equal to $\frac{1}{12}$ th of the mass of one carbon atom ($_6C^{12}$). In kilograms,

$$1\ a.m.u. = 1.66 \times 10^{-27}\ \text{kg}$$

Q.8. What are the Fundamental units and Derived units ?

Ans. Fundamental Units : In mechanics, all the physical quantities can be expressed in terms of *length, mass* and *time*. Therefore length, mass and time are called fundamental or basic quantities. The units used to measure the fundamental quantities *i.e.,* length , mass and time are called Fundamental units.

Derived Units : The units which can be derived from fundamental units are called Derived units. For example the unit of speed can be derived from the fundamental units of length and time, therefore it is a derived unit as,

$$\text{Speed} = \frac{\text{Distance (length)}}{\text{Time}}$$

Q.9. Explain S.I. Units.

Ans. S.I. units is the abbreviated form for SYSTEM INTERNATIONAL D' UNITS. This International System of Units was introduced in 1960 by the International Conference of Weights and Measures.

This international System of Units is based upon *seven fundamental or basic units* and *two supplementary units*.

1. **Seven Fundamental Units and their symbols are :**
   - (*i*) Metre (*m*) for unit of length.
   - (*ii*) Kilogram (*kg*) for unit of mass.
   - (*iii*) Second (*s*) for unit of time.
   - (*iv*) Kelvin (*K*) for unit of temperature.
   - (*v*) Candela (*cd*) for unit of luminous intensity.
   - (*vi*) Ampere (*A*) for unit of electric current.
   - (*vii*) Mole (*mol*) for unit of quantity of matter.
2. **Two supplementary Units with their symbols are ;**
   - (*i*) Radian (*rad*) for unit of plane angle.
   - (*ii*) Steradian (*sr*) for unit of solid angle.

Q.10. Give SI units of pressure, moment of Inertia, specific heat capacity, intensity of illumination and quantity of electricity. Also give the symbolic notation for the units.

Ans.

Quantity	**SI Units**	**Symbolic notation**
(i) Pressure	$N\,m^{-2}$	$kg\,m^{-1}\,s^{-2}$
(ii) Moment of Inertia	$kg\,m^2$	$kg\,m^2$
(iii) Specific heat	joule per kilogram per kelvin	$J\,kg^{-1}\,K^{-1}$
(iv) Intensity of illumination	candela per sq. metre	$cd\,m^{-2}$
(v) Quantity of electricity	ampere-second (coulomb)	$A\,s$

Q.11. What is a Coherent system of units ?

Ans. A coherent system of units is that system, which is based on certain fundamental units from which all other units can be derived by multiplying or dividing them without introducing numerical factors. SI units is a coherent system of units.

TYPE II. MATCH THE FOLLOWING :

1.	(*a*)	Force	(*i*)	Kg/m^3
	(*b*)	Density	(*ii*)	N/m
	(*c*)	Surface Tension	(*iii*)	Joule
	(*d*)	Heat	(*iv*)	Newton
2.	(*a*)	Co-efficient of thermal conductivity	(*i*)	$[M^1L^1T^{-1}]$
	(*b*)	Torque	(*ii*)	$[M^1L^{-1}T^{-2}]$
	(*c*)	Impulse	(*iii*)	$[M^1L^2T^{-2}]$
	(*d*)	Modulus of elasticity	(*ii*)	$[M^1L^{-1}T^{-1}]$
3.	(*a*)	Centripetal Force	(*i*)	$\sqrt{\frac{E}{d}}$
	(*b*)	Kinetic energy	(*ii*)	$2\pi\sqrt{\frac{l}{g}}$
	(*c*)	Velocity of sound	(*iii*)	$\frac{mv^2}{r}$
	(*d*)	Time period of a simple pendulum	(*iii*)	$\frac{1}{2}mv^2$

TYPE III. FILL IN THE BLANKS :

1. The SI units of luminous intensity is .... (mole/candela/photon)

2. The dimension of momentum are ....

($[M^1L^1T^{-1}]$ / $[M^1L^1T^{-2}]$ / $[M^{-1}L^3T^{-2}]$)

3. $[M^1L^2T^{-2}]$ is the dimensional formula for ....

(Force/Energy/Power)

4. According to the principle of homogeneity of dimensions, the dimensions of all the terms on both sides must be the .... (same/different)

5. The value of velocity is 72 km/hour. Its value in m/s will be...............

(30/15/20/25)

TYPE IV : MULTIPLE CHOICE QUESTIONS

1. Name the fundamental quantity :
   (a) Force (b) Length
   (c) Velocity (d) Area
2. Choose the derived unit :
   (a) Mass (b) Length
   (c) Force (d) Time
3. The unit of angular acceleration in the SI system is :
   (a) $N\ kg^{-1}$ (b) ms^{-2}
   (c) rad s^{-2} (d) $K\ m\ kg^{-1}$
4. Newton × sec. is the unit of :
   (a) Momentum (b) Impulse
   (c) Power (d) Work
5. Co-efficient of viscosity has dimensions :
   (a) $ML^{-1}T^{-1}$ (b) MLT^{-2}
   (c) ML^0T^{-2} (d) MLT^{-1}
6. The dimensional formula for latent heat is :
   (a) $M^0L^2T^{-2}$ (b) ML^2T^2
   (c) MLT^{-2} (d) ML^2T^{-1}
7. The dimensional formula of coefficient of thermal conductivity is :
   (a) $ML^2T^{-2}\theta^{-2}$ (b) MLT^{-2}
   (c) $MLT^{-3}\theta^{-1}$ (d) $ML^2T^{-3}\theta^{-1}$
8. Of the following quantities, which one has dimensions different from the remaining three :
   (a) Energy per unit volume
   (b) Force per unit area
   (c) Product of voltage and charge per unit volume
   (d) Angular momentum per unit mass
9. The dimensional formula for impulse is same as the dimensional formula for :
   (a) Momentum
   (b) Energy
   (c) Rate of change of momentum
   (d) Torque
10. Which of the following is dimensionally correct :
    (a) Pressure = Energy per unit area
    (b) Pressure = Energy per unit volume
    (c) Pressure = Force per unit volume
    (d) Pressure = Momentum per unit volume per unit time
11. The error in the measurement of the radius of a sphere is 0.2%, then the percentage error in the calculation of volume is :
    (a) 0.3 (b) 0.4
    (c) 0.6 (d) 0.06
12. The density of a cube is measured by measuring its mass and the length of its side. If the maximum errors in the measurement of mass and length are 3% and 2% respectively, the maximum error in the measurement of the density is :
    (a) 3% (b) 5%
    (c) 7% (d) 9%
13. The density of wood is 0.5 gm/cc in CGS system of units. The corresponding value in SI units is :
    (a) 500 (b) 0.5
    (c) 5×10^{-2} (d) 500000
14. A pressure of 10^6 dynes/cm^2 is equivalent to :
    (a) 10^5 N/m^2 (b) 10^6 N/m^2
    (c) 10^7 N/m^2 (d) 10^8 N/m^2
15. The percentage errors in the measurement of mass and speed are 2% and 3% respectively. How much will be the maximum error in the estimate of kinetic energy obtained by measuring mass and speed :
    (a) 11% (b) 8%
    (c) 5% (d) 1%

16. If the physical quantity X is represented by, $X = M^aL^bT^{-c}$ and the maximum percentage errors in measuring M, L and T are $\alpha\%$, $\beta\%$ and $\gamma\%$ respectively, then the total percentage error in X is :

(a) $(\alpha a + \beta b - \gamma c)\%$
(b) $(\alpha a + \beta b + \gamma c)\%$
(c) $(\alpha a - \beta b - \gamma c)\%$
(d) None of these

ANSWERS

Type II :

1. (*a*) (*iv*) ; (*b*) (*i*) ; (*c*) (*ii*) ; (*d*) (*iii*)
2. (*a*) (*iv*) ; (*b*) (*iii*) ; (*c*) (*i*) ; (*d*) (*ii*)
3. (*a*) (*iii*) ; (*b*) (*iv*) ; (*c*) (*i*) ; (*d*) (*ii*)

Type III :

1. Candela 2. $[M^1L^1T^{-1}]$ 3. Energy 4. same 5. 20

Type IV :

1. (b) 2. (c) 3. (c) 4. (b) 5. (a) 6. (a)
7. (c) 8. (d) 9. (a) 10. (b) 11. (c) 12. (d)
13. (a) 14. (a) 15. (b) 16. (b)

2. FORCE AND MOTION

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. Explain Scalar and vector quantities with examples.

Ans. Scalar Quantity : *A physical quantity which has magnitude alone and no direction is called a scalar quantity.* For example mass, volume, speed, density, charge, length, time are all scalar quantities.

Vector Quantity : *A physical quantity which has magnitude and direction both is called a vector quantity.* For example velocity, acceleration, force, momentum, electric field are all vector quantities.

Q.2. When a bullet is fired from a gun, the bullet moves forward but the gun kicks backward. Which law of motion is illustrated by the statement.

Ans. The statement illustrates Newton's third law of motion, which states that *to every action there is an equal and opposite reaction.*

Q.3. An empty truck and a loaded truck are moving with same velocities. Which truck stops first if brakes with same retarding forces are applied ?

Ans. Empty truck will stop first. Speed being the same, empty truck with lesser mass has less momentum.

Q.4. A man jumping out of a fast moving bus falls with his head forward. Explain.

Ans. When the man is in the bus, whole of his body is moving with the same velocity (velocity of bus). On jumping, his feet on touching the ground comes to rest, whereas the upper part of the body continues to move forward due to inertia of motion. Therefore, the man falls with his head forward.

Q.5. A force of 100 N makes an angle of 30° with the horizontal. What is the vertical component ?

Ans. Horizontal and vertical components of a force are $F\cos\theta$ and $F\sin\theta$ respectively, Here, $F = 100\text{N}$ and $\theta = 30°$

$$\therefore \quad \text{Vertical component} = 100\sin 30° = 100 \times \frac{1}{2} = 50\text{ N} \textbf{ Ans.}$$

Q.6. A ball is released from the top of a building of height h metres. It takes t seconds to reach the ground. What is the position of the ball at $\frac{t}{2}$ seconds.

Ans. We have, $h = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2$

Let x be the distance through which the ball falls in $\frac{t}{2}$ seconds.

$$\therefore \quad x = \frac{1}{2}g\left(\frac{t}{2}\right)^2 = \frac{1}{2}gt^2 \times \frac{1}{4} = h \times \frac{1}{4} = \frac{h}{4} \text{ metres. Ans.} \qquad \left[\because h = \frac{1}{2}gt^2\right]$$

Q.7. The greatest height to which a man can throw a stone is 100 metres. What is the maximum horizontal distance to which he can throw the stone.

Ans. Maximum height, $H_m = \frac{u^2}{2g} = 100$

$$\therefore \quad u^2 = 2g \times 100.$$

Now maximum horizontal range, $R_m = \frac{u^2}{g} = \frac{2 \times g \times 100}{g} = 200$ **Ans.**

Q.8. The range of a projectile, when thrown at an angle of 15° with the horizontal is 1500 m. What is the range of the projectile when projected at an angle of 45° to the horizontal ?

Ans. Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

When $\theta = 15°$, $\quad R_1 = \frac{u^2 \sin 2 \times 15}{g} = \frac{u^2 \sin 30°}{g} = \frac{u^2}{2g}$

When $\theta = 45°$, $\quad R_2 = \frac{u^2 \sin 2 \times 45}{g} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$

$$\therefore \quad R_2 = 2R_1 = 2 \times 1500 = 3000\, m \textbf{ Ans.}$$

Q.9. A roadway bridge over a canal is in the form of a circular arc of radius 18*m*. What is the greatest speed with which a motor cycle can cross the bridge without leaving the ground at the highest point.

Ans. At the highest point, $\frac{mv^2}{r} = mg$

$$\therefore \quad v^2 = rg \quad \text{or} \quad v = \sqrt{rg} = \sqrt{18 \times 9.8} = 13.26\, m/s \textbf{ Ans.}$$

TYPE II. MATCH THE FOLLOWING :

1.	(*a*)	Maximum Height of a projectile, H_m	(*i*)	$\frac{2u \sin \theta}{g}$
	(*b*)	Time of Flight	(*ii*)	$\frac{u^2 \sin 2\theta}{g}$
	(*c*)	Horizontal range, *R*	(*iii*)	$\frac{mv^2}{r}$
	(*d*)	Centripetal force	(*iv*)	$\frac{u^2 \sin^2 \theta}{2g}$
2.	(*a*)	Angle for Maximum Horizontal range	(*i*)	$\tan^{-1} \frac{v^2}{rg}$
	(*b*)	Path followed by a projectile	(*ii*)	Towards Centre
	(*c*)	Direction of Centripetal Force	(*iii*)	45°
	(*d*)	Angle of Banking	(*iv*)	Parabola

TYPE III. FILL IN THE BLANKS :

1. The component of a vector *P* is given as................
(P cosθ / P tanθ / P. cotθ)

2. The rate of change of momentum is proportional to the .... and takes place in the direction of force. (energy/applied force/power)

3. Momentum of a moving body is equal to............... $\left[\frac{m}{v} / m\times v / (m-v)\right]$

4. Resultant of two forces acting in the same direction along the same line is equal to............. of the forces. (difference/sum/product)

5. The path followed by an object thrown in the air by giving some initial velocity is .... (circular/straight/parabola)

6. Drycleaning machine works on the principle of.................
(centripetal force/centrifugal force)

7. At sharp bends, road curves are banked to provide...............
(horizontal force/centripetal force/centrifugal force)

8. While going along a curved path at a fast speed, a cyclist bends................with the vertical to provide necessary centripetal force.
(inwards/outwards/downwards)

9. Rockets work on the principle of conservation of.................
(energy/mass/momentum)

10. A body is thrown vertically upwards, the velocity of the body at the highest point is .... (maximum/zero/equal to the velocity of projection)

TYPE IV : MULTIPLE CHOICE QUESTIONS

1. A body is moving with uniform velocity, its acceleration is :
   - (a) positive
   - (b) negative
   - (c) zero
   - (d) None of these

2. When velocity of a body is increased, its momentum :
   - (a) does not change
   - (b) decreases
   - (c) first increases then decreases
   - (d) increases

3. Which of the following quantities is not a vector :
   - (a) Velocity
   - (b) Mass
   - (c) Force
   - (d) Acceleration

4. A body starts from rest with a uniform acceleration. If it travels distance s_1 in the first 2 seconds and s_2 in the next 2 seconds, then the relation between s_1 and s_2 will be :
   - (a) $s_1 = s_2$
   - (b) $2s_1 = s_2$
   - (c) $3s_1 = s_2$
   - (d) $3s_1 = 2s_2$

5. A ball is thrown vertically upwards in air. If the air resistance cannot be neglected then the acceleration of the ball at the highest point is :
   - (a) equal to g
   - (b) greater than g
   - (c) less than g
   - (d) equal to zero

6. The displacement of x of a particle along a straight line at time t is given by $x = a_0 - a_1 t + a_2 t^2$. The acceleration of the particle is :
   - (a) a_0
   - (b) $2a_1$
   - (c) $2a_2$
   - (d) a_2

7. A passenger travels along the straight road for half the distance with velocity v_1 znd the remaining half distance with velocity v_2. Then average velocity is given by :
   - (a) $v_1 v_2$
   - (b) v_2^2/v_1^2
   - (c) $(v_1 + v_2)/2$
   - (d) $2v_1 v_2 (v_1 + v_2)$

8. A bullet is fired from a gun. The gun recoils. The K.E. of gun in comparison to the K.E. of bullet is :
   - (a) less
   - (b) more
   - (c) equal
   - (d) None of these

9. For a body moving in a horizontal circle with constant speed, which of the following quantities remain unchanged :
   - (a) Kinetic energy
   - (b) Velocity
   - (c) Acceleration
   - (d) Velocity & Kinetic energy

10. A player kicks up a ball at an angle θ with the horizontal. The horizontal range is maximum when θ is equal to :
    - (a) 30°
    - (b) 45°
    - (c) 60°
    - (d) 75°

11. A particle moves in a plane with a constant acceleration in a direction different from the direction of initial velocity. The path of the particle is :
    - (a) Straight line
    - (b) Arc of a circle
    - (c) Parabola
    - (d) Ellipse

12. When a particle at rest is acted upon by two equal forces at 90° to each other, the path of the particle will be :

(a) a hyperbola

(b) a zig-zag curve

(c) a straight line

(d) a parabola

13. An artificial satellite revolving round the earth in circular orbit will move with :

(a) without any force

(b) constant speed

(c) constant acceleration

(d) constant velocity

14. A force produces an acceleration of 1 ms^{-2} in a mass of 1 kg. The force is called one :

(a) kg.wt.

(b) gm.wt.

(c) Newton

(d) dyne

15. The property of a body due to which it can not change its state of rest or uniform motion by itself is called :

(a) Inertia

(b) Force

(c) Momentum

(d) Impulse

16. An athlete runs through some distance before taking a long jump. He does so because :

(a) Running makes him jump comfortably

(b) It warms him up

(c) Inertia of motion of his body at the time of jumping helps him to go through longer distance

(d) He does not slip at the time of jumping.

17. The mass of an astronaut would be :

(a) greater on the moon than on earth

(b) same on earth and moon

(c) lesser on moon than earth

(d) None of these

ANSWERS

Type II :

1. (*a*) (*iv*) ; (*b*) (*i*) ; (*c*) (*ii*) ; (*d*) (*iii*)

2. (*a*) (*iii*) ; (*b*) (*iv*) ; (*c*) (*ii*) ; (*d*) (*i*)

Type III :

1. $P\cos\theta$ **2.** applied force **3.** $m \times v$ **4.** sum **5.** parabola **6.** centrifugal force **7.** centripetal force **8.** inwards **9.** mometum **10.** zero

Type IV :

1. (c) **2.** (d) **3.** (b) **4.** (c) **5.** (a) **6.** (c)

7. (d) **8.** (a) **9.** (a) **10.** (b) **11.** (c) **12.** (c)

13. (b) **14.** (c) **15.** (a) **16.** (c) **17.** (b)

3. ROTATIONAL MOTION

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. What is needed to keep a body rotating ?

Ans. Torque is needed to keep a body rotating. Torque produces angular acceleration.

Q.2. Why rim of a fly wheel is made thick (*i.e.* most of the mass is concentrated at the rim) ?

Ans. In a fly wheel, most of the mass is concentrated at the rim to increases the moment of inertia. Greater moment of inertia of the fly wheel helps to maintain uniform speed.

Q.3. A hard boiled egg and a raw egg are rotated on the table top by applying the same torque in both the cases. How will you distinguish between the two ?

Ans. The hard boiled egg acts a solid mass while rotating, whereas raw egg is a liquid matter. During rotation, liquid mass tends to move away from the centre. Thus in case of raw egg, moment of inertia increases. The raw egg with more moment of inertia spins with less angular acceleration. Hence when same torque is applied, the hard boiled egg spins faster.

Q.4. A circular platform is revolving at a uniform angular speed and a person is sitting near the edge. What will happen if he starts moving from the edge towards the centre of the platform ?

Ans. The angular momentum of system remains constant. When the person is sitting near the edge, the moment of inertia is more. When the person starts moving towards the centre, the moment of inertia decreases therefore, the angular velocity increases.

Q.5. The torque acting on a system is zero. What parameter is constant ?

Ans. When torque acting is zero, the system moves with uniform angular velocity. Therefore angular momentum is constant.

Q.6. A stone attached to one end of a string is revolved around a stick so that the string winds upon the stick and gets shortened. What is conserved and what happens to the speed of rotation ?

Ans. Kinetic energy of rotation is conserved. As the string shortens, the moment of inertia decreases. Therefore, speed of rotation increases.

Q.7. Without weighing how will you distinguish between the two identical spheres of the same material, one being solid and the other being hollow.

Ans. The spheres can be distinguished by rolling them down an inclined plane. The solid sphere will roll down faster.

TYPE II. MATCH THE FOLLOWING :

1.	(*a*)	Moment of Inertia	(*i*)	$I\omega$
	(*b*)	Angular Momentum	(*ii*)	$I\alpha$
	(*c*)	Kinetic Energy of rotation	(*iii*)	MK^2
	(*d*)	Torque	(*iv*)	$\frac{1}{2} I\omega^2$

2. (*a*)	Force	(*i*)	$I\omega$
(*b*)	Mass	(*ii*)	$\tau\omega$
(*c*)	$F \times v$	(*iii*)	M.I.
(*d*)	$m \times v$	(*iv*)	Torque

TYPE III. TRUE AND FALSE

1. If the torque acting on a rotating body is zero, the angular momentum is conserved.
2. Radius of gyration of a body is the radius of the circle in which the body is rotating.
3. M.I. of a wheel decreases on increasing its diameter.
4. Rolling is a combination of rotational and translational motion.
5. Torque in rotational motion has the same significance as force in linear motion.

TYPE IV : MULTIPLE CHOICE QUESTIONS

1. The moment of momentum is called :
   - (a) couple
   - (b) torque
   - (c) impulse
   - (d) angular momentum
2. When a steady torque is acting on a body, the body :
   - (a) continues in its state of rest or uniform motion along a straight line
   - (b) gets linear acceleration
   - (c) gets angular acceleration
   - (d) rotates at a constant speed
3. The moment of inertia of a circular ring of radius R and mass M about a tangent in its plane, is :
   - (a) MR^2
   - (b) $MR^2/2$
   - (c) $3/2\ MR^2$
   - (d) $2MR^2$
4. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along :
   - (a) the radius
   - (b) the tangent to orbit
   - (c) line at an angle of 45° to the plane of rotation
   - (d) the axis of rotation
5. Angular momentum of a body is defined as the product of :
   - (a) mass and angular velocity
   - (b) centripetal force and radius
   - (c) linear velocity and angular velocity
   - (d) moment of inertia and angular velocity.
6. When a torque acting upon a system is zero, which of the following will be constant :
   - (a) Force
   - (b) Linear momentum
   - (c) Angular momentum
   - (d) Linear impulse
7. A dancer on ice spins faster when she folds her arms. This is due to :
   - (a) increase in energy and increase in angular momentum
   - (b) decrease in friction at the skates
   - (c) conservation of angular momentum
   - (d) increase in energy and decrease in angular momentum
8. If I, α and τ are the moment of inertia, angular acceleration and torque respectively of a body rotating about any axis with angular velocity ω, then :
   - (a) $\tau = I\alpha$
   - (b) $\tau = I\omega$
   - (c) $I = \tau\omega$
   - (d) $\alpha = \tau^2\omega$

9. A hollow cylinder and a solid cylinder having the same mass and same diameter are released from rest simultaneously from the top of an inclined plane. Which will reach the bottom first :

(a) the solid cylinder

(b) the hollow cylinder

(c) both will reach the bottom together

(d) the one having greater density

10. Energy of a rotating body is :

(a) $I\omega$ (b) $I\omega^2$

(c) $\frac{1}{2} I\omega^2$ (d) $I^2\omega$

11. A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is :

(a) 2/5 (b) 2/7

(c) 3/5 (d) 3/7

12. A spherical solid ball of 1 kg mass and radius 3 cm is rotating about an axis passing through its centre with an angular velocity of 50 radians/s. The kinetic energy of rotation is :

(a) 4500 J (b) 90 J

(c) 910 J (d) 9/20 J

13. The mass of a flywheel is concentrated near its rim as to have :

(a) a large moment of inertia

(b) a small moment of inertia

(c) a stable moment of inertia

(d) an unstable equilibrium

14. A fan of moment of inertia 0.6 kg × metre^2 is to run upto a working speed of 0.5 revolution per second. Indicate the correct value of the angular momentum of the fan :

(a) $0.6\,\pi \text{ kg} \times \frac{\text{metre}^2}{\text{sec}}$

(b) $6 \text{ kg} \times \frac{\text{metre}^2}{\text{sec}}$

(c) $3 \text{ kg} \times \frac{\text{metre}^2}{\text{sec}}$

(d) $\frac{\pi}{6} \text{ kg} \times \frac{\text{metre}^2}{\text{sec}}$

15. The unit of torque is :

(a) $\text{kg} \times \frac{\text{metre}^2}{\text{sec}^2}$

(b) kg × metre

(c) $\text{kg} \times \frac{\text{metre}}{\text{sec}^2}$

(d) $\frac{\text{kg}}{\text{metre}^2 \times \text{sec}^2}$

ANSWERS

Type II :

1. (*a*) (*iii*) ; (*b*) (*i*) ; (*c*) (*iv*) ; (*d*) (*ii*)
2. (*a*) (*iv*) ; (*b*) (*iii*) ; (*c*) (*ii*) ; (*d*) (*i*)

Type III :

1. True 2. False 3. False 4. True 5. True

Type IV :

1. (d) **2.** (c) **3.** (c) **4.** (d) **5.** (d) **6.** (c)
7. (c) **8.** (a) **9.** (a) **10.** (c) **11.** (b) **12.** (d)
13. (a) **14.** (a) **15.** (a)

4. SIMPLE HARMONIC MOTION AND VIBRATIONS (SHM)

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. What is periodic Motion ? What is the condition for a periodic motion to be a Simple Harmonic Motion ?

Ans. **Periodic Motion :** Any motion which repeats itself after equal intervals of time (however small) is called periodic motion.

Condition of SHM : The restoring force must be proportional to the displacement of the particle from the mean position.

Q.2. A particle is executing SHM between two points A and B. C is the mid point. Name the points where acceleration is maximum and the points where velocity is maximum.

Ans. A and B are the extreme positions and C is the mean position. Therefore, acceleration is maximum at the points A and B. Velocity is maximum at point C.

Q.3. A particle is executing SHM. At what displacement, Kinetic and potential energies are equal ?

Ans. When a particle executes SHM, its energy changes between K.E. and P.E. such that sum of the two remains constant. At any particular instant say at displacement y,

$$\text{K.E.} = \frac{1}{2} m\omega^2(A^2 - y^2) \text{ where A is the amplitude}$$

and $$\text{P.E.} = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2$$

When, $$\text{K.E.} = \text{P.E.}$$

We have, $$\frac{1}{2} m\omega^2(A^2 - y^2) = \frac{1}{2} m\omega^2 y^2$$

Or $2y^2 = A^2$ Or $y = \frac{A^2}{\sqrt{2}}$

$\therefore$ At displacement $y = \frac{A^2}{\sqrt{2}}$, K.E. = P.E.

Q.4. A loaded bus is more comfortable than an empty bus. Why ?

Ans. The body of the bus is supported on a number of springs (shockers) to absorb sudden shocks. During jerks the springs set up the body to vibrate. The frequency of vibrations is given by the relation.

$$\text{Frequency, } n = \frac{1}{2\pi}\sqrt{\frac{K}{\text{mass of the bus}}}$$

Where K is the spring constant.

(i) **When empty :** When bus is empty, the body vibrates with larger frequency and the body is subjected to quicker up and down movements, which is uncomfortable.

(ii) **When loaded :** On loading the bus, mass becomes more and the frequency decreases as is clear from the above relation. The bus therefore is subjected to slow up and down movements which is not uncomfortable.

Therefore, loaded bus is more comfortable than an empty bus.

Q.5. A rattling sound is heard when an automobile picks up certain particular speed. Why ?

Ans. When the frequency of vibrations of the engine becomes equal to the natural frequency of some loose part in the body of the automobile, resonance takes place and the loose part begins to vibrate producing rattling sound.

Q.6. Marching troops are asked to break steps while crossing a suspension bridge. Why ?

Ans. If the frequency of the steps of marching troops coincides with the natural frequency of the suspension bridge, resonance takes place. As a result, the amplitude of vibrations of the bridge increases and there is fear that the bridge may collapse.

TYPE II. FILL IN THE BLANKS :

1. Time period of a seconds pendulum is.............seconds. (1/3/2)
2. Acceleration of a particle in SHM at mean position is .... (zero/maximum)
3. In SHM restoring force is maximum at................. (mean position/extreme position)
4. P.E. of a body in SHM is maximum at............... (any position/mean position/extreme position)
5. Energy of particle in SHM is proportional to............... (square of amplitude/time period/displacement)

TYPE III. MULTIPLE CHOICE QUESTIONS

1. A particle is moving in a circle with uniform speed. Its motion is............

(a) periodic and simple harmonic

(b) periodic but not simple harmonic

(c) parabolic

(d) None of the above

2. A simple pendulum is made of a body which is a hollow sphere containing mercury suspended by means of a wire. If a little mercury is drained off, the period of pendulum will :

(a) remain unchanged

(b) increase

(c) decrease

(d) become erratic

3. The total energy of a particle executing SHM is proportional to :

(a) displacement from equilibrium position

(b) frequency of oscillations

(c) velocity in equilibrium position

(d) square of amplitude

4. The period of a simple pendulum is doubled when :
   - (a) its length is doubled
   - (b) the mass of the bob is doubled
   - (c) its length is made four times
   - (d) the mass of the bob and the length of the pendulum are doubled

5. The total energy of a particle executing simple harmonic motion with amplitude a is proportional to :
   - (a) a^2
   - (b) a^{-2}
   - (c) a
   - (d) $1/a$

6. A body of mass 5 gm is executing simple harmonic motion about a point O with an amplitude of 10 cms. Its maximum velocity is 100 cm/sec. Its velocity will be 50 cms/sec. at a distance:
   - (a) 5 cms
   - (b) $5\sqrt{2}$ cms
   - (c) $5\sqrt{3}$ cms
   - (d) $10\sqrt{2}$ cms

7. A pendulum suspended from the ceiling of a train has a period T when the train is at rest. When the train is accelerating with a uniform acceleration a, the period of oscillation will :
   - (a) become zero
   - (b) change
   - (c) remain unaffected
   - (d) become infinite

8. A simple pendulum performs simple harmonic motion about $X = 0$ with an amplitude A and time period T. The speed of the pendulum at $X = A/2$ will be :
   - (a) $\frac{\pi A\sqrt{3}}{T}$
   - (b) $\frac{\pi A}{T}$
   - (c) $\frac{\pi A\sqrt{3}}{2T}$
   - (d) $\frac{3\pi^2 A}{2T}$

9. For a particle executing simple harmonic motion, the kinetic energy K is given by :

   $K = K_0 \cos^2 \omega t$

   The maximum value of potential energy is :
   - (a) K_0
   - (b) zero
   - (c) $K_0/2$
   - (d) None of these

10. If the period of oscillation of mass M suspended from a spring is one second, then the period of $4M$ will be :
    - (a) $\frac{1}{2}$ sec
    - (b) $\frac{1}{4}$ sec
    - (c) 2 sec
    - (d) 4 sec

11. A second's pendulum is placed in a space laboratory orbiting around the earth at a height 3R from earth's surface where R is earth's radius. The time period of the pendulum will be :
    - (a) zero
    - (b) $2\sqrt{3}$ s
    - (c) 4 s
    - (d) infinite

12. The length of the second's pendulum on the surface of earth is 1 metre. The length of second's pendulum on the surface of moon, where g is $\frac{1}{6}$ th of the value of g on the surface of earth is :
    - (a) $\frac{1}{6}$ metre
    - (b) 6 metre
    - (c) $\frac{1}{36}$ metre
    - (d) 36 metre

13. A body is executing SHM. When its displacement from the mean position is 4 cm and 5cm, the corresponding velocity of the body is 10 cm per sec and 8 cm per sec. Then, the time period of the body is :
    - (a) 2π sec
    - (b) $\frac{\pi}{2}$ sec
    - (c) π sec
    - (d) $\frac{3\pi}{2}$ sec

ANSWERS

Type II :

1. 2
2. zero
3. extreme position
4. extreme position
5. square of amplitude

Type IV :

1. (b)
2. (c)
3. (d)
4. (c)
5. (a)
6. (c)
7. (b)
8. (a)
9. (a)
10. (c)
11. (d)
12. (a)
13. (c)

5. SURFACE TENSION & VISCOSITY

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. Small insects can walk on the surface of still water. How ?

Ans. Free surface of water at rest behaves like a stretched membrane and tends to contract due to surface tension. When insects walk on the surface, the surface layer is depressed and thus surface area is increased. In a tendency to contract the free surface, stretched membrane supports the weight of the insects. Therefore they can walk on the surface of water.

Q.2. A drop of a liquid is spherical in shape. Why ?

Ans. Due to surface tension, free surface of a liquid tends to contract or is in tendency to have minimum surface area. For a given volume, sphere has the minimum surface area. Thus to have minimum surface area, liquid drops acquire spherical shape if there is no external force acting.

Q.3. Why a steel needle placed gently on the surface of water can float ?

Ans. When a small steel needle is placed gently on the free surface of water at rest, the surface of water below the needle gets slightly depressed. The surface area of water below the needle is increased and becomes curved. The force of surface tension acts along the tangent to the curved surface. The vertical component of the force of surface tension balances the weight of the needle and hence the needle floats.

Q.4. Explain why, two droplets of a liquid (mercury or water etc.) when brought in contact are pulled together to form a bigger drop ?

Ans. Two droplets of a liquid when brought in contact are pulled together due to force of cohesion and form a bigger drop due to surface tension. The surface area of the single drop is less than the total surface area of the two drops.

Q.5. An oil drop placed on the surface of water spreads out and form a thin film, whereas a drop of water placed in oil contracts to a spherical shape. Why ?

Ans. Force of cohesion between oil molecules is less than the force of adhesion between water molecules and oil molecules and surface tension of oil is very small. Therefore, oil spreads out on the surface of water in the form of a thin layer.

Force of cohesion between water molecules is much greater than the adhesive force between water molecules and oil molecules and surface tension of water is very large. Therefore, water drop acquires spherical shape and contracts in the oil.

Q.6. Explain the cleaning action of soap or detergent.

Ans. With the addition of soap or detergents into water, its surface tension is reduced. Detergent or soap solution therefore can penetrate into the pores of the clothes and can dissolve the dirt or grease particles which can be washed away by running water. Ordinary water due to high value of surface tension can not pierce into the pores, therefore cleaning becomes difficult.

Q.7. A cricket ball when thrown in air without spin, follows parabolic path, whereas spinning cricket ball does not follow parabolic path. Why ?

Ans. Spinning ball carries some air along the direction of spin. Therefore, the relative speeds of air above and below the ball become different. According to Bernoulli's theorem, there is created some pressure difference above and below the ball. Due to this pressure difference, the ball does not follow parabolic path and its direction can not be assessed.

Q.8. Why do the clouds float in air ?

Ans. Clouds are very fine water droplets suspended in air. The terminal velocity of these water droplets in clouds is very small, because it is directly proportional to the square of the radius of the droplets. The droplets acquire terminal velocity much before reaching the earth (rain) and are seen to float in the air in the form of clouds.

TYPE II. TRUE AND FALSE.

1. Dew drops are cubical in shape.
2. Surface energy of a surface film is the work done to increase the area of the surface film through unity.
3. Angle of contact for a liquid which does not wet the surface is less than 90°.
4. Water reaches the leaves and branches of the tree by capillary action.
5. Surface tension of water is decreased by dissolving soap in it.
6. Blood pressure of a person (standing) is greater near the feet than near the head.
7. A small rain drop falls faster than a big rain drop.
8. Mercury rises in a capillary tube.
9. Lubricating action of an oil decreases with rise in temperature.
10. If the diameter of a capillary tube is doubled, the rise of water in it is also doubled.

TYPE III. FILL IN THE BLANKS

1. Surface tension of a liquid ....with the rise in temperature. (increases/decreases)
2. Kerosene rises up in the wick of a lamp due to................ (Surface tension/Viscosity)
3. Two capillaries of different radii are held vertically in a liquid. The height of liquid will be more in the tube of...............radius. (larger/smaller)
4. A good lubricant should have.................viscosity. (high/low)

5. Soap..............the surface tension of water. (increases/decreases)

6. Fields are ploughed to preserve..............for longer period. (moisture/crops/air)

TYPE IV : MULTIPLE CHOICE QUESTIONS

1. A liquid does not wet the surface of a solid if the angle of contact is :
   (a) 0° (b) < 90°
   (c) 45° (d) > 90°

2. Meniscus of mercury in a capillary is :
   (a) Concave (b) Convex
   (c) Plane (d) None of these

3. The velocity of a falling rain drop attains limited value because of :
   (a) surface tension
   (b) upward thrust of air
   (c) viscous force of air
   (d) air currents

4. Two water droplets merge to form a larger drop. In this process,
   (a) energy is released
   (b) energy is absorbed
   (c) no change takes place
   (d) None of these

5. A good lubricant should have :
   (a) high viscosity (b) low viscosity
   (c) high density (d) low B.P.

6. Stream line motion is that motion in which there is :
   (a) Radial velocity gradient
   (b) Longitudinal velocity gradient
   (c) Both the above
   (d) None of these

7. The rate of flow of a liquid through a capillary tube in an experiment to determine the viscosity of a liquid will increase :
   (a) If the pressure of the tube is increased
   (b) If the length of the tube is increased
   (c) If the radius of tube is increased
   (d) None of these

8. Which of the following substances has the greatest viscosity :
   (a) Water (b) Hydrogen
   (c) Mercury (d) Air

9. The lift in an aeroplane is based on :
   (a) Law of gravitation
   (b) Inverse square law
   (c) Torricelli's theorem
   (d) Bernoulli's principle

ANSWERS

Type II :

1. False 2. True 3. False 4. True 5. True
6. True 7. False 8. False 9. True 10. False

Type III :

1. decreases 2. Surface Tension 3. smaller 4. high
5. decreases 6. moisture

Type IV :

1. (d) 2. (b) 3. (c) 4. (a) 5. (a) 6. (b)
7. (b) 8. (c) 9. (d)

6. ELASTICITY

MULTIPLE CHOICE QUESTIONS

1. The property of a body due to which the body tends to regain its original length, volume or shape is called :
   (a) Plasticity (b) Elasticity
   (c) Density (d) Resistivity

2. Force acting per unit area of a body or rod or wire is called :
   (a) Strain
   (b) Density
   (c) Stress
   (d) Surface density

3. The ratio of change in length to the original length is :
   (a) Stress
   (b) Tension
   (c) Volumetric strain
   (d) Longitudinal strain

4. According to Hooke's law, within elastic limits, we have :
   (a) Force is proportional to strain
   (b) Stress is proportional to strain
   (c) Stress is inversely proportional to strain
   (d) None of these

5. A copper wire of area of cross-section 2×10^{-2} cm^2 is fixed at one end and loaded at the other end with a mass of 400 gm. The stress developed is : ($g = 10$ m/s^2)
   (a) 2.0×10^6 N/m^2
   (b) 4×10^6 N/m^2
   (c) 2.5×10^7 N/m^2
   (d) zero

6. The highest value of stress for which Hooke's law holds good is called :
   (a) elasticity
   (b) strain
   (c) elastic limit
   (d) Young's modulus

7. A body which regains its original shape, size or volume after removing the deforming forces is called :
   (a) Plastic body (b) Hard body
   (c) Soft body (d) Elastic body

8. A metal can be drawn into long fine wires. Due to this property, the metal is called :
   (a) malleable (b) ductile
   (c) brittle (d) plastic

9. Force required to stretch a wire of unit area of cross-section to double its length is numerically equal to :
   (a) Young's modulus
   (b) Bulk Modulus
   (c) Strain
   (d) Modulus of rigidity

10. Steel is more elastic than rubber because for a given stress, strain produced in steel is :
    (a) greater than that in rubber
    (b) equal to that in rubber
    (c) less than that in rubber
    (d) None of these

11. A glass sheet breaks into pieces when struck with hammer. Due to this property glass is said to be :
    (a) ductile (b) elastic
    (c) malleable (d) brittle

12. Which of the following substances is not elastic ?
    (a) Steel rod
    (b) Butter
    (c) Wooden rod
    (d) Steel sheet

ANSWERS

1. (b) 2. (c) 3. (d) 4. (b) 5. (a) 6. (c)
7. (d) 8. (b) 9. (a) 10. (c) 11. (d) 12. (b)

EXPLANATION

1. The bodies tend to regain their original length, volume, shape etc. due to the propert of elasticity.

2. Stress = Force/Area

3. Longitudinal strain $= \dfrac{\text{Change in length}}{\text{Original length}}$

4. Within elastic limits, stress $\propto$ strain.

5. Stress $= \dfrac{F}{A} = \dfrac{mg}{\text{area}} = \dfrac{0.4 \times 10}{2 \times 10^{-6}} = 2.0 \times 10^{6}\ \text{N/m}^2$.

6. The maximum force within which a body regains its original configuration whe deforming forces are removed is called elastic limit.

7. In an elastic body the strain is less and the body regains its original shape or siz quickly.

8. The metals which can be drawn into fine wires are called ductile.

9. $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F/A}{l/L} = \dfrac{F/1}{1/1} =$ F, the force.

10. For equal length of steel and rubber, equal forces will produce smaller increase i length in steel than in rubber.

11. A substance which breaks into pieces on applying external force is said to be brittle

12. Butter does not regain its original configuration when deforming force is removed Hence butter is said to be plastic.

HEAT

1. HEAT AND TEMPERATURE

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. State any three effects othe temperature of a normal man is 98.4°. The water i very hot while the man is not hot to touch. Why ?

Ans. Water boils at 100°C (Celsius) and the normal temperature of the man is 98.4° (Fahrenheit). 98.4°F = 36.8°C which is very small as compared to 100°C. So the ma is not hot to touch.

Q.3. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which one will respond quickly to temperature changes ?

Ans. Thermometer with cylindrical bulb. The reason is that the area of cylindrical surface is greater than spherical surface and hence the mercury in cylindrical bulb reaches the temperature of the surroundings earlier.

Q.4. What are the different scales of measuring temperature ? What is the relation between them.

Ans. ***Different scales of thermometers are :***

(i) Celsius scale (°C)

(ii) Fahrenheit scale (°F)

(iii) Reumur scale (°R)

(iv) Kelvin scale or Absolute scale (°K)

Relationship :

$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{R-0}{80-0} = \frac{K-273}{373-273}$$

or $$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4} = \frac{K-273}{5}$$

Q.5. How will you convert temperature from Celsius scale into Kelvin scale ?

Ans. Celsius temperature can be converted into temperature on the Kelvin scale by using the relationship.

$$T°K = 273 + t°C$$

where t is the reading on centigrade scale.

Q.6. Name the thermometers employed to measure the temperature of ranges :

(i) – 30°C to 350°C

(ii) – 40°C to 500°C

(iii) – 200°C to 1600°C

Ans. (i) Mercury thermometer

(ii) Bimetallic thermometer

(iii) Platinum Resistance thermometer.

Q.7. What is the principle of Platinum Resistance thermometer ?

Ans. **Principle of Resistance thermometer :** If a platinum wire is heated, its resistance increases and the resistance decreases if it is cooled. The resistance increases uniformly with the increase in temperature :

If R_0 is the resistance at 0°C, then resistance at any temperature t°C is given by :

$$R_t = R_0(1 + \alpha t)$$

where α is the temperature co-efficient of resistance.

Q.8. The top of a lake is frozen. Air in contact is at –15°C. What do you expect the maximum temperature of water (*i*) in contact with the lower surface of ice and (*ii*) at the bottom of the lake ?

Ans. (*i*) 0°C

(*ii*) 4°C

TYPE II : MULTIPLE CHOICE QUESTIONS

1. A 'Celsius' is a unit :
   - (a) of current
   - (b) of heat
   - (c) equivalent to degree centigrade
   - (d) equivalent to degree Kelvin

2. The temperature at which centigrade and Fahrenheit scales give the same reading :
   - (a) 40°C (b) 60°C
   - (c) – 40°C (d) 40°F

3. In the celsius temperature scale, the absolute zero of temperature is at :
   - (a) 0°C (b) –32°C
   - (c) 100°C (d) –273.15°C

4. Water boils at 100°C in Bombay. In Simla, it will boil at :
   - (a) 100°C
   - (b) more than 100°C
   - (c) Less than 100° C
   - (d) None of the above

5. Mercury thermometer may be used to measure temperatures upto nearly :
   - (a) 100°C (b) 212°C
   - (c) 350°C (d) 700°C

6. Water will boil at a temperature higher than 100°C :
   - (a) at mountains
   - (b) in a pressure cooker
   - (c) Both in (a) and (b) above
   - (d) neither (a) nor (b) above

7. Bimetallic thermometer is based upon the effect of heat that :
   - (a) Gases expand on heating
   - (b) Liquids expand on heating
   - (c) Resistance of metals decreases on heating
   - (d) Solids expand by different amounts on heating.

8. Thermo-electric thermometer is based upon :
   - (a) Liquids expand on heating
   - (b) Different solids expand by different amounts
   - (c) See-Beck effect
   - (d) Substances change their states on heating

9. The thermometer suitable to measure 2000°C is :
   - (a) Gas thermometer
   - (b) Mercury thermometer
   - (c) Vapour pressure thermometer
   - (d) Total radiation Pyrometer

10. 104°F corresponds to what temperature on the Kelvin scale :
    - (a) 313 (b) 308
    - (c) 377 (d) 418

11. 104°F corresponds to what temperature on the centigrade scale :
    - (a) 40 (b) 35
    - (c) 104 (d) 145

12. The temperature of a body is – 40°C. It is equal to :
    - (a) – 40°F (b) 22°F
    - (c) –180°F (d) 40°F

13. The temperature of the Sun is measured by :

(a) Thermoelectric thermometer

(b) Bimetallic thermometer

(c) Pyrometer

(d) Gas thermometer

14. Mercury thermometer is based upon the effect :

(a) Solids expand on heating

(b) Liquids expand on heating

(c) Gases expand on heating

(d) None of these

15. A difference of temperature of 25°C is equivalent to a difference of :

(a) 45°F (b) 72°F

(c) 32°F (d) 25°F

16. Temperature measures the :

(a) amount of heat contained in a body

(b) amount of heat required to heat it

(c) degree of hotness of the body

(d) Average K.E. of molecules

TYPE III. FILL IN THE BLANKS

1. $\frac{C}{5} = \frac{F-32}{.....-32}$
2. – 40°C = ...°F.
3. Absolute zero = ...°C
4. Range of platinum resistance thermometer is.................
5. The temperature range –200°C to 1600°C can be measured by ....
6. Bimetallic thermometer is neither..................thermometer nor............. in response.
7. In a thermoelectric thermometer, the energy is converted from ....into..........
8. A thermometer is more ....than a pyrometer.
9. Heat is the..............and temperature is its...............
10. Effects of Heat are used as..............for the construction and working of..............
11. 98.8°F = ...°C.
12. –32°F = ...°C

TYPE IV. TRUE AND FALSE

1. Mercury thermometer can be used to measure temperature upto 500°C (Given B.P. of Hg = 357°C).
2. Bimetallic thermometer is a very sensative thermometer.
3. Thermo-electric thermometer is very sensative and can record quick variations of temperature at a point.
4. Heat is the cause and temperature is its effect.
5. $\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4} = \frac{T°K-273}{5}$
6. Bimetallic thermometer can be used to measure the temperature of a man.

7. Water is a better thermometric substance than mercury.
8. The thermometric substance should have a high specific heat.
9. Disappearing filament optical Pyrometer is based upon the fact that two bodies having equal temperatures have the same colour.

ANSWERS

Type II :

1. (c) 2. (c) 3. (d) 4. (c) 5. (c) 6. (b)
7. (d) 8. (c) 9. (d) 10. (a) 11. (a) 12. (a)
13. (c) 14. (b) 15. (a) 16. (d)

Note : (5). Hg (Mercury) boils at 360°C.

Type III :

1. 212 2. – 40 3. –273 4. –200°C to 1200°C
5. Thermo-electric thermometer 6. sensative, quick 7. heat, electricity
8. sensative 9. cause, effect 10. principle, thermometers 11. 37
12. –35.6

Type IV :

1. False 2. False 3. True 4. True 5. True
6. False 7. False 8. False 9. True

2. EXPANSION OF SOLIDS

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. Why do clocks run slower in Summer ?

Ans. We know that $T = 2\pi\sqrt{\frac{l}{g}}$. In summers, the length of the pendulum increases due to thermal expansion and hence the time period increases. The clock thus run slower in summer.

Q.2. Why do clocks run faster in Winter ?

Ans. We know that $T = 2\pi\sqrt{\frac{l}{g}}$. In winters, the length of the pendulum decreases due to contraction on fall of temperature. So the time period decreases. The clocks then run faster in Winter.

Q.3. The difference between length of a certain brass rod and that of steel rod remains constant at all temperatures. Is this possible ?

Ans. Yes, when $l_1\alpha_1 = l_2\alpha_2$ where l_1 and l_2 are the lengths α_1 and α_2, the coefficients of linear expansion of brass and steel respectively.

Change in length of brass rod $= l_1\alpha_1 t$

and, Change in length of steel rod $= l_2\alpha_2 t$

But, $l_1\alpha_1 t = l_1\alpha_1 t$

∴ Under these conditions, the difference between lengths remain constant.

Q.4. Explain why a beaker full of water at 4°C overflows if the temperature is decreased or increased ?

Ans. The reason is the anomalous behaviour of water. The density of water is maximum at 4°C. So water expands whether it is cooled below 4°C or heated above 4°C.

Q.5. A long cylindrical Vessel having a linear coefficient of expansion α is filled with a liquid upto a certain level. On heating, it is found that the level of the liquid in the cylinder remains the same. What is the volume co-efficient of expansion of liquid ?

Ans. Coefficient of cubical expansion of liquid

= 3 × Coefficient of linear expansion of solid.

$= 3\alpha$.

Q.6. Two identical rectangular strips of Copper and Steel are rivetted to form a bimetallic strip. What will happen on heating ?

Ans. The coefficient of linear expansion of copper is more than that of steel. Therefore, on heating, the bimetallic strip will bend in such a way that copper strip remains outside *i.e.* forms the Convex surface.

Q.7. Why is invar used in making clock pendulum ?

Ans. Invar is an alloy of nickel and steel and has negligible expansion. The net change in the length of the pendulum in Summer or Winter is negligible. So the pendulum clocks are provided with invar pendulum to keep correct time.

Q.8. Why a small gap is left between two rails while laying them ?

Ans. This is done to allow for the expansion of the rails during summer when the temperature rises.

If there are no gaps, the expansion of the rails will make them bend and can result in a big accident due to derailment.

Q.9. What is a thermostat ? Enlist its uses.

Ans. Thermostat is a device which is used to control the temperature. It consists of a bimetallic strip with iron on its lower side and brass on the other side. When the strip gets heated, it bends away from the circuit contact points and thus the circuit become open or the circuit breaks. Further heating stops and temperature falls slowly and slowly. When the strip again becomes straight – the contact with the circuit point is again made.

Thermostats are used in automatic presses, refrigerators, automatic fire alarms, Geysers etc.

TYPE II : MULTIPLE CHOICE QUESTIONS

1. Co-efficient of Volume expansion is :
   (a) Equal to coefficient of linear expansion
   (b) Twice the coefficient of linear expansion
   (c) Thrice the coefficient of linear expansion
   (d) None of the above

2. A ring shaped piece of metal is heated, if the material expands, the hole will :
   (a) expand
   (b) contract
   (c) expand or contract depending upon the width of the ring
   (d) expand or contract depending upon the value of the coefficient of expansion

3. A bimetallic strip is heated, it will :
   (a) bend towards the metal with lower thermal expansion coefficient
   (b) bend towards the metal with smaller thermal expansion coefficient
   (c) Twist itself into a helix
   (d) have no bending

4. When water is heated from 0°C to 10°C, its volume :
   (a) increases
   (b) decreases
   (c) does not change
   (d) first decreases and then increases

5. By increasing the temperature of a liquid, its :
   (a) volume and density decrease.
   (b) volume and density increase.
   (c) volume increases and density decreases.
   (d) volume decreases and density increases.

6. Two spheres of same size are made of the same metal but one is hollow and the other is solid. They are heated to the same temperature, then :
   (a) Both spheres will expand equally.
   (b) Hollow sphere will expand more. than the solid one
   (c) Solid sphere will expand more than the hollow one.
   (d) None of the above

7. When Vapour condenses into liquid :
   (a) it absorbs heat.
   (b) it liberates heat.
   (c) its temperature rises.
   (d) its temperature decreases.

8. On heating a liquid of coefficient of cubical expansion γ in a container having coefficient of linear expansion $\frac{\gamma}{3}$, the level of the liquid in the container will :
   (a) rise
   (b) fall
   (c) will remain almost stationary
   (d) it is difficult to say

9. A metallic scale is $1m$ in length at 30°C and $1.00024m$ at 60°C. Its length at 50°C will be :
   (a) $1.00016\ m$ (b) $1.00040\ m$
   (c) $1.00020\ m$ (d) $1.00015\ m$

10. Which is the correct relation ?
    (a) $\gamma = 3\alpha = \frac{3}{2}\beta$
    (b) $\gamma = 3\alpha = 2\beta$
    (c) $\beta = 2\alpha = 3\gamma$
    (d) $\beta = 2\alpha = 2\gamma$

11. Density of water at 4°C is :

(a) maximum

(b) minimum

(c) Both of these

(d) None of these

12. Cement flours crack during summer :

(a) Due to expansion

(b) Due to contraction

(c) Both of these

(d) None of these

TYPE III. FILL IN THE BLANKS

1. A Copper disc with a circular hole in it is heated, the diameter of the hole will...............
2. Maximum density of water is at...............
3. A steel yard will measure...............on a hot day and ....on a cold day than its actual measurement.
4. $\gamma =$ ....α.
5. $\beta =$ ....α.
6. The density of a substance...............on heating.
7. When water is heated from 0°C to 10°C. It first...............upto 4°C and then............... upto 10°C.
8. The thermostat is used to...............the temperature.
9. Two bodies of different dimensions at the same temperature have...............amounts of heat.
10. Two bodies of different dimensions having the same amounts of heat have............... temperature.

ANSWERS

Type II :

1. (c) 2. (a) 3. (a) 4. (d) 5. (c) 6. (a)
7. (b) 8. (c) 9. (a) 10. (a) 11. (a) 12. (a)

Type III :

1. increase 2. 4°C 3. more, less 4. 3 5. 2
6. decreases 7. decreases and increases 8. controll 9. different
10. different

3. TRANSFER OF HEAT

TYPE – I. SHORT ANSWER TYPE QUESTIONS

Q.1. Pieces of copper and glass are heated to the same temperature. Why does the piece of copper feel hotter on touching ?

Ans. Copper is better conductor of heat than glass. When piece of copper is touched, it transmits heat rapidly to the hand, while glass transmits heat slowly.

2. At what common temperature would a block of wood and block of metal feel equally cold or equally hot when touched ?

ns. When the temperature of an object is equal to that of the human body, no heat is transmitted from the object to the body and vice-versa. When the temperature of both, the block of wood and the block of metal is equal to 36.8°C. They appear equally hot or cold when touched.

Q.3. Stainless steel cooking pans are preferred with extra copper bottom. Why ?

Ans. We know that the conductivity of copper is more as compared to the conductivity of steel. Therefore, when a cooking pan fitted with extra copper sheet is placed over a flame, more heat passes. This causes early cooking of food.

Q.4. Why two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double the thickness ?

Ans. This is so because the air enclosed between two layers of cloth prevents the transmission of heat from our body to outside as air is a bad conductor of heat.

Q.5. In winter why the birds sit with their wings spread out ?

Ans. When a bird sits with wings spread out, the bird encloses a lot of air which is a bad conductor of heat. The air does not allow the internal heat of the body to escape.

Q.6. On a winter night, you feel warmer when clouds cover the sky than when the sky is clear. Why ?

Ans. We know that earth absorbs heat during the day and radiates it at night. When the clouds cover the sky, the heat radiated by earth is reflected back while the heat escape into space when the sky is clear. Therefore, we feel warmer on a cloudy night than on night with clear sky.

Q.7. Air and wood are bad conductors of heat. The heat emitted by a hot body is not able to reach us. When a wooden screen is interposed, but this is not the case when air is present. Why ?

Ans. When air is present, the heat is communicated by convection and radiation but this is not the case with wooden screen.

Q.8. What is a black body ?

Ans. A black body is defined *as a body which completely absorbs all the heat radiations falling upon it without reflecting or transmitting any part of it*. When such a body is heated, it emits radiations of all wave-lengths depending upon the temperature of the black body.

Q.9. What is a steady state ?

Ans. When the temperature of various points of the bar (body) is changing, this state is said to be a variable state. After some time a state is reached when the temperature of each cross-section (part) becomes steady. This state is known as steady state. At this state any heat received by any cross-section is partly conducted to the next section and partly radiated *i.e.* no heat is absorbed by any cross-section.

Q.10. What is temperature gradient ?

Ans. Temperature gradient is defined as *the rate of change of temperature with distance*.

$$\text{Temperature gradient} = \frac{\text{Change of temperature}}{\text{Distance}}$$

TYPE II : MULTIPLE CHOICE QUESTIONS

1. Thermal Conductivity of a rod depends on :
   - (a) length
   - (b) mass
   - (c) area of X-section
   - (d) material of the rod

2. In a room containing air, heat can go from one place to another :
   - (a) by conduction only
   - (b) by convection only
   - (c) by radiation only
   - (d) by all the three modes

3. A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of increase of temperature of the solid is proportional to :
   - (a) $T_2 - T_1$ (b) $T_2^2 - T_1^2$
   - (c) $T_2^3 - T_1^3$ (d) $T_2^4 - T_1^4$

4. Two bodies A and B having equal surface areas are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by A and B are in the ratio :
   - (a) 1 : 16 (b) 1 : 2
   - (c) 1 : 4 (d) 1 : 5

5. A black body does not :
   - (a) emit radiation
   - (b) absorb radiation
   - (c) reflect radiation
   - (d) refract radiation

6. In Searle's method for finding conductivity of metals, the temperature gradient along the bar :
   - (a) is greater near the hot end
   - (b) is greater near the cold end
   - (c) is the same at all points along the bar
   - (d) increases as we go from hot end to cold end

7. Five identical balls made of different materials (Lead, Brass, Iron, Zinc and Copper) are heated upto 100°C in boiling water. The heat contained in all the balls will be :
   - (a) equal
   - (b) different
   - (c) equal in Zinc and Lead balls
   - (d) equal in Copper and Iron balls

8. Good absorbers of heat are :
   - (a) poor emitters
   - (b) non-emitters
   - (c) good emitters
   - (d) highly polished

9. A body at high temperature 1 Kelvin radiates heat at a rate proportional to :
   - (a) T^4 (b) T^{-4}
   - (c) T (d) T^2

10. What is the relationship between the energy E radiated by a black body per unit time and the absolute temperature T ?
    - (a) $E \alpha T$ (b) $E \alpha T^2$
    - (c) $E \alpha T^3$ (d) $E \alpha T^4$

11. The SI unit of Stefan's constant is :
    - (a) $Nm^{-2} K^{-1}$ (b) $J m^{-2}K^{-1}$
    - (c) $J m^{-2}S^{-1}K^{-4}$ (d) $W m^{-2}K^{-1}$

12. Two bodies A and B are placed in an evacuated vessel maintained at temperature of 27°C. The temperature of A is 327°C and that of B is 227°C. The ratio of heat loss from A and B is about :
    - (a) 2 : 1 (b) 1 : 2
    - (c) 4 : 1 (d) 1 : 4

13. According to Wien's law :

(a) $\lambda_m \times T =$ Constant

(b) $\frac{\lambda_m}{T} =$ Constant

(c) $\frac{T}{\lambda_m} =$ Constant

(d) $T + \lambda_m =$ Constant

14. The temperature of a black body becomes half of its original value, the amount of radiation emitted by the body will reduce to :

(a) $\frac{1}{16}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) remains unchanged

TYPE III. TRUE AND FALSE

1. Two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double thickness.
2. As the heat is continuously generated by an electric heater, the temperature of heater rises continuously.
3. Two rods A and B are of equal lengths. Each rod has its ends at temperature T_1 and T_2. Then rate of flow of heat through the rod A and B is equal.
4. A black body is good absorber of heat but is a bad radiator.
5. Radiation is the fastest mode of heat transfer.
6. On a winter night, we feel warmer when clouds cover the sky than when the sky is clear.
7. The spheres of the same material have radii 1m and 4m and temperatures 4000K and 2000K respectively. The energy radiated per second by the first sphere is greater than that by the second.
8. Thermal conductivity of air is less than that of felt. We prefer felt to air for thermal insulations.

ANSWERS

Type II :

1. (d) 2. (d) 3. (d) 4. (a) 5. (c) 6. (c)
7. (b) 8. (c) 9. (a) 10. (d) 11. (d) 12. (a)
13. (a) 14. (a)

Type III :

1. True 2. False– the temperature becomes constant after some time.

3. False – rate is equal when $K_1A_1 = K_2A_2$.

4. False 5. True 6. True 7. False – $E : \sigma T^4 \times 4\pi R^2 \alpha T^4 R^2$

$$\therefore \frac{E_1}{E_2} = (T_1^4 . R_1^2)/T_2^4 R_2^2$$

8. True

Chapter UD

Units and Dimensions

Lord Kilvin once said, ''When you can **measure** what you are speaking about, and express it in **numbers**, you have advanced to the stage of science, but when you cannot express it in numbers, your knowledge is of a megre and unsatisfactory kind.''

It is a long way to the sun. It is a long way to U.S.A. It is a long way to Delhi. May be, but in these cases the distances are measured in millions, thousands and hundreds of kilometers respectively. Similarly time is often described with equal lack of precision. As, ''There will be solar eclipse soon''. 'The building will be completed soon'. 'Dinner will be ready soon'. In each case, we have to guess from the context whether *soon* refers to years, weeks, or minutes. Sometimes, the context is not helpful. For example, the assurence, 'I will be ready soon' is ambiguous. In this case, the speaker is probably concerned more with appeasement than with measurement.

Qualitative description must be replaced by quantitative measurement. Measurement is a counting process. Physics deals with accurate measurement, yet the degree of accuracy aimed at depends on the purpose for which the measurements are being made. There is no such thing as absolute accuracy. No physical measurement is absolutely correct.

The laws of physics are expressed in terms of basic quantities that require a clear definition. For example, physical quantities as force, velocity, volume, and acceleration can be described in terms of more fundamental quantities which themselves are defined in terms of comparison with established standards (units). In mechanics, the three fundamental quantities are length (L), mass (M), and time (T). All other physical quantities are derived from these.

The word **dimension** has a special meaning in physics. It denotes the qualitative nature of a physical quantity. Whether a distance is measured in feet or metres or centimeters it is a distance. We say, it has dimensions in *length*. The symbols used to specify length, mass and time are L, M and T respectively. We use brackets [ ] to denote the dimensions of physical quantity. For example, the dimensions of velocity are written as $[v] = L/T = [LT^{-1}]$. In many situations, we may be required to derive or check a specific formula.

In this chapter we shall learn a useful procedure called **dimensional analysis** which can be used to assist in the derivation or to check the expressions.

UD—1. PHYSICAL QUANTITY :

All quantities in terms of which laws of physics can been expressed and which can be measured directly or indirectly are called **physical quantities.** In other words. *Any quantity which can be measured is called a* **physical quantity.** For example, force, work, time, mass, length, electric current are all physical quantities because they can be measured in terms of base units.

Measurement. *Measurement is the process of comparison of an unknown quantity with a known fixed unit quantity of the similar nature.*

Every measurement, whether it be a distance, a weight, an interval of time or any other physical quantity requires two things :

***(i)* Unit or standard :** The choosen standard of same kind taken as reference in order to measure a physical quantity is called the unit of that quantity. The meaning of the measurement of physical quantity is to find out the number of lines, its unit contained in that physical quantity.

***(ii)* A number of Numerical factor :** The number which indicates that how many times a unit is repeating in a physical quantity is called numerical factor or numerical value of physical quantity.

For example if we measure mass of an object as 10 Kg. This indicates that 1 Kg unit is repeating 10 times in the physical quantity. Similarly if we measure length of an object as 5m. This indicates that 1m unit of length is repeating 5 times in the physical quantity etc.

Hence in general we can write

Measurement of a physical quantity = Numerical value of the physical quantity (n) × unit (4)

Physical quantity= $n \times 4$.

UD—2. CHARACTERISTICS OF A STANDARD UNITS

A unit selected for measuring a physical quantity should have the following properties :

(i) It should be well defined *i.e.,* its concept should be clear

(ii) It should be of suitable size *i.e.,* neither too large nor too small

(iii) It should be easily reproducible at all places

(iv) It should not change with time and from place to place

(v) It should not change with change in its physical condition like temperature, pressure, humidity etc

(vi) It should be easily accesible

UD—3. FUNDAMENTAL AND DERIVED UNITS

Fundamental Quantity. *A fundamental quantity is a quantity which connot be expressed in terms of any other physical quantity.*

In physics, the quantities like **length, mass** and **time** are chosen as fundamental quantities. The reason for this being that these quantities connot be expressed in terms of any other fundamental quantity. They represent the most basic notions of scientific quantities.

Fundamental units. *The units selected for measuring the fundamental quantities* (*Mass, length* and *time*) *are called fundamental units.*

All the physical quantities can be expressed in terms of these fundamental units

For measuring mass, length and time there are independent unity such as kilogram (Kg), metre (m) and second (s) Hence kilogram (Kg), metre (m) and second (s) are called **fundamental units.**

Derived units. *The units of all physical quantities which can be expressed in terms of fundamental units are called* ***derived units.***

All the physical quantities can be expressed in terms of fundamental quantities and are known as **derived quantities** and the units in which these quantities are measured are called **derived units.**

For example,
$$\text{Area} = \text{Length} \times \text{Length} = (\text{Length})^2$$

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} = \frac{\text{Length}}{\text{Time}^2}$$

$$\text{Force} = \text{Mass} \times \text{Acc.} = \frac{\text{Mass} \times \text{Length}}{(\text{Time})^2}$$

Thus the units of area, velocity and force are derived from the Fundamental units of mass, length and time etc.

Unit of Mass : Mass of a body is the quantity of matter contained in the body. It is an essential property of material body and hence can never be zero. It is also not affected by the presence of other bodies near the body.

The internationally accepted unit of mass is kilogram (Kg).

The general conference of weight and measures defined kilogram as the mass of Platinum–irridium cylinder kept at the International Bureau of Weight and Measures at Severs near Paris, France.

Unit of Length : Length of an object may be defined as the distance of separation between its two ends. Internationally accepted unit of length is metre (m). General Conference of weight and Measures defined metre as the distance between two lines marked as a platinum–irridium rod kept at a constant temperature of 273·16 K at International Bureau of Weight and Measures at Sevres near Paris, France.

Unit of Time : According to Einstein, time is simply what a clock reads.

Internationally accepted unit of time is second (s). Internationally accepted definition of second is given below :

A second or better called a solar second is $\frac{1}{24 \times 60 + 60}$ th or $\frac{1}{86400}$ th part of a mean solar day.

Solar day is defined as the time that elapses between noons of two consecutive days and mean solar day is the average of the solar days in a complete one year.

UD–4. SYSTEMS OF FUNDAMENTAL UNITS

The following systems of fundamental units have been in use :

1. *C.G.S.* **System.** In this system the unit of length is centimetre, the unit of mass is gram and the unit of time is second.

The centimetre is one-hundredth part of a metre.

The Gram is one thousandth part of a kilogram.

The Second is defined as $\frac{1}{86400}$ th part of a mean solar day.

2. *F P S* **System.** In this system the unit of length is foot, the unit of mass is a pound and the unit of time is the second.

The foot is one third of the distance between two lines on a platinum–irridium alloy bar at a temperature of 62°F kept at the standard office of the Board of Trade, London.

The pound is the mass of a platinum–irridium cylinder kept at the standard office of B O T, London.

Second – same as above.

3. *M K S* **System.** The unit of length in this system is metre, the unit of mass is kilogram and the unit of time is second.

4. *S I* **(System International) Units.** French name of this system is *System International Units,* abbreviated as *S I* Units. This system is an improved and extended version of *M K S* system of units.

With the development of science and technology the three fundamental quantities like *mass, length* and *time* were not sufficient as many other physical phenomenon like light, heat, electricity etc. came up.

These physical phenomenon required more fundamental units in addition to the existing units of mass, length and time. Therefore, *M K S* system was modified with the addition of four other fundamental quantities. This gave rise to International System of Units abbreviated as **S I** for **System International.**

Thus *S I* units are based on seven fundamental units or base units. The seven base units selected are :

(i) The *metre* (*m*) as the unit of **length.**

A metre *is equal to 1,650,763·73 wavelengths in vacuum of orange light (radiations) emitted by an isotope of krypton–86.*

(ii) The **kilogram** (*kg*) as the unit of **mass.**

A kilogram *is equal to the mass of platinum–irridium alloy cylinder of diameter equal to its height.* It is kept at the International Bureau of Weights and Measures at Severs near Paris.

(iii) The *second* (*s*) as the unit of **time.**

A second *is the duration of 9,192,631,770 vibrations of the radiations emitted by ground state of the Caesium–133 atom.*

(iv) The *ampere* (*A*) as the unit of **electric current.**

An **ampere** *is that constant current which when flowing through two straight parallel conductors of infinite length and negligible area of cross section and placed one metre apart in vaccum would produae between the conductors a force equal to* 2×10^{-7} *newton per metre length of the conductors.*

(v) The Kelvin (*K*) as the unit of **temperature.**

The Kelvin is the fraction 1/273·16 of the thermodynamic temperature of the triple point of water.

(vi) The *candela* (*cd*) as the unit of **luminous intensity.**

The candela is the luminous intensity in the perpendicular direction of a surface of 1/600,000 square metre of a black body at the temperature of freezing platinum under a pressure of $101325\ N/m^2$.

(vii) The *mole* (mol) as the unit for **quantity of matter.**

A mole *is the amount of a substance which contains as many elementary units as there are carbon atoms in exactly 0·012 kg of* C^{12}.

In addition to the seven fundamental units, there are the following two **supplementary units.**

(i) The *radian* (red) as the unit of **plane angle.**

A radian *is the angle subtended at the centre of a circle (between two radii) by an arc of the circle of length equal to radius of the circle.*

$$\text{Plane angle in radian } \theta = \frac{\text{Length of arc } (l)}{\text{radius of circle } (r)}$$

(ii) The *steradian* (sr) as the unit of **solid angle.**

A steradian *is a solid angle subtended at the centre of a sphere by an area of the surface of the sphere, equal to that of a square whose each side is equal to the radius of the sphere.*

$$\text{Solid angle in steradian} = \frac{\text{area cut out from the surface of sphere}}{(\text{radius})^2}$$

UD—5. S I BASE UNITS AND SYMBOLS :

The seven base units and two supplementary units with their internationally accepted symbols are given below :

TABLE OF S I BASE UNITS

Sr.No.	*Physical quantity*	*Names of S I Unit*	*Abbreviations for S I units (symbols)*
1.	Length	metre	*m*
2.	Mass	kilogram	*kg*
3.	Time	second	*s*
4.	Electric Current	ampere	*A*
5.	Luminous Intensity	candela	*cd*
6.	Thermodynamic temperature	Kelvin	*K* (not °*K*)
7.	Amount of substance	mole	mol.

TABLE OF SUPPLEMENTARY S I UNITS

Sr.No.	*Quantity*	*Name of S I Unit*	*Symbol*
1.	Plane angle	radian	*rad*
2.	Solid angle	steradian	*sr*

Note. 1. Mole is a unit of quantity of matter which is used in chemistry.

2. The two supplementary *S I* units are kept separate from the base units because they are dimensionless.

UD—6. RULES AND CONVENTIONS FOR THE USE AND SPACING OF SYMBOLS IN S I UNITS

The following rules must be taken into consideration for the use and spacing of symbols of units in *S I* system.

1. Full names of units, which are named after a person, are not written with a capital initial letter *e.g., kelvin* not *Kelvin; newton* not *Newton; joule* not *Joule; ampere* not *Ampere* etc.

2. The Symbol for a unit, named after a person, is written by a capital initial letter *e.g. N for newton, W for watt, J for joule and A for ampere.*

3. Symbols for the units which are not named after a person, are written with small initial letters, *e.g., m for metre, kg for kilogram* and *s* for *second.*

4. Symbols for units do not take a plural from with added '*s*'. The symbols merely name the unit in which the preceeding magnitude is measured *e.g., 100 kg* and not *100 kgs, 4·2 J* and not *4·2 Js, 43 N* and not *43 Ns.*

5. Units may be written out in full or using agreed symbols only but no other abbreviations should be used *e.g.* for ampere symbol is *A* and not Amp., the symbol for newton is *N* and not Nt.

6. No fullstops or other punctuation marks be used within or at the ends of symbols for units to indicate abbreviations, *e.g. 3m* and not *3m. S I units* and not *S I units.*

7. While expressing the magnitude of a physical quantity, a space is left between the numeral and the symbol for the unit for example 80 *kg* and not 80 *kg*, 36·81 *N* and not 36·81 *N*.

8. Similarly space is also left between the symbols for compound units (*Derived Units*) *e.g. N m* for newton × metre and not *N m* ; 1 $kg\ m\ s^{-1}$ for kilogram × metre × per second square and not $kg\ m\ s^{-2}$.

This reduces the risk of confusion when an index notation is used. For example, velocity in metre per second is written as $m\ s^{-1}$ instead of *m/s* but $m\ s^{-1}$ may mean per millisecond. Similarly the unit of work in symbolic notation may be written as $kg\ m^2\ s^{-2}$ and not $kg\ m^2\ s^{-2}$.

UD—7. MULTIPLE AND SUB–MULTIPLES OF UNITS

The choice of the metre as base unit of length compared to centimetre (*c.g.s*) system, yields derived units of more practical values. All the coherent units are not of a convenient size for all applications in engineering and technology. The metre, for instance, is a convenient unit for measuring the dimensions of fields but is too large for precision measurements and too small for expressing large distance between towns. Therefore, multiples and sub-multiples of the units had to be used. The list of multiple and sub-multiple with their symbols and values is given below :

TABLE OF MULTIPLE AND SUB-MULTIPLE FACTORS

Prefix	*Symbol*	*Multiplier Factor* (*value*)	
tera	*T*	1 000 000 000 000	$= 10^{12}$
giga	*G*	1 000 000 000	$= 10^{9}$
mega	*M*	1 000 000	$= 10^{6}$
kilo	*k*	1 000	$= 10^{3}$
hecto	*h*	100	$= 10^{2}$
deca	*da*	10	$=10^{1}$
deci	*d*	0·1	$= 10^{-1}$
centi	*c*	0·01	$= 10^{-2}$
milli	*m*	0·001	$= 10^{-3}$
micro	μ	0·000 001	$= 10^{-6}$
nano	*n*	0·000 000 001	$= 10^{-9}$
pico	*p*	0·000 000 000 001	$= 10^{-12}$
femto	*f*	0·000 000 000 000 001	$= 10^{-15}$
atto	*a*	0·000 000 000 000 000 001	$= 10^{-18}$

According to the above prefixes used to identify multiples and sub-multiples of units, 1 millimetre (*mm*) = 10^{-3} metre, 1 nanosecond (*ns*) = 10^{-9} second, 1 mega watt (*MW*) = 10^{6} watts, 1 micro-coulomb (μC) = 10^{-6} coloumb etc.

While writing the prefixes with the units, no space is left between the symbol for the prefix and the symbol of unit for example (*mm*) for millimetre and not (*m m*), (*kg*) for kilogram and not (*k g).*

Advantages of S I System : *(i)* It is a rational system of units *i.e.,* it uses only one unit for one physical quantity, *e.g.,* it uses as unit Joule (J) for all types of energies (heat, light, mechanical)

(ii) It is closely related to cgs system of units

(iii) It is extended form of mks system of units

(iv) It is a metric system of units *i.e.,* its multiples and submultiples can be expressed in powers of 10

(v) It is coherent system of unit.

UD–8. CONCEPT OF DIMENSIONS.

The word **dimensions** has a special meaning in physics. It denotes the qualitative nature of a physical quantity. A distance may be measured in any units *i.e.*, miles, metres, centimetres, it is a distance. We say that it has dimensions in *length*. Similarly a physical quantity which can be measured in units of mass is said to have dimensions in mass.

Thus the concept of dimensions permits us to characterise all physical quantities as composed of a few base dimensions like mass, length, time etc. All quantities can be expressed in terms of these base dimensions.

Mass, length, time etc. are considered to be the base dimensions and are denoted by bracketed capital letters $[M]$, $[L]$, $[T]$ respectivly. $[M]$, $[L]$, $[T]$ indicate the nature of the dimensions and not the magnitudes.

To represent a physical quantity completely, all the base dimensions should be taken into consideration.

For example area is expressed as the product of length and breadth. As length and breadth both have dimensions in length, therefore, the dimensions of area will be $[L] \times [L]$ or $[L^2]$ and volume is represented by $[L] \times [L] \times [L]$ or $[L^3]$. A quantity which does not depend upon any one base dimension is said to have **zero dimensions** of the unit upon which the quantity does not depend. For example volume does not depend upon mass and time hence the dimensions of mass and time in volume will be zero. Therefore, dimensions of volume in general are written as $[M^0L^3T^0]$ and that of area are written as $[M^0L^2T^0]$.

We can express the dimensions of area and volume as below :

(i) **Area $[M^0L^2T^0]$.** the dimensions of area are *zero* in *mass, 2 in length* and *zero* in *time*.

(ii) **Volume $[M^0L^3T^0]$.** The dimensions of volume are *zero* in *mass, 3* in *length* and *zero* in *time*.

Definition of Dimensions. *The dimensions of a derived physical quantity may be defined as the defined as the powers to which its base units must be raised to represent it completely.*

Example. *(i)* Velocity $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{\text{length}}{\text{time}} = \dfrac{[L]}{[T]} = [M^0L^1T^{-1}]$

∴ Velocity has dimensions zero in mass, 1 in length and – 1 in time.

(ii) Acceleration $= \dfrac{\text{Velocity}}{\text{Time}} = \dfrac{\text{length}}{(\text{time})^2} = \dfrac{[L]}{[T^2]} = [M^0L^1T^{-2}]$

∴ Acceleration has dimensions zero in mass, 1 in length and – 2 in time.

(iii) Force = mass × acceleration $= [M] \times \dfrac{[L]}{[T^2]} = [M^1L^1T^{-2}]$

∴ Force has dimensions 1 in mass, 1 in length and – 2 in time.

In general if a derived physical quantity depends upon the *a*th power of mass unit, *b*th power of length unit and *c*th power of time unit then the dimensions of the quantity are written as

$$[M^a]\ [L^b]\ [T^c] \quad \text{or} \quad [M^a L^b T^c]$$

UD—9. DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION :

Dimensional Formula. *A dimensional formula is an expression which shows how (with what powers) and which of the fundamental units enter into the units of a physical quantity.*

To find the dimensional formula of a physical quantity we must first express that quantity in terms of the physical quantities of which the dimensions in mass, length and time are known to us.

For example, the dimensional formula for force $[M^1L^1T^{-2}]$ indicates the relation between derived unit (force) and the fundamental units of mass, length and time.

Dimensional Equation. *An equation which indicates the relation between the fundamental units of mass, length and time with their powers and the derived units is called a **dimensional equation.***

For example, dimensional formula for power is $[M^1L^2T^{-3}]$. Therefore, dimensional equation for power is written as

$$\text{Power} = [M^1L^2T^{-3}]$$

If a physical quantity 'X' has dimensions x in mass, y in length and z in time, then its dimensional equation can be written as,

$$x = [M^xL^yT^z]$$

Ex. UD—1. Derive the dimensional formulas of the following quantities and write down their dimensions.

Ans. *(i)* $\text{Work} = \text{Force} \times \text{distance} = \text{Mass} \times \text{acc.} \times \text{distance}$

$$= \text{Mass} \times \frac{\text{length}}{\text{time}^2} \times \text{Length} = [M^1L^2T^{-2}]$$

Dimensions. 1 in mass, 2 in length and – 2 in time.

(ii) $$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{mass} \times \text{Acc.}}{\text{Area}} = \frac{\text{Mass} \times \text{length}}{\text{Area} \times \text{Time}^2}$$

$$= [M^1L^1T^{-2}]$$

Dimensions. 1 in mass, – 1 in length and – 2 in time.

(iii) Gravitational Constant, Here $F = G\dfrac{M_1 \times M_2}{d^2}$

$$\therefore \text{Gravitational Constant, } G = \frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}} = \frac{[M^1L^1T^{-2}] \times [L^2]}{[M] \times [M]}$$

$$= [M^{-1}L^3T^{-2}]$$

Dimensions. – 1 in mass, 3 in length and – 2 in time.

(iv)
$$\text{Momentum} = \text{Mass} \times \text{Velocity} = \text{Mass} \times \frac{\text{distance}}{\text{time}}$$

$$=[M]\times\frac{[L]}{[T]} = [M^1L^1T^{-1}]$$

Dimensions. 1 in mass 1 in length and – 1 in time.

Ex. UD—2 Derive the dimensional formula for *(i)* Density, *(ii)* Power, *(iii)* Moment of a force, *(iv)* Co–efficient of viscosity, *(v)* Angle *(vi)* angular velocity and *(vii)* Co–efficient of thermal conductivity.

Ans. *(i)*
$$\textit{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{[M]}{[L^3]} = [M^1L^{-3}T^0]$$

(ii)
$$\textit{Power} = \frac{\text{Work}}{\text{time}} = \frac{\text{Force}\times\text{distance}}{\text{time}} = \frac{[M^1L^1T^{-2}]\times[L]}{[T]}$$

$$= [M^1L^2T^{-3}]$$

(iii) *Moment of a Force*
$$= \text{Force} \times \perp \text{distance}$$
$$= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$$

(iv) *Co–efficient of Viscosity,* (η)
$$(\eta) = \frac{\text{Force}\times\text{distance}}{\text{Area}\times\text{Velocity}}$$

$$= \frac{\text{Mass}\times\text{acceleration}\times\text{distance}}{(\text{length})^2\times\text{distance / time}}$$

$$= \frac{[M]\times[LT^{-2}]\times[L]}{[L^2]\times[LT^{-1}]}\ [M^1L^{-1}T^{-1}]$$

(v)
$$\textit{Angle} = \frac{\text{Arc (length)}}{\text{radius (length)}} = \frac{[L]}{[L]} = [M^0L^0T^0] = \text{No dimensions}$$

(vi) *Angular Velocity,* (ω)
$$(\omega) = \frac{\text{Angle}}{\text{time}} = \frac{M^0L^0T^0}{T} = [\mathrm{M^0L^0T^{-1}}]$$

(vii) *Co–efficient of thermal conductivity,* (*K*)

$$= \frac{\text{Heat}\times\text{Thickness}}{\text{Area}\times\text{time}\times\text{temperature}} = \frac{\text{Energy}\times\text{length}}{\text{Area}\times\text{time}\times\text{temperature}}$$

$$\frac{=[M^1L^2T^{-2}]\times[L]}{[L^2]\times[T]\times[K]} = [M^1L^1T^{-3}K^{-1}]\ (\text{S I units})$$

$$K\ \frac{=[M^1L^2T^{-2}]\times[L]}{[L^2]\times[T]\times[L^0T^0]} = [M^1L^1T^{-3}]\ (\text{MKS units})$$

Ex. UD—3. Derive the dimensional formula for the following quantities :

***(i)* Energy *(ii)* Relative density *(iii)* Torque *(iv)* Surface tension *(v)* Moment of Inertia *(vi)* Impulse *(vii)* Couple *(viii)* Centripetal force and *(ix)* Angular Momentum.**

Ans. *(i) Energy :*

(*a*) Kinetic Energy or (Heat) $= \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$

$$= [M] \times [LT^{-1}]^2 = [M^1L^2T^{-2}]$$

(*b*) Potential Energy = Force × displacement (vertical)

= Mass × acc. due to gravity × height

$[M] \times [LT^{-2}] \times [L] = [M^1L^2T^{-2}]$

(ii) Relative Density

$$(\textit{Specific gravity}) = \frac{\text{Density of a substance}}{\text{Density of water at } 4^\circ C}$$

$$= \frac{[M^1L^{-3}T^0]}{[M^1L^{-3}T^0]} = [M^0L^0T^0] = \text{No dimensions}$$

(iii) Torque (Moment of a force) = Force × ⊥ distance

$$[M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$$

(iv) *Surface tension* $= \frac{\text{Force}}{\text{length}} = \frac{[M^1L^1T^{-2}]}{[L]} \; [M^1L^0T^{-2}]$

(v) *Moment of Inertia* = Mass × (radius of gyration)^2

= Mass × (length)^2

$= [M] \times [L^2] = M^1L^2T^0]$

(vi) Impulse = Force × time $= [M^1L^1T^{-2}] \times [T] = [M^1L^1T^{-1}]$

(vii) Couple = Force × Distance $= [M^1L^1T^2] \times [L] = [M^1L^2T^{-2}]$

(viii) *Centripetal force* $= \frac{\text{mass} \times (\text{velocity})^2}{\text{radius}}$

$$= \frac{[M] \times [LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$$

(ix) Angular Momentum = Moment of Inertia × Angular velocity

$$= I\omega = \text{Mass} \times (\text{length})^2 \times \frac{\text{angle}}{\text{time}}$$

$$= [M] \times [L^2] \times [M^0L^0T^{-1}] = [M^1L^2T^{-1}]$$

Ex. UD—4. Derive the dimensional equations for the following :

(i) Heat (ii) Specific heat (iii) Latent heat (iv) Temperature (v) Stress (vi) Strain (vii) Elasticity (viii) Frequency (ix) Force constant (x) Surface Energy (xi) Plank's constant (h).

Ans. (i) *Heat* (Kinetic Energy) $= \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$

$$= [M] \times [LT^{-1}]^2 = [M^1L^2T^{-2}]$$

(ii) *Specific heat,* here $Q = m\,s\theta$ or $s = \frac{Q\,(\text{Heat energy})}{m\,(\text{mass}) \times \theta\,(\text{temperature})}$

$$= \frac{M^1L^2T^{-2}}{[M] \times [K]} = [M^0L^2T^{-2}K^{-1}]$$

(iii) *Latent heat,* $Q = mL$

or $L = \frac{Q\,(\text{heat energy})}{m\,(\text{mass})} = \frac{[M^1L^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$

(iv) *Temperature* = average *K.E.* of molecules.

$$= \frac{\text{Energy (Kinetic)}}{\text{mass}} = \frac{[M^1L^2T^{-2}]}{[M]}\ [M^0L^2T^{-2}]$$

(v) *Stress* $= \frac{\text{Force}}{\text{Area}} = \frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$

(vi) *Strain*

(longitudinal) $= \frac{\text{Change in length}}{\text{Original length}} = \frac{[L]}{[L]} = (M^0L^0T^0)$ = No dimensions

(vii) *Elasticity*

(*Modulus of elasticity*) $= \frac{\text{Stress}}{\text{Strain}} = \frac{[M^1L^{-1}T^{-2}]}{[M^0L^0T^0]}\ [M^1L^{-1}T^{-2}]$

(viii) *Frequency* $= \frac{\text{No. of Vibrations}}{\text{time}} = \frac{\text{No}}{[T]} = \frac{[M^0L^0T^0]}{[T]} = [M^0LT^{-1}]$

(ix) *Force constant,* $F = Kma$

or $K = \frac{F}{m \times a} = \frac{\text{Force}}{\text{mass} \times \text{Acc.}} = \frac{[M^1L^1T^{-2}]}{[M^1] \times [LT^{-2}]} = [M^0L^0T^0]$

(x) *Surface energy* $= \frac{\text{Energy}}{\text{area}} = \frac{[M^1L^2T^{-2}]}{[L^2]} = [M^1L^0T^{-2}]$

(*xi*) Plank's constant (h) $= \dfrac{\text{Energy}}{\text{Frequency}} = \dfrac{[M^1L^2T^{-2}]}{[L^{-1}]} = [M^1L^2T^{-1}]$

Ex. UD—5. Explain which of the following pairs of physical quantities have the same dimensions :

(*i*) Work and Power, (*ii*) Stress and Pressure, (*iii*) Momentum and Impulse (*iv*) Moment of a force and angular momentum. (*Pb. Nov., 1988*)

Ans. (*i*) Dimensions of work = Force × distance = $[M^1L^2T^{-2}]$

Dimensions of Power = Work/Time = $[M^1L^2T^{-3}]$

(*ii*) Dimensions of Stress $= \dfrac{\text{Force}}{\text{area}} = \dfrac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$

Dimensions of Pressure $= \dfrac{\text{Force}}{\text{area}} = [M^1L^{-1}T^{-2}]$

∴ Stress and Pressure have the same dimensions.

(*iii*) Dimensions of Momentum = mass × velocity = $[M^1L^1T^{-1}]$

Dimensions of Impulse = Force × time = $[M^1L^1T^{-1}]$

∴ Momentum and Impulse have the same dimensions.

(*iv*) Dimensions of Moment of a force = Force × ⊥ distance.

$$= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$$

Dimensions of angular Momentum = Moment of Inertia × Ang. velocity

$$= [M^1L^2T^0] \times [M^0L^0T^{-1}] = [M^1L^2T^{-1}]$$

Dimensions are not the same.

UD—10. DERIVATION OF S I UNITS OF PHYSICAL QUANTITIES IN SYMBOLIC NOTATIONS FROM THE BASIC DEFINITIONS :

Derivation of units. By using dimensional equations we can derive the *S I* units of various physical quantities from their basic definitions.

From the basic definition, write the dimensional formula of the physical quantity. To obtain the units in the given system, substitute the unit values of $[M]$, $[L]$ and $[T]$ in that system, in the dimensional formula.

For example :

1. Velocity. *Velocity is the rate of change of position and its dimensional formula is* $[M^0L^1T^{-1}]$.

∴ Unit of velocity $= [1\,kg]^0 \times [1m] \times 1s]^{-1} = m\,s^{-1}$ (*metre per sec.*)

2. Acceleration *is the rate of change of velocity and its dimensional formula is* $[M^0L^1T^{-2}]$

∴ Unit of acceleration $= [1\,kg]^0\,[\,1m]\,[1s]^{-2} = m\,s^{-2}$ (*metre per second*^{2})

3. Force. *Force is a push or pull which produces or tends to produce stops or tends to stop motion in a body.*

$$\text{Force} = \text{mass} \times \text{Acc.} = [M] \times \frac{[L]}{[T^2]} = [M^1L^1T^{-2}]$$

$$\therefore \quad \text{The unit of Force } [1\ kg]^1 \times [1m]^1 \times [1s]^{-2} = kg\ m\ s^2$$

$$= \text{N } (newtons)$$

4. Work. $= \text{Force} \times \text{distance} = [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

$$\therefore \text{The units of work in } S\ I \text{ units} = [1\ kg]^1 \times [1m]^2 \times [1s]^{-2}]$$

$$= kg\ m^2\ s^{-2} = N\ m = J\ (joules)$$

5. $$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{\text{Force} \times \text{distance}}{\text{time}} = \frac{M^1L^1T^{-2} \times L}{T} = [M^1L^2T^{-3}]$$

$$\therefore \text{The units of power in } S\ I \text{ units} = [1\ kg]^1\ [1m]^2\ [1s^{-3} = kg\ m^2\ s^{-3}$$

$$= kg\ m^2\ s^{-2}.\ s^{-1} = J\ s^{-1} = W\ (watts)$$

Ex. UD—6. Give the S I units of Pressure, Moment of Inertia, co-efficient of elasticity, co-efficient of thermal conductivity, specific heat.

Ans. *(i)* **Pressure** is defined as the force acting per unit area and its dimensional formula is $[M^1L^{-1}T^{-2}]$

$$\therefore \quad \text{Units of Pressure} = [1\ kg]^1\ [1m]^{-1}\ [1s]^{-2} = kg\ m^{-1}\ s^{-1}$$

$$kg\ m\ s^{-2} \times m^{-2} = N\ m^{-2} \text{ (newton per square meter)}$$

(ii) **Moment of Inertia** is defined as the product of mass and square of radius of gyration and its

$$\text{Dimensional Formula} = [M^1L^2T^0]$$

$$\therefore \quad \text{Units of } M.I. = [1\ kg]^1\ [1m]^2\ [1s]^0 = kg\ m^2.$$

(iii) Co-efficient of elasticity or modulus of elasticity is the ratio of stress to strain and its $D\ F = [M^1L^{-1}T^{-2}]$.

∴ Units of co-efficient of elasticity or modulus of elasticity are

$$= [1\ kg]^1\ [1m]^{-1}\ [1s]^{-2} = kg\ m^{-1}\ s^{-2}.$$

(iv) **Co-efficient of thermal conductivity** is the amount of heat that flows per second through a rod of unit area and of unit length and having temp. difference of 1°C.

$$\text{Dimensions of } (K) = [M^1L^1T^{-3}K^{-1}]$$

$$\therefore \quad \text{Units of } K = [1\ kg]^1\ [1m]^1\ [1s]^{-3}\ [1K]^{-1}$$

$$= kg\ m\ s^{-3}\ K^{-1}$$

$$= W\ m^{-1}\ K^{-1}.$$

(v) **Specific heat** is the amount of heat required to raise the temp. of 1 *gm* of substance through 1°C. Dimensions of sp. heat $= [M^0L^2T^{-2}K^{-1}]$

$$\therefore \quad \text{Units of sp. heat} = [1\ kg]^0\ [\ 1m]^2\ [1\ s]^{-2}\ [1K]^{-1}$$

$$= J\ kg^{-1}\ K^{-1}$$

Imp.

UD–11. S I UNITS, DIMENSIONAL FORMULAE, RELATIONSHIP WITH OTHER PHYSICAL QUANTITIES OF SOME NON–MECHANICAL AND MECHANICAL QUANTITIES. S I UNITS AND DIMENSIONAL FORMULAE OF :

(i) Some Mechanical Quantities

Sr. No.	Physical quantity	Relation with other Physical quantities	Dimensional formulae	Name of S I units	Symbolic notation
1.	Force	mass × acceleration	$[M^1L^1T^{-2}]$	newton (*N*)	$kg\ m\ s^{-2}$
2.	Work	Force × distance	$[M^1L^2T^{-2}]$	joule (*J*)	$kg\ m^2\ s^{-2}$
3.	Power	$\frac{\text{Work}}{\text{time}}$	$[M^1L^2T^{-3}]$	watt (*W*)	$kg\ m^2\ s^{-3}$
4.	Energy (all forms)	Stored work	$[M^1L^2T^{-2}]$	joule (*J*)	$N\ m$
5.	Pressure, Stress	$\frac{\text{force}}{\text{area}}$	$[M^1L^{-1}T^{-2}]$	$N\ m^{-2}$	$kg\ m^{-1}\ s^{-2}$
6.	Momentum	mass × velocity	$[M^1L^1T^{-1}]$	$kg\ m\ s^{-1}$	$kg\ m\ s^{-1}$
7.	Moment of force (Moment of couple) (Torque)	Force × distance	$[M^1L^2T^{-2}]$	$N\ m$	$kg\ m^2\ s^{-2}$
8.	Impulse	Force × time (change in momentum)	$[M^1L^1T^{-1}]$	$N\ s$	$kgm\ s^{-1}$
9.	Strain	$\frac{\text{Change in dimensions}}{\text{Original dimensions}}$	$[M^0L^0T^0]$	No units	—
10.	$\frac{\text{Modulus of}}{\text{elasticity}}$	$\frac{\text{Stress}}{\text{Strain}}$	$[M^1L^{-1}T^{-2}]$	newton per sq. $m\ N\ m^{-2}$	$kg\ m^{-1}\ s^{-2}$
11.	Surface energy	$\frac{\text{energy}}{\text{area}}$	$[M^1L^0T^{-2}]$	joule/sq. *m*, ($J\ m^{-2}$)	$kg\ s^{-2}$
12.	Surface tension	$\frac{\text{force}}{\text{length}}$	$[M^1L^0T^{-2}]$	newton per meter $N\ m^{-1}$	$kg\ s^{-2}$
13.	Co–eff. of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{area} \times \text{velocity}}$	$[M^1L^{-1}T^{-1}]$	newton per sq. *m*	$kg\ m^{-1}\ s^{-1}$
14.	Moment of Inertia	mass × (radius of gyration)^2	$[M^1L^2T^0]$	kilogram (metre)^2	$kg\ m^2$
15.	Angular velocity	$\frac{\text{angle}}{\text{time}}$	$[M^0L^0T^{-1}]$	rad. per second	rad s^{-1}

16.	Frequency	$\frac{1}{\text{timeperiod}}$	$[M^0L^0T^{-1}]$	per second	Hz
17.	Area	Length × Breadth	$L \times L = L^2$ $= [M^0L^2T^0]$	metre^2	m^2
18.	Volume	Length × Breadth × height	$L \times L \times L$ $= L^3\ [M^0L^3T^0]$	metre^3	m^{-3}
19.	Density	Mass/Volume	$\frac{M}{L^3}$ $= [ML^{-3}T^0]$	$\frac{\text{kilogram}}{\text{metre}^3}$	Kg m^3
20.	Speed or velocity	velocity/time	$\frac{L}{T}\ [M^0LT^{-1}]$	metre/second	$-ms^{-1}$
21.	Acceleration	velocity/time	$\frac{LT^{-1}}{T}$ $= [M^0LT^{-2}]$	metre second^2	second^2-ms^{-2}
22.	Pressure	Force/Area	$\frac{MLT^{-2}}{L^2}$ $= [ML^{-1}T^{-2}]$	Newton/ metre^2	Nm^2
23.	Moment of force (Torque)	Force × ⊥r distance	$[MLT^{-2}]$ $=[L]\ [ML^2T^{-2}]$	Newton /metre	Nm
24.	Velocity gradient	Velocity /distance	$\frac{[LT^{-1}]}{[L]}$ $= [M^0L^0T^{-1}]$	second^{-1}	s^{-1}
25.	Angle	Arc/radius	$\frac{[L]}{[L]}$ = No dimensions	radian	rad.
26.	Angular acceleration	$\frac{\text{Angular Velocity}}{\text{Time}}$	$\frac{T^{-1}}{T}$ $= [M^0L^0T^{-2}]$	$\frac{\text{radian}}{(\text{second})^2}$	rad. s^{-2}
27.	Angular momentum	Moment of inertia × Angular velocity	$[ML^2]\ [T^{-1}]$ $= [ML^2T^{-1}]$	$\frac{\text{kilogram metre}^2}{\text{second}}$	Kg $m^2\ s^{-1}$
28.	Temperature	$\frac{\text{Heat(energy)}}{\text{mass}}$	$[M^0L^2T^{-2}]$	kelvin	K
29.	Co-eff. of thermal	$\frac{\text{Heat} \times \text{thickness}}{\text{Area} \times \text{temp.} \times \text{time}}$	$[M^1L^{-1}T^1]$	watt per metre per kelvin	$W\ m^{-1}\ K^{-1}$
30.	Latent heat	$\frac{\text{Heat(energy)}}{\text{mass}}$	$[M^0L^2T^{-2}]$	joule per kelvin	jK^{-1}

31.	Specific heat	$\frac{\text{Heat(energy)}}{\text{mass} \times \text{temp.}}$	$[M^0L^2T^{-2}K^{-1}]$	joule per kilogram *per* kelvin	$J\,kg^{-1}\,K^{-1}$
32.	Intensity of illumination	$\frac{\text{Luminous intensity}}{\text{Area}}$	$[cd\,M^0L^{-2}T^0]$	candela per sq.	$cd\,m^{-2}$
33.	Quantity of electricity (Charge)	electric current × time	metre $[M^0L^0A^1T^1]$	ampere (coulomb)	second $A\,s$
34.	Electric Potential	$\frac{\text{Work}}{\text{Charge}}$	$[M^1L^2T^{-3}A^{-1}]$	volt	WA^{-1}
35.	Resistance	$\frac{\text{Pot.diff}}{\text{Current}}$	$[M^1L^2T^{-3}A^{-2}]$	ohm	VA^{-1} or NV^{-1}
36.	Capacitance	$\frac{\text{Charge}}{\text{Pot. diff}}$	$[M^{-1}L^{-2}T^4A^2]$	farad	$As\,V^{-1}$

UD—12. PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

Principle of homogeneity of dimensions states that the dimensions of all the terms on both sides on an equation must be the same. According to the principle of homogeneity, the comparison, addition and subtraction of all physical quantities is possible only if they are of the *same nature i.e.*, they have the *same dimensions.*

Example. A physical relation must be dimensionally homogeneous *i.e.*, all the terms on both sides of the relation must have the same dimensions.

In the equation $S = ut + \frac{1}{2}at^2$

The length (S) has been equated to velocity (u) and time (t), which at first seems to be meaningless. But if this equation is dimensionally homogeneous *i.e.*, the dimensions of all the terms on both sides are the same, then it has physical meaning.

Now dimensions of various quantities in the equation are :

distance, $S = [L^1]$

velocity, $u = [L^1T^{-1}]$

time $t = [T^{-1}]$

acceleration, $a = [L^{-1}T^{-2}]$

½ is a constant and has no dimensions. Thus the dimensions of the term on *L.H.S.* $S = [L^1]$

and dimensions of terms on *R.H.S.*

$$ut + \frac{1}{2}at^2 = [L^1T^{-1}]\,[T^1] + [L^1T^{-2}]\,[T^2]$$

$$= [L^1] + [L^1]$$

Here the dimensions of all the terms on both sides of the equation are the same. *Therefore, the equation is dimensionally homogeneous.*

UD—13. DIMENSIONAL ANALYSIS.

A careful examination of the dimensions of various quantities involved in a physical relation is called **dimensional analysis.** The analysis of the dimensions of a physical quantity is of great help to us in a number of ways as discussed under the uses of dimensional equations :

Uses of dimensional equations. Following are the three chief uses of dimensional equations :

1. *To convert a physical quantity from one system of units into another.*
2. *To check the correctness of physical relations.*
3. *To drive physical relations for various quantities.*

1. Conversion of a physical quantity from one system of units into another. Conversion from one system of units into another means to express the value of some physical quantity in one system of units when its value in another system of units is given.

The value of a physical quantity is the product of a *numeral* (n) and the *unit* (u) and is the same ($= nu$) on all the systems of units. this enables us to convert from one system of units to another.

Let n_1 and n_2 be the numerical values of a given physical quantity X and u_1 and u_2 be the units respectively in two different systems of units, then

$$X = n_1u_1 = n_2u_2.$$

Let the physical quantity X has dimensions x in mass, y in length and z in time, therefore, the dimensional formula will be $[M^xL^yT^z]$.

If n_1 is the numerical value of the physical quantity in a system in which unit values of fundamental units of mass, length and time are M_1, L_1 and T_1. Then the value of the physical quantity X in this system is given as

$$X = n_1\,[M_1{}^xL_1{}^yT_1{}^z]. \qquad ...(i)$$

Again if its numerical value be n_2 in *another* system of units in which unit values of fundamental units are $[M_2]$, $[L_2]$ and $[T_2]$ then

$$X = n_2\,[M_2]^x\,[L_2]^y\,[T_2]^z \qquad ...(ii)$$

From *(i)* and *(ii)*, we have,

$$n_2\,[M_2]^x\,[L_2]^y\,[T_2]^z = n_1\,[M_1]^x\,[L_1]^y\,[T_1]^z$$

$$\therefore \qquad n_2 = n_1\frac{[M_1]^x[L_1]^y[T_1]^z}{[M_2]^x[L_2]^y[T_2]^z}$$

or

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^x\left(\frac{L_1}{L_2}\right)^y\left(\frac{T_1}{T_2}\right)^z$$

where $\left(\frac{M_1}{M_2}\right), \left(\frac{L_1}{L_2}\right)$ and $\left(\frac{T_1}{T_2}\right)$ are conversion factors for mass, length and time respectively.

Thus knowing the fundamental units in the two systems and the numerical value of the quantity in one of them, its numerical value in the other system can be calculated.

The above relation should be applied after expressing the given quantity in absolute units.

Note. *During conversion take the system of units in which the numerical value of the quantity is given as the first system and the system in which the numerical value is to be determined as the second system.*

Ex. Convert a force of 100 daynes into newtons.

Sol. Conversion is of force from *C.G.S.* to *M.K.S.* system. Dimensional formula for force = $[M^1L^1T^{-2}]$

$\therefore\ x = 1,\ y = 1$ and $z = -2$

C.G.S. System	*S.I. System (M.K.S.)*
$n_1 = 100$	$n_2 = ?$
$M_1 = 1\ g$	$M_2 = 1\ kg$
$L_1 = 1\ cm$	$L_2 = 1\ m$
$T_1 = 1\ s$	$T_2 = 1\ s$

Now,
$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

or
$$n_2 = 100\left[\frac{1g}{1kg}\right]^1\left[\frac{1cm}{1m}\right]^1\left[\frac{1s}{1s}\right]^{-2}$$

$$= 100\left[\frac{0\cdot 001kg}{1kg}\right]^1\left[\frac{0\cdot 01m}{1m}\right]^1[1]^{-2}$$

$$= 100 \times 10^{-3} \times 10^{-2} \times 1 = 100 \times 10^{5} = 10^{-3}$$

$\therefore$ 100 dynes $= 10^{-3}\ N$ (Ans.)

Ex. UD—7. If the density of mercury is 13·6 gm/cm³, convert its value into kg/m³ using dimensional equations. *(Pb. Nov., 1989)*

Sol. conversion is of density from *C.G.S.* to *M.K.S.* system. Dimensional formula for density = $[M^1L^{-3}T^0]$

$\therefore\ x = 1, y = -3$ and $z = 0$

C.G.S. System	*M.K.S. System*
$n_1 = 13\cdot 6$	$n_2 = ?$

$$\begin{array}{ll|ll} M_1 & = 1\ g & M_2 & = 1\ kg \\ L_1 & = 1\ cm & L_2 & = 1\ m \\ T_1 & = 1\ s & T_2 & = 1\ s \end{array}$$

Now,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$= 13\cdot 6\left[\frac{1g}{1kg}\right]^1\left[\frac{1cm}{1m}\right]^{-3}\left[\frac{1s}{1s}\right]^0$$

$$= 13\cdot 6\left[\frac{0\cdot 001kg}{1kg}\right]^1\left[\frac{0\cdot 01m}{1m}\right]^{-3}\times[1]^0$$

$$= 13{\cdot}6 \times 0{\cdot}001 \times 10^6 \times 1 = 13{\cdot}6 \times 10^3$$

∴ Density of $13{\cdot}6\ gm/cm^3 = 13{\cdot}6 \times 10^3\ kg/m^3$.

Ex. UD—8. Convert the value of joule into ergs using dimensional equations.

Sol. Conversion is of work from *S I* to *C.G.S.* system.

Dimensional formula for work = $[M^1L^2T^{-2}]$

∴ $x = 1,\ y = 2$ and $z = -2$

S I (M.K.S.) System C.G.S. System

$$\begin{array}{ll|ll} n_1 & = 1 & n_2 & = ? \\ M_1 & = 1\ kg & M_2 & = 1\ gm \\ L_1 & = 1\ m & L_2 & = 1\ cm \\ T_1 & = 1\ s & T_2 & = 1\ s \end{array}$$

Now,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$= 1\left[\frac{1kg}{1g}\right]^1\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$= 1\left[\frac{1000g}{1g}\right]^1\left[\frac{100cm}{1cm}\right]^2\times[1]^{-2}$$

$$= 1 \times 1000 \times 10000 \times 1 = 10^7$$

∴ **Work of 1 joule = 10^7 ergs. Ans.**

Ex. UD—9. Convert a velocity of 72 km/hr into m/s with the help of dimensions.

Sol. Conversion is of velocity from *km/hr* to *m/s.* Dimensional formula of velocity = $[M^0L^1T^{-1}]$

$\therefore \quad x = 0, \quad y = 1 \quad$ and $\quad z = -1$

Ist System	*IInd System*
$n_1 = 72\ km/hr$	$n_2 = ?$
$M_1 = 1\ kg$	$M_2 = 1\ kg$
$L_1 = 1\ km$	$L_2 = 1\ m$
$T_1 = 1$ hour	$T_2 = 1\ s$

Now,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$= 72\left[\frac{1kg}{1g}\right]^0\left[\frac{1km}{1m}\right]^1\left[\frac{1\text{hour}}{1s}\right]^{-1}$$

$$= 72[1]^0\left[\frac{1000m}{1m}\right]^1\left[\frac{3600s}{1s}\right]^{-1}$$

$$= 72 \times 1 \times 1000 \times \frac{1}{3600} = 20$$

$\therefore$ Velocity of 72 km/hr = 20 m/s. **Ans.**

Ex. UD—10. Convert one newton into dynes, newton and dyne are the units of force on the M.K.S. and C.G.S. system respectively.

Sol. Conversion is of force *M.K.S.* to *C.G.S.* system. The dimensional formula for force = $[M^1L^1T^{-2}]$

$\therefore x = 1, y =$ and $z = -2$

Different values of the fundamental units are :

M.K.S. System : $\quad M_1 = 1\ kg,\ L_1 = 1\ m,\ T_1 = 1\ s$

C.G.S. System : $\quad M_2 = 1\ g,\ L_2 = 1\ cm,\ T_2 = 1\ s$

Also $\quad 1\ kg = 1000\ g$ and $1\ m = 100\ cm$,

Substituting the values in the equation

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^x\left(\frac{L_1}{L_2}\right)^y\left(\frac{T_1}{T_2}\right)^z$$

$$n_2 = n_1\left(\frac{1kg}{1g}\right)^1\left(\frac{1m}{1cm}\right)^1\left(\frac{1s}{1s}\right)^{-2}$$

$$= 1\left(\frac{1000g}{1g}\right)^1\left(\frac{100cm}{1cm}\right)^2 \times (1s)^{-2} \qquad [\because n_1 = 1\ N]$$

$$= 1 \times 1000 \times 100 \times 1 = 100{,}000 = 10^5$$

$\therefore$ Force of $1\ N = 10^5$ *dynes* **Ans.**

Ex. UD—11. Find the number of watts in one horse power. Watt is the unit of power on the M.K.S. system and horse power that of on the F.P.S. system.

Sol. 1 $H.P. = 550$ ft. lb/sec. $= 550 \times 32{\cdot}2$ ft. lbs/sec.

$= 17710$ ft. lbs/sec. (Absolute units)

So, here $n_1 = 17710$ and $n_2 = ?$

The dimensional formula for power is $[M^1L^2T^{-3}]$

$\therefore \quad x = 1, \quad y = 2 \quad$ and $\quad z = -3$

Different values of fundamental units are :

F.P.S. system : $M_1 = 1$ lb, $L_1 = 1$ ft. $T_1 = 1$ sec.

M.K.S. system : $M_2 = 1\ kg$, $L_2 = 1\ m$, $T_2 = 1\ s$

Also 1 lb $= 0{\cdot}4536\ kg$ and 1 $ft = 0{\cdot}3048\ m$

Substituting the values in the equation,

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)^x\left(\frac{L_1}{L_2}\right)^y\left(\frac{T_1}{T_2}\right)^z$$

$$\therefore \quad n_2 = 17710\left(\frac{1lb}{1kg}\right)\left(\frac{1ft}{1m}\right)^2\left(\frac{1s}{1s}\right)^{-3}$$

$$= 17710\left(\frac{0\cdot4536kg}{1kg}\right)^1\left(\frac{0\cdot3048m}{1m}\right)^2\left(\frac{1s}{1s}\right)^{-3}$$

$$= 17710 \times 0{\cdot}4536 \times (0{\cdot}3048)^2 = 746{\cdot}4$$

$\therefore$ **1 H.P. = 746·4 watts.**

Also **I.H.P.** $= 0{\cdot}7464\ kW$ (kilowatts)

2. To check the correctness of Physical relations.

According to Principle of Homogeneity of dimensions a physical relation or equation is correct, if the dimensions of all the terms on both sides of the equation are the same. If the dimensions of even one term differs from those of others, the equation is not correct.

Ex. UD—12. Check the accuracy of the relation, S = ut + $\frac{1}{2}$ at² where S is the distance covered by a body in time t, moving with initial velocity u and acc. 'a'.

Sol. Dimensions of the term on the *L.H.S.*

S (distance) $= [L^1]$

Taking the terms on the *R.H.S.* we have,

(*i*) dimensions of u (*initial velocity*) $= [L^1T^{-1}]$

(*ii*) dimensions of t (*time*) $= [T^1]$

(iii) dimensions of a (*acceleration*) = $[L^1T^{-2}]$

$\frac{1}{2}$ being a constant has no dimensions

∴ Dimensions of the terms on *R.H.S.*

$$ut + \frac{1}{2}at^2 = [L^1T^{-1}]\,[T^{-1}] + [L^1T^{-2}]\,[T^{-1}]^2$$

$$= [L^1] + [L^1]$$

Thus the dimensions of each term on the *R.H.S.* are equal to the dimensions of the term on *L.H.S.* Hence the given relation is correct.

Ex. UD—13. Check the correctness of the following formulae by dimensional analysis.

(i) $\mathbf{F = \frac{mv^2}{r}}$ ***(ii)*** $t = 2\pi\sqrt{\frac{l}{g}}$

where all the letters have their usual meanings

Sol. *(i)* $F = \frac{mv^2}{r}$

Dimensions of the term on *L.H.S.*

Force, $F = [M^1L^1T^{-2}]$

Dimensions of the term on *R.H.S.*

$$\frac{mv^2}{r} = \frac{[M^1][L^1T^{-1}]^2}{[L]} = \frac{[M^1L^2T^{-2}]}{[L]} = [M^1L^1T^{-2}]$$

The dimensions of the term on the *L.H.S.* are equal to the dimensions of the term on the *R.H.S.* Therefore, the relation is correct.

(ii) $t = 2\pi\sqrt{\frac{l}{g}}$

Here, Dimensions of *L.H.S.* $t = [T^1] = [M^0L^0T^1]$

Dimensions of the terms on *R.H.S.*

(i) Dimensions of (*length*) = $[L^1]$

(ii) Dimensions of g (*acc. due to gravity*) = $[L^1T^{-2}]$

2π being constant, has no dimensions.

Hence the dimensions of terms $2\pi\,\frac{l}{g}$ on the *R.H.S.*

$$= \left(\frac{L^1}{L^1T^{-2}}\right)^{\frac{1}{2}} = [T^1] = [M^0L^0T^1]$$

Thus the dimension of the terms on both sides of the relation are the same *i.e.*, $[M^0L^0T^1]$. Therefore, the relation is correct.

Ex. UD—14. Check the correctness of the following equation on the basis of dimensional analysis, $V = \sqrt{\frac{E}{d}}$ V is the velocity of sound, E is the elasticity and d is the density of the medium.

Sol. Here,

Dimensions of the term on *L.H.S.* $V = [M^0L^1T^{-1}]$

Dimensions of elasticity, $E = [M^1L^1T^{-2}]$

and Dimensions of density, $d = [M^1L^{-3}T^0]$

∴ Dimensions of the term on *R.H.S.*

$$\sqrt{\frac{E}{d}} = \left[\frac{M^1L^{-1}T^{-2}}{M^1L^{-3}T^0}\right]^{\frac{1}{2}} = [M^0L^1T^{-1}]$$

Thus dimensions on both sides are the same, therefore, the equation is correct.

Ex. UD—15. Check the accuracy of the following relations :

(i) $\mathbf{E = mgh + \frac{1}{2} mv^2}$

(ii) $\mathbf{v^3 - u^2 = 2\ a\ s^2}$. *(Pb. Nov., 1989)*

Sol. *(i)* $E = mgh + \frac{1}{2} mv^2$

Here, dimensions of the term on *L.H.S.*

Energy, $E = [M^1L^2T^{-2}]$

Dimensions of the terms on *R.H.S.*

Dimensions of the term, $mgh = [M] \times [LT^{-2}] \times [L] = [M^1L^2T^{-2}]$

Dimensions of the term, $\frac{1}{2} mv^2 = [M]\ [LT^{-1}]^2 = [M^1L^2T^{-2}]$

Thus dimensions of all the terms on both sides of the relation are the same, therefore, the relation is correct.

(ii) The given relation is, $v^3 - u^2 = 2\ a\ s^2$

Dimensions of the terms on *L.H.S.*

$$v^3 = [M^0L^1T^{-1}]^3 = [M^0L^3T^{-3}]$$

$$u^2 = [M^0L^1T^{-1}]^2 = [M^0L^2T^{-3}]$$

Dimensions of the terms on *R.H.S.*

$$2\ a\ s^2 = [M^0L^1T^{-2}] \times [M^0L^1T^0]^2$$

$$= [M^0L^3T^{-2}]$$

Substituting the dimensions in the relations $v^3 - u^2 = 2as^2$, we get,

$$[M^0L^3T^{-3}] - [M^0L^2T^{-2}] = [M^0L^3T^{-2}]$$

The dimensions of all the terms on both sides of the relation are not the same. Therefore, the relation is wrong.

Ex. UD—16. Using Principle of Homogeneity of dimensions, check the correctness of equation, h = $\frac{2Td}{rg\cos\theta}$

Sol. the given formula is, $h = \frac{2Td}{rg\cos\theta}$

Dimensions of term on *L.H.S.*

$$\text{height } (h) = [M^0L^1T^0]$$

Dimensions of terms on *R.H.S.*

$$T = \text{surface tension} = [M^1L^0T^{-2}]$$

$$d = \text{density} = [M^1L^{-3}T^0]$$

$$r = \text{radius} = [M^0L^1T^0]$$

$$g = \text{acc. due to gravity} = [M^0L^1T^{-2}]$$

$$\cos\theta = [M^0L^0T^0] = \text{no dimensions}$$

$$\therefore \text{Dimensions of } \frac{2Td}{rg\cos\theta} = \frac{[M^1L^0T^{-2}]\times[M^1L^{-3}T^0]}{[M^0L^1T^0]\times[M^0L^1T^{-2}]} = [M^2L^{-5}T^0]$$

Dimensions of terms on *L.H.S.* are not equal to dimensions on *R.H.S.* Hence' formula is not correct.

3. Derivation of equations for physical quantities.

The principle of homogeneity of dimensions, helps us to derive expressions for different **physical** quantities. Knowing the factors on which a physical quantity may be expected to depend, an expression for it can be obtained in terms of these factors, such that the dimensions of the terms on both the sides of the expression are the same. Take into account all possible factors on which are result may be expected to depend. The factors on which the result does not depend get eliminated later.

Ex. UD—17. Using dimensional analysis, derive an expression for the time period of a simple pendulum.

Sol. The time period t may possibly depend upon

(i) Mass m of the bob of the pendulum.

(ii) Length l of the pendulum.

(ii) Acceleration due to gravity g at the place where pendulum is suspended.

(iv) Angle of swing θ.

Again, let the time period depends upon *a*th power of mass, *b*th power of length, *c*th power of g and *d*th power of angle θ

$\therefore$ (i) $t \propto m^a$, (ii) $t \propto l^b$ (iii) $t \propto g^c$ (iv) $t \propto \theta^d$. Combining all the factors, we get

$$t \propto m^a\, l^b\, g^c\, \theta^d$$

or
$$t = k\, m^a\, l^b\, g^c\, \theta^d \qquad \ldots(A)$$

where k is a dimensionless constant.

Writing the dimensions of t, m, l, g and θ in (A), we get

$$[M^0L^0T^1] = [M^1L^0T^0]^a\, [M^0L^1T^0]^b\, [M^0L^1T^{-2}]^c\, [M^0L^0T^0]^d$$

θ being a ratio has no dimensions *i.e.*, $d = 0$ and k is the dimensionless constant.

$$\therefore \qquad [M^0L^0T^1] = [M^aL^{b+c}T^{-2c}]$$

According to the principle of homogeneity of dimensions, the powers of M, L and T on both sides must be equal. Therefore, comparing the powers of M, L and T on both sides.

$$a = 0,\ b + c = 0 \text{ and } 2c = 1$$

or
$$c = -\frac{1}{2} \quad \text{and} \quad b - \frac{1}{2} = 0, \quad \text{or} \quad b = \frac{1}{2}$$

Hence,
$$t = k\, m^0 l^{\frac{1}{2}}\, g^{-\frac{1}{2}} \theta^0$$

or
$$t = k\sqrt{\frac{l}{g}}$$

The value of k can be determined experimentally and if it comes out to be 2π.

The required relation is, $t = 2\pi\sqrt{\dfrac{l}{g}}$

Ex. UD—18. If velocity of sound in a medium depends on its elasticity and density. Derive the relation for the velocity of sound in a medium by the method of dimensions.

Sol. The velocity of sound V depends upon :

(i) The elasticity of the medium (E), and

(ii) The density of the medium (D).

$\therefore$ Let $\quad V \propto E^a$

and $\quad V \propto D^b$.

Combining the two : $\quad V \propto E^a D^b$

or $\quad V = k\, E^a D^b \qquad \ldots(i)$

where k is a constant.

Now dimensions of

Velocity, $\quad V = [M^0L^1T^{-1}]$

Elasticity, $\quad E = [M^1L^{-1}T^{-2}]$

Density, $D = [M^1L^{-3}T^0]$

Writing the dimensions of V, E and D in *(i)*, we get,

$$[M^0L^1T^{-1}] = [M^1L^{-1}T^{-2}]^a\,[M^1L^{-3}T^0]^b$$

or $$[M^0L^1T^{-1}] = [M^{a+b}L^{-a-3b}T^{-2a}]$$

Applying the principle of homogeneity of dimensions, the powers of M, L and T on both sides must be equal.

Therefore, comparing the powers of M, L and T on both sides,

$$a + b = 0, \quad -a - 3b = 1 \quad \text{and} \quad -2a = -1$$

$$\therefore \qquad a = \frac{1}{2} \quad \text{and} \quad b = -a = -\frac{1}{2}$$

Hence $$V = kE^{\frac{1}{2}}.D^{-\frac{1}{2}}$$

or $$V = k\sqrt{\frac{E}{D}}$$

The value of k can be determined experimentally and if it comes to be 1.

Then, the required relation is, $V = \sqrt{\frac{E}{D}}$

Ex. UD—19. Derive an expression for the potential energy possessed by a body when raised through a certain height, by making use of dimensional equations.

Sol. The potential energy E possessed by a body depends upon :

(i) Mass m of the body

(ii) Acceleration due to gravity, g and

(iii) Height h through which the body is raised.

Again, let and potential energy depends upon xth power of mass yth power of g and zth power of h.

$\therefore$ *(i)* $E \propto m^x$, *(ii)* $E \propto g^y$, *(iii)* $E \propto h^z$.

Combining all the factors, we get

$$E \propto m^x g^y h^z$$

or $$E = k\,m^x g^y h^z \qquad \ldots(A)$$

where k is dimensionless constant.

Writing the dimensions of E, m, g and h in (A), we get,

$$(M^1L^2T^{-2}) = (M^1L^0T^0)^x\,(M^0L^1T^{-2})^y\,(M^0L^1T^0)^z$$

or $$[M^1L^2T^{-2}]\ [M^xL^{y+z}T^{-2y}]$$

According to the principle of homogeneity of dimensions the powers of M, L and T on both sides must be equal.

Therefore. comparing the powers of M, L and T on both sides.

$$x = 1,\ y + z = 2 \quad \text{and} \quad -2y = -2$$

$$\therefore \quad x = 1,\ y = 1 \quad \text{and} \quad z = 1$$

Hence, $$E = k\,m^1\,g^1\,h^1 = k\,m\,g\,h$$

The value of k is determined experimental and if it comes out to be 1.

The required equation, is E = mgh.

Ex. UD—20. A body of mass (m) is moving with uniform speed (*v*) in a circle of radius (r). Find an expression for centripetal force F by the method of dimensions.

Sol. The centripetal force F acting on a body moving in a circle depends upon :

(i) Mass m of the body

(ii) Uniform speed v and

(iii) Radius r of the circle.

Again let the centripetal force depends upon xth power of mass, yth power of velocity and zth power of radius.

$\therefore$ *(i)* $F \propto m^x$ *(ii)* $F \propto v^y$ *(iii)* $F \propto r^z$.

Combining all the factors, we get

$$F \propto m^x\,v^y\,r^z$$

or $$F = k\,m^x\,v^y\,r^z.$$

k being dimensionless constant.

Writing the dimensions of F, m, v and r in *(i)*, we get

$$[M^1L^1T^{-2}] = [M^1L^0T^0]^x\,[M^0L^1T^{-1}]^y\,[M^0L^1T^0]^z$$

or $$[M^1L^1T^{-2}] = [M^xL^{y+z}T^{-y}].$$

According to the principle of homogeneity of dimensions, the power of M, L and T must be equal on both sides.

Therefore, comparing the powers of M, L and T

$$x = 1,\ y + z = 1 \quad \text{and} \quad -y = -2,$$

$$\therefore \quad x = 1\ y = 2 \text{ and } z = 1 - y = 1 - 2 = -1$$

Hence $$F = k\,m^1\,v^2\,r^{-1}. = k\frac{mv^2}{r}$$

The value of k is determined experimentally and if it comes out to be 1.

$\therefore$ The required expression is,

$$F = \frac{mv^2}{r}$$

Ex. UD—21. A body of mass (m) is moving in a horizontal circle of radius (r) with angular velocity (ω). Find the centripetal force acting on it using method of dimensions.

Sol. Let the centripetal force depends upon :

*a*th power of mass, *b*th power of radius and *c*th power of angular velocity.

$\therefore$
$$F \propto m^a$$
$$F \propto r^b$$
$$F \propto \omega^c.$$

Combining all the factors, we get
$$F \propto m^a\, r^b \propto^c$$

or
$$F = k\, m^a\, r^b \propto^c. \qquad \ldots(i)$$

where k is dimensionless constant.

Writing the dimensions of F, m, r and ω in *(i)*, we get
$$[M^1L^1T^{-2}] = [M^1]^a\, [L]^b\, [T^{-1}]^c$$
$$[M^1L^1T^{-2}] = [M^aL^bT^{-c}].$$

Applying the principle of Homogeneity of dimensions or comparing the powers of M, L and T on both sides.
$$1 = a,\ 1 = b \quad \text{and} \quad -2 = -c$$

Hence,
$$a = 1,\ b = 1 \quad \text{and} \quad c = 2$$

substituting the values of a, b and c in *(i)*, we get
$$F = k\, m^1\, r^1\, \omega^2$$

or
$$F = k\, m\, r\, \omega^2.$$

The value of k can be determined experimentally and if it comes out to be 1.

The required relation is, $F = m\, r\, \omega^2$.

Ex. UD—22. If the frequency of stretched string depends upon the length of the string, the tension in the string and mass per unit length of the string. Derive a relation for the frequency by the method of dimensions.

Sol. Let the frequency (n) is directly proportional to the xth power of length, l, yth power of tension T and zth power of mass per unit length ρ.

$\therefore$ *(i)* $n \propto l^x$, *(ii)* $n \propto T^y$ and *(iii)* $n \propto \rho^z$.

Combining all the factors
$$n \propto l^x\, T^y \rho^z$$

or
$$n = k l^x\, T^y \rho^z \qquad \ldots(A)$$

where k is dimensionless constant.

Writing the dimensions of n, l T and ρ in (A), we get
$$[M^0L^0T^{-1}] = [L^1]^x\, [M^1L^1T^{-2}]^y \left[\frac{M}{L}\right]^z$$

or
$$[M^0L^0T^{-1}] = [M^{y+z}L^{x+y-z}T^{-2y}]$$

equating the powers of M, L and T, we get,

$$y + z = 0 \; (i), \; x + y - z = 0 \; (ii) \text{ and } -2y = -1 \; (iii)$$

From the equations (i), (ii) and (iii), we get,

$$y = \frac{1}{2}, \; z = -\frac{1}{2} \text{ and } x = -1.$$

Substituting the values of x, y and z in (A)

$$n = k\, l^{-1}\, T^{\frac{1}{2}}\, \rho^{-\frac{1}{2}}$$

or $$n = k\frac{1}{l}\sqrt{\frac{T}{\rho}}.$$

The value of k can be determined experimentally.

Ex. UD—23. The wavelength λ associated with a moving electron depends upon its mass m, its velocity v and Plank's constant h. Prove dimensionally that $\lambda \propto \frac{h}{mv}$.

Sol. Let *(i)* $\lambda \propto m^a$ *(ii)* $\lambda \propto v^b$ and *(iii)* $\lambda \propto h^c$

Combining all the factors

$$\lambda \propto m^a\, v^b\, h^c$$

or $$\lambda = k\, m^a\, v^b\, h^c. \qquad \ldots(1)$$

where k is dimensionless constant.

Writing the dimensions on either side of (1), we get

$$[M^0L^1T^0] = [M^1]^a\, [LT^{-1}]^b\, [M^1L^2T^{-1}]^c$$

or $$[M^0L^1T^0] = [M^{a+c}L^{b+2c}T^{-b-c}]$$

Equating the powers of M, L and T both sides

$$a + c = 0 \quad \ldots(i), \qquad b + 2c = 1 \quad \ldots(ii),$$

$$-b-c = 0 \ldots(iii)$$

Solving *(i)*, *(ii)* and *(iii)*, $a = -1$, $b = -1$, $c = 1$.

$$b = -1, c = 1.$$

$\therefore$ From equation (1) we get,

$$\lambda = k\, m^{-1}\, v^{-1}\, h^1 \quad \text{or} \quad \lambda = k\frac{h}{mv}.$$

Hence $$\lambda \propto \frac{h}{mv}$$

Ex. UD—24. Check the correctness of the relation $v = \frac{\sqrt{2GM}}{R}$ where v is velocity, G is universal constant, R is radius and M is mass.

Sol. Dimensional formula for L.H.S. is $v = [M^0LT^{-1}]$

In the R.H.S. dimensional formula for $G = [M^{-1}L^3T^{-2}]$

dimensional formula for $M = [M]$

dimensional formula for $R = [L]$

2 is a dimensionless constant. Substituting the dimensions of G, M, R in the

$$\frac{\sqrt{2GM}}{R}$$

$$\text{We get dimensions of R.H.S.} = \left[\frac{[M^{-1}L^3T^{-2}][M]}{[L]}\right]^{\frac{1}{2}} = M^0LT^{-1}]$$

Hence dimensions of L.H.S. = dimensions of R.H.S.

So the given equation is correct.

Ex. UD—25. The velocity of a particle is given in terms of time t by the equation.

$$v = \mathrm{A}t + \frac{\mathrm{b}}{\mathrm{t+c}}$$

What are the dimensions of a, b and c ?

Sol. Dimensional formula for L.H.S. $v = \{LT^{-1}]$

In the R.H.S. dimensional formula of at $[LT^{-2}]\,[T] = [LT^{-1}]$

$t + c =$ time, c has dimensions of time and hence is added in t.

Dimensions of $t + c\,[T]$

Now $$\frac{b}{t+c} = v$$

$$\Rightarrow \qquad b = v\,(t + c) = [LT^{-1}]\,[T] = [L]$$

There dimensions of a = $[LT^{-2}]$, Dimensions of $b = [L]$ and that of $c = [T]$

Ex. UD—26. The velocity of a body which has fallen freely under gravity varies as g^xh^y, where g is acceleration due to gravity at a place and h is height. Find the value of x and y using dimensional analysis.

Sol. Let $v \alpha\ g^xh^y$ or $v = k.\ g^xh^y$ ...(1)

Where k is dimensionless constant.

Dimensional formula for $v = [M^0LT^{-1}]$

Dimensional formula for $g = [M^0LT^{-2}]$

Dimensional formula of $h = [M^0LT^0]$

Substituting these dimensions in equation n (1) we get,

$$[M^0LT^{-1}] = [M^0LT^{-2}]^x\,[M^0LT^0]^y$$

$$[M^0LT^{-1}] = [M^0L^nT2^x]\,[M^0L^yT^0]$$

$$[M^0LT^{-1}] = [M^0L^{x+y}T^{-2x}]$$

Comparing the powers of M, L and T we get.

$$1 = x + y \quad \text{(Powers of } L) \qquad \ldots(2)$$

$$-1 = -2x \quad \text{(Powers of } T)$$

$$\Rightarrow \qquad x = \frac{1}{2} \qquad \ldots(3)$$

Put on (3) in on (2) we get

$$y = \frac{1}{2} \qquad \ldots(4)$$

Hence $x = \frac{1}{2}, y = \frac{1}{2}$ **Ans.**

Ex. UD—27. In the gas equation $\left(P+\frac{a}{v^2}\right)(v - b) = RT$, where T is the absolute temperature, P is pressure and v is volume of gas. What are dimensions of a and b ?

Sol. Like quantities are added or substracted from each other *i.e.*,

$\left(P+\frac{a}{v^2}\right)$ has dimensions of pressure $= [ML^{-1}\,T^{-2}]$

hence $\frac{a}{v^2}$ will be dimensions of pressure $= [ML^{-1}T^{-2}]$

$$a = [ML^{-1}T^{-2}]\,[\text{Volume}]^2 = [ML^{-1}T^{-2}]\,[L^3]^2$$

$$a = [ML^{-1}T^{-2}]\,[L^6]\,[ML^5T^{-2}]$$

Dimensions of $a = [ML^5T^{-2}]$

$(v - b)$ have dimensions of volume *i.e.*, b will have dimensions of volume *i.e.*, $[L^3]$

$$b = [L^3] = [M^0L^3T^0]$$

UD—14. LIMITATIONS OF DIMENSIONAL EQUATION

The method of dimensions has the following limitations :

1. It does not help us to find the value of dimensionless constants involved in various physical relations. The value, of such constant have to be determined by some experiments or mathematical investigations.

2. This method fails to derive formula of a physical quantity which depends upon more than three factors. Because only three equations are obtained by comparing the powers of M, L and T.

However, in *S I* units formulas for physical quantities depending upon more than three quantities can be derived because there are seven base units.

3. This method is applicable only in case of power functions. It fails to derive relations of quantities involving exponential and trigonometric functions.

4. The method cannot be directly applied to derive relations which contain more than one terms on one side or both sides of the equation, such as $v = u + at$ or $S = ut + \frac{1}{2} at^2$ etc. However, such relations can be derived indirectly.

5. A dimensionally correct relation may not be true physical relation because the dimensional equality is not sufficient for the correctness of a given physical relation.

UD—15. ERRORS

The difference between the observed value, during measurement, and the actual value of a physical quantity is called an error.

Or

The uncertainity in a measurement is called an error.

Types of errors. Errors are of a number of types, some of which are given below :

1. **Systematic Errors.** These errors are due to known causes. The important causes are :

(*a*) **Temperature of measuring scales.** A metal scale is correct at a temperature at which it is graduated. If measurements are made with such a scale at a temperature lower or higher than the temperature at which it was graduated, there occurs an error.

(*b*) **Buoyancy of air.** While determining the weight of a body of large volume an error is caused due to buoyancy of air due to which there is decrease in weight.

(*c*) **Radiations.** Due to radiation or absorption of heat there is loss or gain of heat in experiments on calorimetery.

Methods of elimination :

(*a*) Error due to the temperature of measuring scale can be calculated and should be applied to the observed values.

(*b*) The error due to buoyancy can be calculated from the volume of the substance and the density of air.

(*c*) Radiation error can be minimised by covering the radiating surface with some insulating material or by adjusting the temperature.

2. **Instrumental Errors.** These errors are due to faulty or inaccurate instruments. The important faults in instruments are given below ;

(*a*) **Zero Error.** A measuring instrument is said to have zero error if the pointer does not lie on the zero of the scale or in case of vernier zero of vernier scale may not coincide with the zero of main scale.

(*c*) **Back-lash Error.** This error is due to loose fittings or wear and tear in screw gauges. Due to this defect if the screw is adjusted by rotating it in one direction and then turned in the opposite direction, the head may be rotated through some number of divisions but the screw does not move along its axis.

(*c*) In measuring time, error is caused if the watch is running fast or too slow.

Methods or elimination :

(*a*) Zero error is determined at the start of experiment and zero correction is applied in the observed values.

(*b*) Back–lash error is avoided by rotating the screw always in the same direction.

(*c*) The error in a watch may be determined by comparison with a standard chronometer and then applied during measurement.

3. Random Error. These errors are due to unknown causes.

(*a*) **Human Errors.** Sometimes the errors in measurements are due to human qualities of the person performing the experiments. These errors arise due to the lack of *attentiveness, poor sight and peculiar habits of the observer.* As an example, the observer may be in the habit of holding his head always a bit to the left or right while reading a needle on the scale.

(*b*) Some times repeated measurements of a quantity by the same person and the same instrument give values which are slightly different from each other. These errors have no set pattern and depend upon chance. So we can not depend upon single observation.

Methods of elimination :

(*a*) In order to eliminate personal errors, the measurements should be repeated by different observers.

(*b*) The random errors can be minimised by repeating the measurements many times and taking the arithmatic mean of all the measurements.

4. Proportional Errors. These errors occur in case of quantities which cannot be measured directly, but have to be calculated from those, that can be so measured. In such cases, the slightest inaccuracy in the measurement of each factor involved will contribute an error in the final result. As an example, in the measurement of volume of a sphere, a small error in the measurement of radius will introduce an appreciable error in the volume.

Such errors can be minimised by observing more than the usual care in making measurements of quantities whose higher powers are to be used in the calculations of the final result.

UD—16. DEGREE OF ACCURACY

The extent of accuracy in measurement is called degree of accuracy.

As an example, a metre rod can measure accurately upto 0·1 cm, whereas a vernier calliper can measure correct upto 0·01 cm. Clearly the degree of accuracy of a vernier calliper is more as compared to a metre rod.

In another example, if the actual value of acceleration due to gravity is 9·81 $m\ s^{-2}$ and a student A determines its value to be 9·79 $m\ s^{-2}$ and a student B determines its value to be 9·84 $m\ s^{-2}$. In this case degree of accuracy of measurement of student A is more than that of student B.

Degree of accuracy depends upon the following main factors :

1. Least count of the measuring instrument.
2. Accurate estimation of the zero error of the measuring instrument.
3. Personal factors.

UD—17. ABSOLUTE ERROR

Absolute error of measurement is the magnitude of the difference between the actual value of the quantity and its experimentally observed value.

∴ Absolute error = *Actual value – Observed value* (in the actual value)

Suppose the specific heat of copper as determined by a student in the lab. is 0·087 where as the actual value is 0·095, then

Absolute error = 0·095 – 0·087 = 0·008 in 0·095.

UD—18. PERCENTAGE OR RELATIVE ERROR

Percentage error or relative error is defined as the ratio of absolute error to the actual value of the quantity being measured, expressed per 100. Percentage error is expressed as :

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{Actual value}} \times 100$$

It is the *percentage error* and not the absolute error which is a true index of the *degree of accuracy* of measurement.

In the measurement of a large quantity percentage error is less, whereas in the measurement of small quantities %age error is more. Therefore, in the measurement of a small quantity special technique should be devised during measurent to keep the error low.

Suppose the atmospheric pressure as determined by Boyle's law apparatus in the laboratory comes out to be 74·5 cms of mercury, whereas the actual value as noted from Fortin's Barometer is 75·8 cms of mercury.

∴ Absolute error = 75·8 – 74·5 = 1·3 cms.

$$\text{And percentage error} = \frac{1\cdot 3}{75\cdot 8} \times 100 = 1\cdot 71\%$$

Again let the specific heat of copper determined by a student in the laboratory is 0·087, whereas the actual value is 0·095.

∴ Absolute error = 0·095 – 0·087 = 0·008

$$\text{And percentage error} = \frac{0\cdot 008}{0\cdot 095} \times 100 = 8\cdot 42\%$$

It is clear from the above examples that though the absolute error in the second case is much less than in the first result, yet the percentage error in the second case is more. Therefore, the first result is more accurately determined because the percentage error is less. Hence the accuracy of results is judged by the percentage error alone.

UD—19. PERCENTAGE ERROR

Percentage error is defined as the ratio of absolute error to the actual value of the quantity expressed per 100.

Calculation of percentage error in the value of 'g'. The value of '*g*' is determined in the lab. using simple pendulum and is given by the relation,

$g = 4\pi^2 \frac{l}{t^2}$ where l is the length of the pendulum and t is the time period. Therefore, in the determination of value of g in the lab, we are required to measure two quantities l and t. The percentage error in the value will depend upon the errors in the measurements of the values of l and t.

Let us consider that actual values of l and t are A and B respectively, and $A \pm \Delta A$ and $B \pm \Delta B$ are their respective measured values. Here, ΔA and ΔB are the absolute errors in A and B respectively. Again, let the actual value of g be Z. Then we are to calculate the error ΔZ or Z *i.e.*, the value of g.

$$\therefore \qquad Z = \frac{A}{B^2} \qquad \text{(value of constant omitted)}$$

$$\therefore \qquad Z \pm \Delta Z = \frac{A \pm \Delta A}{(B \pm \Delta B)^2} = (A \pm \Delta A)\,(B \pm \Delta B)^{-2}$$

$$= (A \pm \Delta A)\frac{1}{B^2}\left(1 \pm \frac{\Delta B}{B}\right)^{-2}$$

$$= (A \pm \Delta A)\frac{1}{B^2}\left[1 \pm \frac{2\Delta B}{B} + \ldots\right]$$

$$= \frac{1}{B^2}(A \pm \Delta A)\frac{(B \pm 2\Delta B)}{B}$$

or
$$Z \pm \Delta Z = \frac{1}{B^3}\left[AB \pm B.\Delta A + 2A.\Delta B + 2\Delta A.\Delta B\right]$$

Dividing *L.H.S.* by Z and *R.H.S.* by $\frac{A}{B^2}$ we get,

$$1 \pm \frac{\Delta Z}{Z} = \frac{1}{B^3}\left[\frac{AB \pm B.\Delta A \pm 2A.\Delta B \pm 2A.\Delta B}{\frac{A}{B^2}}\right]$$

or
$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} + \frac{2\Delta B}{B} + \frac{2\Delta A}{A} \times \frac{\Delta B}{B}$$

Since $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are very small, therefore neglecting the product $\frac{\Delta A}{A} \times \frac{\Delta B}{B}$, we have,

Error in Z,
$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + 2\frac{\Delta B}{B}$$

Thus, percentage error is given as

$$\frac{\Delta Z}{Z} \times 100 = \frac{\Delta A}{A} \times 100 + 2\frac{\Delta B}{B} \times 100$$

Thus, the percentage error in the determination of g is the sum of the percentage errors in the measurement of length (l) and twice in the measurement of time (t).

Therefore, the time period should be measured more accurately because the error in the measurement of time will be double in the final result.

Ex. UD—24. Which is the most accurate measurement out of the following measurements of the same length : 3 metre, 3·0 metre, 3·00 metre and 3·000 metre. Illustrate your answer by giving full calculations.

Sol. The length measured with a metre a metre scale is 3 m. The least count of metre scale is 0·1 cm.

$$\therefore \qquad \Delta l = 0{\cdot}1 \text{ cm} = 0{\cdot}001 \text{ m.}$$

$$\text{Hence, possible \% age error is } = \frac{0 \cdot 001}{3} \times 100 = 0 \cdot 033\%$$

Similarly in case when the length is 3·0 metres

$$\Delta l = 0{\cdot}01 \text{ cm} = 0{\cdot}0001 \text{ m.}$$

$$\therefore \text{ Possible percentage error is } = \frac{0 \cdot 0001}{3 \cdot 0} \times 100 = 0 \cdot 0033\%$$

Again in case the length measured is 3·00 metres

$$\Delta l = 0{\cdot}001 \text{ cm} = 0{\cdot}00001 \text{ m.}$$

$$\therefore \qquad \text{Possible error is } = \frac{0 \cdot 00001}{3 \cdot 00} \times 100 = 0 \cdot 00033\%$$

Also when length measured is expressed as 3·000 metre

$$\Delta l = 0{\cdot}0001 \text{ cm} = 0{\cdot}000001 \text{ m.}$$

$$\therefore \text{ The possible error is } = \frac{0 \cdot 000001}{3 \cdot 000} \times 100 = 0 \cdot 000033\%$$

Clearly the degree of accuracy in case of measurement of length expressed as 3·000 m is more. Therefore, measurement 3·000 metre is most accurate.

UD—20. SIGNIFICANT FIGURES

The reasonably trustworthy digits that are used to represent the magnitude of a physical quantity with proper accuracy are called ***significant figures.***

Or

The digits which tell us the number of figures we are reasonably sure of having counted in making a measurement are called significant figure.

The accuracy of any measurement is always limited due to the limit of accuracy of the measuring instruments. When we say that the length of glass strip is 1·453 cm. If metre rod is used for measurement, it can measure accurately upto 0·1 cm so a trustworthy result is the length = 1·4 cm and we have no knowledge of other figures *i.e.,* 5 and 3. Therefore, the two digits 1 and 4 are called significant figures because, they indicate the accuracy of measurement. The digit 1 is reliable while the second figure 4 in the result 1·4 cm is the first unreliable digit. Again if measurements are made with vernier callipers, the reliable result will be 1·45, since least count of vernier callipers is 0·01 cm. In this case first unreliable digit is 5. The number of significant figures is three. *i.e.,* 1·45. *Thus the greater the number of significant figures obtained while making measurement, the more accurate is the measurement.*

Following are the rules for determining the number of significant figures :

1. All non–zero digits are significant. As 142·85 *m* has five significant figures.

2. All zeros between two non–zero digits are significant. As an example 805·007 contains six significant figures.

3. Unless indicated all zeros to the left of an understood decimal point but to the right of a non–digit are not significant. As in 307000 *m* there are only three significant figures.

4. All zeros to the left of an expressed decimal point and to the right of a non–zero digit are significant. For example, 307000·0 *m* has six significant figures.

5. All zeros to the right of a decimal point but to the left of a non–zero digits are not significant. As in example 0·000475 *m*, it contains only three significant figures.

6. All zeros to the right of decimal point and to the right of a non–zero digit are significant. As in 0·03040 *m* and 40·03 *m* both contain four significant figures.

Role of significant figures. The greater the number of significant figures obtained when making a measurement, the more accurate is the measurement. On the other hand a measurement made only to a few significant figures is not very accurate even if all care is taken to make the measurement.

QUESTIONS FOR SELF TEST

1. (*a*) Defferentiate between Fundamental and Derived units.

(*b*) What are *SI* units. Mention the *SI* base units along with their symbols.

2. Write the *SI* units of the following quantities :–

(i) Work *(ii)* Pressure *(iii)* Impuls *(iv)* Moment of Inertia *(v)* Specific heat *(vi)* Charge *(viii)* Intensity of illumination.

3. What is meant by the dimensions of a physical quantity ?

Write the dimensional formula of the following physical quantities.

(i) Torque *(ii)* Momentum *(iii)* Co–eff. of viscosity *(iv)* Surface tension *(v)* Gravitational constant *(vi)* Temperature *(vii)* Modulus of elasticity *(viii)* Co–eff. of thermal conductivity.

4. Sate and explain principle of Homogeneity of Dimensions. How will you apply it to check the correctness of the relations.

(i) $V = \sqrt{\frac{E}{d}}$, *(ii)* $t = 2\pi\sqrt{\frac{l}{g}}$,

(iii) $E = mgh + \frac{1}{2}mv^2$, *(iv)* $P = h\,d\,g.$

(*v*) $S = ut + \frac{1}{2}at^2$, (*vi*) $h \frac{2T\cos\theta}{rdg}$

5. Use principle of homogeneity of dimensions to derive the relations :

(*i*) $F \frac{mv^2}{r}$, (*ii*) $KE = \frac{1}{2}mv^2$, (*iii*) Modulus of elasticity, (*iv*) Heat quantity (Q),

(*v*) $t = 2\pi\sqrt{\frac{l}{g}}$

PROBLEMS

1. A body of mass *m kg* has a weight *W* newtons. The acceleration due to gravity is $g\ m\ s^{-2}$. Find the equation connecting the three quantities with the help of dimensions. ($W = mg$)
2. Convert a velocity of 12 *cm/s* into *km/hr.* (0·432 *km/hr*)
3. Find the number ergs in 35 joules of energy. (35×10^7 ergs)
4. The acceleration of a body is 15 $m\ s^{-2}$. What is the value of acceleration in $cm\ s^{-2}$. (1500 cm/s^2)
5. The value of $G = 6{\cdot}67 \times 10^{-8}$ $dyre\ cm^2/gm^2$. What will be its value in *SI* units ? ($6{\cdot}67 \times 10^{11}\ Nm^2\ kg^{-2}$)
6. Establish dimensionally the expression for the centripetal force. Facting on a particle of mass *m* moving with a velocity *v* in a circle of radius *r*.

   $\left(F = \frac{KmV^2}{e}\right)$
7. Velocity *v* of sound depends on the pressure *P* and density ρ of the medium. So that $v = \sqrt{\frac{P}{P}}$.
8. Using dimensional formula of force, calculate number of Newton's in one dyne.

   (Ans. 10^{-5} *N***)**
9. Find the value of 10J on a system which has 10 cm 1000g and 30 seconds are fundamental units. **(Ans.** 9×10^5**)**
10. Convert velocity 126 *km/hr* into *m/s.* **(Ans.** 35 *m/s***)**

Chapter

FM

Force and Motion

FM–1. INTRODUCTION

Scalar Quantities. *A scalar quantity is a physical quantity that is completely specified by a number with appropriate units.* That is a scalar quantity has **only magnitude** and no direction.

Examples of scalar quantities are volume, mass, speed, density, charge, energy, temperature, length, time etc.

Scalar quantities obey the laws of algebra *i.e.,* they can be added algebrically.

Vector Quantities. *A vector quantity is a physical quantity that is completely specified by its direction as well as its magnitude.*

To describe completely the force acting on an object, we must specify both, the direction of the applied force and a *number with unit* to indicate the *magnitude* of force. Similarly to describe the motion of an object completely, we must specify both, how fast it is moving and the direction of its motion.

A few examples of vectors are displacement, velocity, acceleration, impulse, force, momentum, electric field etc.

Vectors do not follow the laws of Algebra. Therefore, they are added by vector addition.

FM–2. REPRESENTATION OF A VECTOR

A vector is represented by a line with an arrow head. The length of the line gives the magnitude and the arrow head represents the direction of the vector. On a paper, vector quantity is represented by placing an arrow on the corresponding letter as $\vec{A}$.

The magnitude of the vector is called the modulus (mod) of vector A and is represented by $|A|$

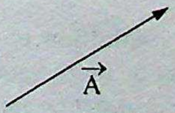
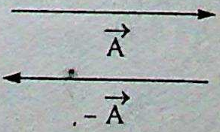


Fig. FM—1

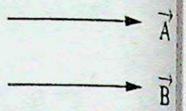
Type of Vectors : Vectors can be divided into the following types.

1. Negative Vectors : The negative of a vector is defined as another vector having same magnitude drawn in opposite direction.

i.e., any vector $\vec{A}$ and its negative vector $(-\vec{A})$ are as shown :



2. Equal Vectors : Two or more vectors are said to be equal, if they have same magnitude and direction. If $\vec{A}$ and $\vec{B}$ are two equal vectors then,

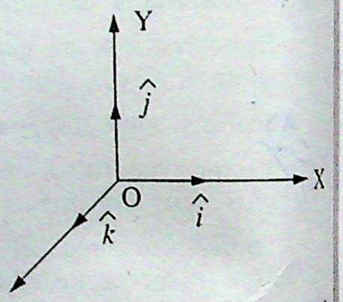


$$\vec{A} = \vec{B}$$

3. Unit Vector : A vector divided by its magnitude is called a unit vector. It has magnitude one unit and direction same as the direction of given vector. It is denoted by $\hat{A}$

$$\hat{A} = \frac{\vec{A}}{A}$$

Unit vectors along the three mutually perpendicular axis are represented by $\hat{i},\hat{j}$ and $\hat{k}$.



$\hat{z}$ is the unit vector along x–axis.

$\hat{j}$ is the unit vector along y–axis.

$\hat{k}$ is the unit vector along z–axis.

$\hat{z},\hat{j}$ and $\hat{k}$ are mutualy perpendicular to each other *i.e.*, angle between any pai is 90°.

4. Co–initial Vectors : Two or more vectors having same initial point are calle co–initial vectors.

5. Collinear Vectors : Two or more vectors having equal or unequal magnitudes, which act either along the same line of along the parallel lines are called colllinear vectors.

6. Zero Vector : A vector having zero magnitude and arbitrary direction i called zero vector. It is dented by $\hat{o}$.

Addition of Vectors. Addition of vectors means to find the resultant vector o two or more vectors. For the addition of two vectors, we use *triangle Law of vecto or parallelogram law of Vectors.*

Triangle Law of Vectors. *It two vectors can be represented both in magnitu and direction by the sides of a triangle taken in the same order, then the resultant i represented completely in magnitude and direction, by the third side of the triang taken in the opposite order.*

Suppose we have to add two vectors $\vec{P}$ and $\vec{Q}$ as shown in fig. FM–2. Place the initial point (tail) of $\vec{Q}$ on the terminal point (head) of $\vec{P}$. Complete the triangle to get a new vector $(\vec{P}+\vec{Q})$ starting from tail of $\vec{P}$ to head of $\vec{Q}$. This new vector is the resultant vector $\vec{R}$.

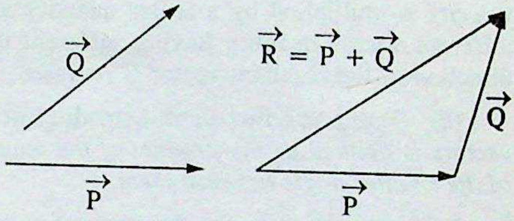


Fig. FM—2

Thus sum of vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ vector.

i.e. $\vec{R} = \vec{P}+\vec{Q}$

According to triangle law of vectors, if three vectors are represented by the three sides of a triangle taken in order, then their resultant is zero. Thus if three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ can be represented completely in magnitude and direction by the three sides of a triangle taken in order, then their vector sum is zero.

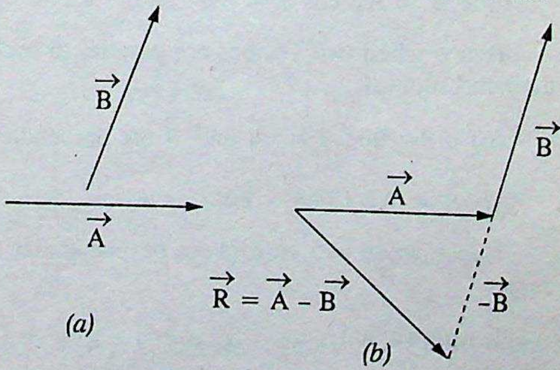
$$\therefore \quad \vec{A}+\vec{B}+\vec{C} = 0.$$

The resultant $-\vec{C}$ vector of $\vec{A}$ and $\vec{B}$ cancels the third vector $\vec{C}$.

Subtraction of Vectors. The process of subtracting one vector from another vector is equivalent to adding vectorially the negative of the vector to be subtracted. If $\vec{A}$ and $\vec{B}$ are two vectors, then

$$\vec{A}-\vec{B} = \vec{A}+(-\vec{B})$$

The subtraction of $\vec{B}$ from $\vec{A}$ as shown in fig. FM–3 is done with the help of triangle law of vectors.



(a) (b)

Fig. FM—3

Multiplication of Vectors. Multiplication of vectors can be of the following three kinds :

(i) **Multiplication of vector by a scalar.** The multiplication of vector by a scalar or a pure number *n* gives a vector in the same direction but *n* times its magnitude. As an example multiplication of a vector $\vec{a}$ with a number *n* gives a vector $\vec{A}$, such that

$$\vec{A} = n\,\vec{a}$$

In case n is a physical scalar quantity having units, then the units of $\vec{A}$ will be **changed** and will not be the same as that of $\vec{a}$. As for example if vector quantity ***velocity*** is multiplied by a scalar quantity *mass*. Then, the product is momentum, a different vector quantity having different units. In case the scalar is negative, the direction of the resultant vector is reversed.

(ii) **Scalar product or dot product of two vectors.** *The scalar product of two vectors is defined as the product of the magnitudes of the two vectors and the cosine of the smaller angle between them.*

It is also called as the dot product and is represented by a dot between the two vectors.

$$\therefore \qquad (\vec{A}\cdot\vec{B}) = \vec{A}\,\vec{B}\cos\theta = S$$

S being a scalar quantity.

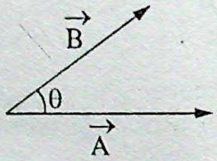


Fig. FM—4

Again the dot product can be written as

$$(\vec{A}\cdot\vec{B}) = \vec{A}(\vec{B}\cos\theta)$$

or

$$(\vec{A}\cdot\vec{B}) = \vec{B}(\vec{A}\cos\theta)$$

Thus scalar product of two vectors may be defined as the product of one vector with the projection (vector × cos θ) of the other vector along the direction of the first vector.

Special Cases : *(i)* If $\theta = 0°$, *i.e.*, $\vec{A}$ and $\vec{B}$ are parallel to each other, then

$$\vec{A}.\vec{B} = AB\cos\theta = AB\,Cos\,0° = AB \qquad [\because \cos 0° = 1]$$

Hence when two vectors are parallel to each other their dot product is eaqual to their magnitudes.

(ii) If $\theta = 90°$, *i.e.*, $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then

$$\vec{A}\cdot\vec{B} = AB\,Cos\theta = AB\,Cos\,90° = Zero \qquad [\because Cos\,90° = 0]$$

Hence when two vectors are perpendicular to each other then their dot product is zero.

It follows that $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)\,(1)\,Cos\,0° = 1$

and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{z} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = \hat{z}\cdot\hat{k} = \hat{k}\cdot\hat{z} = 0$

Because angle between $\hat{i}\cdot\hat{i}, \hat{j}\cdot\hat{j}$ and $\hat{k}\cdot\hat{k}$ is 0° and angle between $\hat{i}, \hat{j}, \hat{k}$ is 90°

(iii) **Vector product or cross product.** *Vector product or cross product of two vectors is defined as the product of the magnitudes of the two vectors and the sine of the angle between them and acts in a direction perpendicular to the plane containing the two vectors.*

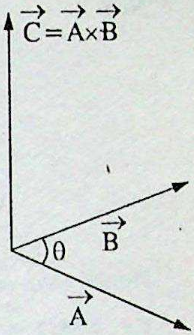


Fig. FM—5

If $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product (*or cross product*) written as $\vec{A} \times \vec{B}$ is a vector $\vec{C}$. Therefore,

$$\vec{C} = \vec{A} \times \vec{B} = A\ B \sin\theta$$

The direction of $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$.

Special Cases : *(i)* If $\theta = 0°$ *i.e.,* $\vec{A}$ and $\vec{B}$ are parellel to each other, then,

$$\vec{A} \times \vec{B} = AB \text{ Sin } \theta = AB \text{ Sin } 0° = \text{Zero}$$

Hence, Cross product of two parallel vector is zero. [$\because$ Sin 0° = 0]

(ii) If $\theta = 90°$, *i.e.,* $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then,

$$\vec{A} \times \vec{B} = AB \text{ Sin } 90° = AB \times 1 = AB \qquad [\because \text{Sin } 90° = 1]$$

Hence when two vectors are perpendicular to each other, their cross product (vector product) is equal to product of their magnitudes.

It follows that $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = (1)\,(1) \text{ Sin } 0° = 0$

and $\qquad \hat{i} \times \hat{j} = \hat{k}, \ \hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{i} = \hat{j}$

If two vectors are written as $\vec{A} = Ax\,\hat{i} + Ay\,\hat{j} + Az\,\hat{k}$

$$\vec{B} = Bx\,\hat{i} + By\,\hat{j} + Bz\,\hat{k}$$

then $\vec{A} \times \vec{B}$ is given by, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$

$$\vec{A} \times \vec{B} = \hat{i}\ (Ay\ Bz - By\ Az) - \hat{j}\ (Ax\ Bz - Bx\ Az) + \hat{k}\ (Ax\ By - Bx\ Ay)$$

Ex. FM-1. Prove that the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}, \ \hat{B} = 2\hat{i} - \hat{j}$$

Sol. The two vectors $\vec{A}$ and $\vec{B}$ are perpendicular, if their dot product $(\vec{A} \cdot \vec{B})$ is zero.

Now $$\vec{A}\cdot\vec{B} = (\hat{i}+2\hat{j}+3\hat{k}).(2\hat{i}-\hat{j}+0\hat{k})$$

$$= 2(\hat{i}\cdot\hat{i}) - 2(\hat{j}\cdot\hat{j}) + 3\times 0(\hat{k}\cdot\hat{k})$$

$$= 2(1) - 2(1) + 0(1) \qquad \left[\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\right]$$

$$\vec{A}\cdot\vec{B} = 2 - 2 + 0 = 0$$

Hence two vectors are perpendicular to each other.

Ex. FM-2. Find the value of m for which vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

$$\vec{A} = \hat{i}+2\hat{j}+m\hat{k}, \quad \vec{B} = 2\hat{i}-\hat{j}+\hat{k}$$

Sol. Two vectors $\vec{A}$ and $\vec{B}$ are perpendicular if $\vec{A}\cdot\vec{B} = 0$

$$\vec{A}\cdot\vec{B} = (\hat{i}+2\hat{j}+m\hat{k}).(2\hat{i}-\hat{j}+\hat{k}) = 0$$

$$= 2(\hat{i}\cdot\hat{i}) + (2)(-1)(\hat{j}\cdot\hat{j}) + m(\hat{k}\cdot\hat{k}) = 0$$

$$= 2(1) + (-2)(1) + m(1) = 0 \qquad \left[\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\right]$$

$$= 2 - 2 + m = 0$$

$$\Rightarrow \qquad m = 0$$

Hence, if $m = 0$ then two vector $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Ex. FM-3. Find angle between vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} = \hat{i}+2\hat{j}-\hat{k}, \quad \vec{B} = -\hat{i}+\hat{j}-2\hat{k}$$

Sol. We know that if θ in angle between $\vec{A}$ and $\vec{B}$, then,

$$\vec{A}\cdot\vec{B} = AB\,\text{Cos}\theta$$

$$\Rightarrow \qquad \text{Cos}\theta = \frac{\vec{A}\cdot\vec{B}}{AB} \qquad \ldots(1)$$

$$\vec{A}\cdot\vec{B} = (\hat{i}+2\hat{j}-\hat{k}).(-\hat{i}+\hat{j}-2\hat{k})$$

$$= -(\hat{i}\cdot\hat{i}) + 2(\hat{j}\cdot\hat{j}) + (2)(\hat{k}\cdot\hat{k})$$

$$= -(1) + 2(1) + 2(1) \qquad \left[\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\right]$$

$$= -1 + 2 + 2$$

$$\vec{A}\cdot\vec{B} = 3 \qquad \ldots(2)$$

Also $$|\vec{A}| = A = \sqrt{(1)^2+(2)^2+(-1)^2}$$

$$A = \sqrt{1+4+1} = \sqrt{6} \qquad \text{...(3)}$$

$$|\vec{B}| = B = \sqrt{(-1)^2+(1)^2+(-2)^2}$$

$$B = \sqrt{1+1+4} = \sqrt{6} \qquad \text{...(4)}$$

Put from (2), (3) and (4) in equation (1) we get,

$$\text{Cos}\theta = \frac{3}{\sqrt{6}\cdot\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\text{Cos}\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 60^\circ \text{ Ans.}$$

Ex. FM-4. If $\vec{A}$ $(3\hat{i}+\hat{j}+4\hat{k})$ **and** $\vec{B} = \hat{i}+2\hat{j}-\hat{k}$ **find the value of** $(\vec{A}+\vec{B}).(\vec{A}-\vec{B})$**.**

Sol. While adding or substracting two vectors coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ are added or substracted from each vector so,

$$\vec{A}+\vec{B} = (3\hat{i}+\hat{j}+4\hat{k})+(\hat{i}+2\hat{j}-\hat{k})$$

$$= (3\hat{i}+\hat{i})+(\hat{j}+2\hat{j})+(4\hat{k}-\hat{k})$$

$$\vec{A}+\vec{B} = 4\hat{i}+3\hat{j}+3\hat{k}$$

and $$\vec{A}-\vec{B} = (3\hat{i}+\hat{j}+4\hat{k})-(\hat{i}+2\hat{j}-\hat{k})$$

$$= (3\hat{i}-\hat{i})+(\hat{j}-2\hat{j})+(4\hat{k}+\hat{k})$$

$$\vec{A}-\vec{B} = 2\hat{i}-\hat{j}+5\hat{k}$$

Now $$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = (4\hat{i}+3\hat{j}+3\hat{k}).(2\hat{i}-\hat{j}+5\hat{k})$$

$$= 8(\hat{i}\cdot\hat{i})-3(\hat{j}\cdot\hat{j})+15(\hat{k}\cdot\hat{k})$$

$$= 8(1) - 3(1) + 15(1)$$

$$= 8 - 3 + 15$$

$$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 20 \text{ Ans.}$$

Ex. FM-5. If $\vec{A} = (\hat{i}-2\hat{j}+3\hat{k})$ **and** $\vec{B} = (4\hat{i}+5\hat{j}+0\hat{k})$ **find** $\vec{A}\times\vec{B}$

Sol.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 4 & 5 & 0 \end{vmatrix}$$

$$= \hat{i}(0-15) - \hat{j}(0-12) + \hat{k}(5+8)$$

$$\vec{A}\times\vec{B} = 15\hat{i} + 12\hat{j} + 13\hat{k} \text{ Ans.}$$

Ex. FM-6. Show that the vectors $\vec{A} = \hat{i} - 5\hat{j}$, $\vec{B} = 2\hat{i} - 10\hat{j}$ are parallel to each other.

Sol.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & 0 \\ 2 & -10 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(-10+10)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{A}\times\vec{B} = \vec{0}$$

Hence two vectors are parallel to each other.

Ex. FM-7. Prove that $|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2$

Sol. $\vec{A}\times\vec{B} = AB \text{ Sin } \theta$ so $|\vec{A}\times\vec{B}|^2 = |\ AB \text{ Sin}\theta\ |^2$

$\vec{A}\cdot\vec{B} = AB \text{ Cos } \theta$ so $|\vec{A}\cdot\vec{B}|^2 = |\ AB \text{ Cos}\theta\ |^2$

Now $|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = (AB \text{ Sin}\theta)^2 + (AB \text{ Cos}\theta)^2$

$$= (AB)^2 \text{ Sin}^2\theta + (AB)^2 \text{ Cos}^2\theta$$

$$= (AB)^2 (\text{Sin}^2\theta + \text{Cos}^2\theta)$$

$$= A^2B^2 (1) \qquad [\because \text{Sin}^2\theta + \text{Cos}^2\theta = 1]$$

$$|\vec{A}\times\vec{B}|^2 + |\vec{A}\cdot\vec{B}|^2 = A^2B^2 \text{ Ans.}$$

Ex. FM-8. Show that the vector $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

Sol. Two vector are parallel if $\vec{A} \times \vec{B} = \vec{0}$

so
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$

$$= \hat{i}(-9+9) - \hat{j}(6-6) + \hat{k}(18-18)$$

$$= 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

Hence $\vec{A}$ and $\vec{B}$ vectors are parallel to each other or in other words angle between them is 0°.

FM–4. COMPONENTS OF A VECTOR.

The projections of a vector along the axes of a rectangular co-ordinate system are called components of a vector.

Rectangular components. If the components of a given vector are perpendicular to each other, then they are called *rectangular components.*

Resolution of a vector into rectangular components.

Let the vector $\vec{OP} = (\vec{R})$ is to be resolved into two rectangular components, $\vec{x}$ and $\vec{y}$.

From the triangle law of vectors, we have in vector triangle *OAP*,

$$\vec{R} = \vec{x} + \vec{y}$$

$\vec{x}$ and $\vec{y}$ are the rectangular components of $\vec{R}$

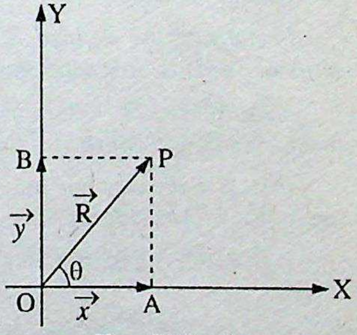


Fig. FM—6

The components $\vec{x}$ and $\vec{y}$ of a vector $\vec{R}$ are the projections of $\vec{R}$ on the co-ordinate axis.

In right-angled triangle, *OAP*

$$R = \sqrt{x^2 + y^2}$$

If θ is the angle which *R* makes with X-axis, then

$$\frac{OA}{OP} = \cos\theta \quad \text{or} \quad \frac{x}{R} = \cos\theta$$

$$\therefore \quad x = R\cos\theta \qquad \ldots(i)$$

Also $$\frac{AP}{OP} = \sin\theta \quad \text{or} \quad \frac{y}{R} = \sin\theta$$

$$\therefore \qquad y = R \sin \theta. \qquad \ldots(ii)$$

Thus in general the components of a vector are found by multiplying the magnitude of the vector by the cosine of the angle that the vector makes with each of the co-ordinate axes.

Ex. FM–9. Determine the vertical component of a vector whose horizontal component is of magnitude 13 units and the angle between vector itself and horizontal component is 60°.

Sol. Let the magnitude of vector $= R$

$\therefore$ Horizontal component, $x = R \cos \theta$

But, Horizontal component, $x = 13$ units and $\theta = 60°$

$$\therefore \qquad 13 = R \cos 60° \quad \text{or} \quad R = \frac{13}{\cos 60°}$$

$$\text{Now, Vertical component, } y = R \sin 60° = \frac{13}{\cos 60°} \times \sin 60°$$

$$= 13 \times \tan 60° = 13 \times \sqrt{3}$$

$$= 18{\cdot}3846 \text{ units.} \quad \textbf{Ans.}$$

FM–5. FORCE

Force may be defined as a pull or push which produces or tends to produce, stops or tends to stop motion in a body.

Examples. *(i)* When a foot ball is kicked, it begins to move. The push which makes the ball to *move* is a *force.*

(ii) A man can not move a wall if he pushes it but the agency which tends to move the wall is a force.

(iii) A moving car is stopped by applying brakes. Here, the motion is stopped by a force.

SI units of force are ***newtons*** **(*N*).** One newton is defined as *the force which, when acting on a 1 kg mass, produces an acceleration of 1 m/s*^{2}, *1* $N = 1$ *kg m s*$^{-2}$.

FM–6. NEWTON'S LAWS OF MOTION.

The laws which govern the motion of a body are said to be Newton's laws of motion and are discussed below.

(i) **First Law of Motion.** *Every body continues to remain in its state of rest or uniform motion in a straight line unless it is compelled by external forces to change its state of rest or uniform motion.*

The first part of the law says that a body at rest will remain at rest unless an external force acts on it to change the state of rest. For example a book lying, on a table remains lying there unless some body applies force to lift or move it.

The second part of the law says that a body moving with uniform speed in a straight line will continue to do so unless a force is applied to stop it. This appears to be contrary to our daily experience. We find that a rolling ball stops after some time when apparently no force is applied to it. Similarly, a scooter stops soon after we cut off the supply of oil.

In fact the ball or the scooter stop because of the external forces like friction, air resistance and gravity which tend to stop them. If all such external forces tending to stop them are eliminated, the ball or the scooter or any body once set to move would go on moving for ever.

(ii) **Second Law of Motion.** *The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.*

This law has two parts. According to first part a certain force acting on a body produces a certain change in momentum. If the force is *doubled,* the change in momentum is doubled. Since momentum is constant, the velocity is *doubled*. Similarly if the force is made *there* times, the velocity will become three times. Therefore momentum becomes three times.

According to the second part, if a force is applied to a body at rest, the body will begin to move in the direction of the force. If a force is applied to a moving body in the direction of motion, the velocity will increase and hence the momentum will also increase. If, however, the force is applied in a direction opposite to the direction of motion, the velocity will decrease and hence the momentum will also decrease. The change in momentum takes place in the direction of force.

(iii) **Third Law of Motion.** *To every action there is an equal and opposite reaction.*

According to third law of motion, when a body A exerts a force (F_{AB}) on another body B, the body B also exerts an equal but opposite force (F_{BA}) on A. The force exerted by A on B *i.e.* F_{AB} is called *action* and an equal and opposite force that B exerts on A *i.e.* F_{BA} is called *reaction*.

$$\therefore \qquad F_{AB} = -F_{BA}.$$

From the equation it is clear that two forces are equal in magnitude but opposite in direction.

Illustrations. The following examples illustrate the third law of motion.

(i) When a rubber ball is struck against the wall, it jumps back due to the reaction of the wall.

(ii) When a man jumps out of a boat, the boat is pushed in the backward direction.

(iii) In order to swim, a person pushes the water backward with his hands and he, in turn, is pushed forward due to the **reaction** of water. Similarly while rowing a boat we push the water backward with the help of oars and due to the reaction of water the boat moves forward.

(iv) When a bullet is fired from a gun, the bullet moves forward whereas the gun *kicks* backward. The momentum of the bullet in the forward direction *i.e.*, action is equal to the momentum of the gun in the backward direction *i.e.*, reaction

(*v*) An arrangement for sprinkling water based upon Newton's third law of motion is shown in Fig. FM–7. As the water issues out of the nozzles, it exerts an equal force in the backward direction causing the arrangement to rotate in the opposite direction in a horizontal plane, the water is scattered in all directions. This arrangement is known as **rotary lawn sprinkler**.

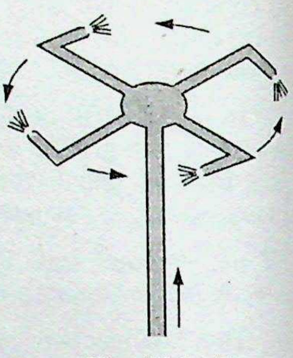


Fig. FM—7

It is clear from the examples given above that *to every action there is an equal and opposite reaction.* The question arises "If action and reaction are equal and opposite" why should a body move at all. The answer to this question is that the action and reaction act on **different** bodies.

FM–7. MOMENTUM :

The quantity of motion possessed by a moving body is called **momentum.** *It is measured by the product of mass and velocity of the body.* It is a matter of common experience that a car moving slowly can be stopped more easily than when it is moving fast. Similarly, a cricket ball hit gently can be caught easily but it is difficult to catch it if it is moving fast. These examples show that greater the **velocity** of a body, the greater is the force required to stop it.

Again a cricket ball and a tennis ball may be moving with the same velocity but a greater force is required to stop the cricket ball than that required to stop the tennis ball. This shows that greater the mass of the body, greater is the force required to stop it.

From the above, we conclude that the force required to stop a moving body or conversely to move a body with a given velocity depends upon two factors (*i*) velocity and (*ii*) mass of the body. *This product of the mass and the velocity of a moving body is known as* its **momentum.**

$$\therefore \quad \text{Momentum} = \text{mass} \times \text{velocity}$$

or

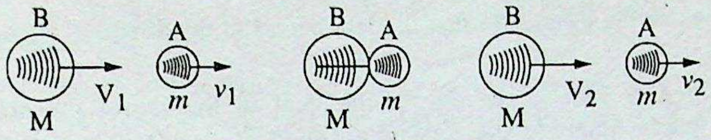
$$M = m \times v$$

$S\,I$ units of momentum are $kg\ m\ s^{-1}$

FM–8. PRINCIPLE OF CONSERVATION OF MOMENTUM :

The algebraic sum of the momenta of bodies in a system along a straight line remains constant and is not altered due to their mutual action and reaction on each other.

Consider two bodies A and B of masses m and M moving in the same direction with velocities, v_1 and V_1 respectively ($V_1 > v_1$). (See fig. FM–8). When the body B collides against body A, the velocities change after collision. Let body A move with velocity v_2 and B with velocity V_2 in the forward direction, then



Before Collision

Collision

After Collision

Fig. FM—8

$$\text{Initial momentum of } A = mv_1$$

$$\text{Initial momentum of } B = MV_1$$

$$\text{Total momentum before collision} = mv_1 + MV_1$$

After the collision,

$$\text{Momentum of body } A = mv_2$$

and

$$\text{Momentum of body } B = MV_2$$

$$\therefore \quad \text{Total momentum after collision} = mv_2 + MV_2$$

If + *F* is the force (*action*) exerted by body *B* on the body *A*, then according to Newton's third law of motion, a force (*reaction*)– *F* is exerted by the body *A* on the body *B* for a short time *t* in which the collision takes place.

∴ Change of momentum of *B* (*action*) *i.e.* + *Ft*,

$$+Ft = MV_2 - MV_1 = -(MV_1 - MV_2) \quad \because V_1 < V_2 \qquad \ldots(i)$$

Also change of momentum of *A* (*reaction*) $= -Ft = -(mv_2 - mv_1)$ ...(*ii*)

According to Newton's third law of motion *action* and *reaction* are equal and opposite, therefore from (*i*) and (*ii*) we have,

$$-(MV_1 - MV_2) = -(mv_2 - mv_1)$$

or

$$MV_1 - MV_2 = mv_2 - mv_1$$

Thus, Momentum before collision = Momentum after collision.

Example to Illustrate the Principle of conservation of momentum :

(*i*) **Recoil of a Gun :** The momentum of the system of bullet and gun before firing is zero, as both are at rest. When bullet is fired from the gun, the bullet moves forward where as the gun recoils or kicks in the backward direction. The momentum of the bullet in the forward direction is equal to the momentum of gun in the backward direction. If *m* is the mass of the bullet and *v* its velocity, M the mass of the gun and V its velocity in the backward direction, therefore is,

$$\text{Momentum of bullet} = mv$$

$$\text{Momentum of gun} = -MV$$

$$\therefore \quad mv = -MV$$

or

$$mv + MV = 0$$

∴ Velocity of recoil of the gun is given as,

$$V = -\frac{m}{M}v$$

The negative sign indicates that the gun moves in the direction opposite to that of the bullet.

(ii) When a man jumps out of the boat to the shore, the boat is pushed backward. Before jumping the total momentum of man and boat is zero. During jump, the man moves in the forward direction and the boat must acquire a momentum equal and opposite to that of man. This explains the motion of the boat away from the shore when the man jumps off to the shore.

(iii) Jet planes and rockets act on the principle of conservation of momentum.

Ex. FM–10. A gun fires a shell of mass 5 kg in a horizontal direction. The gun recoils with a velocity of 4·0 m/s and its mass is 3 tonne. Calculate the velocity of the shell.

Sol. Here, sum of initial momentum of the gun and shell is zero

Mass of gun, $M = 3 \text{ tonne} = 3 \times 1000 \text{ kg}$

Velocity of gun, $V = 0\cdot4 \text{ ms}^{-1}$

$\therefore$ Momentum of gun, $MV = 3000 \times 0\cdot4 = 1200 \text{ kg m/s}$

Mass of shell, $m = 5 \text{ kg}$

Let, Velocity of shell $= v = ?$

$\therefore$ Momentum of shell $= -mv = -5 \times v$

Sum of momentum after firing $= MV - 5v$

$$\therefore \quad 1200 - 5v = 0$$

or

$$v = \frac{12000}{5} = 240 \; m\,s^{-1} \text{ (Ans.)}$$

Ex. FM–11. A gun weighing 10 kg fires a bullet of 0·03 kg with a velocity of 330 m/s. With what velocity does the gun recoil ?

Sol. Mass of gun, $M = 10 \text{ kg}$

Mass of bullet, $m = 0\cdot03 \text{ kg}$

Velocity of bullet, $v = 330 \text{ m/s}$

Let the velocity of recoil of gun be V m/s,

Then according to the principle of conservation of momentum,

$$MV + mv = 0$$

or

$$V = -\frac{mv}{\text{M}} = -\frac{0\cdot03 \times 330}{10}$$

$$= -0\cdot99 \text{ m/s (Ans.)}$$

The negative sign indicates that the gun recoils in the backward direction.

FM–9. IMPULSIVE FORCE :

A force which acts for a very short time but produces an appreciable change of momentum is called **Impulsive force.**

Impulse : *Impulse of a force is defined as the total change in momentum produced by the impulsive force.*

In some cases the force acting on a body is not constant or it acts for a very short time. For example, when a collision takes place between two bodies or when a blow is made with a hammer, it is ordinarily not possible to measure the actual value of the force as its value changes with time. In such cases the *effect of the force is measured by the total change in momentum that it produces in a given time.*

If the velocity of a body of mass m changes from u to v in t seconds, then the total change in momentum during this time is $(mv - mu)$. If F is the average force applied, then

$$F = \frac{mv - mu}{t} = \text{rate of change of momentum.}$$

or $$F \times t = mv - mu = \text{total change in momentum.}$$

Thus impulse *of a force may also be defined as the product of force and time and is equal to the total change in momentum of a body.*

FM–10. MEASUREMENT OF FORCE.

According to the first part of Newton's second law of motion when a force is applied on a body a change in its momentum is produced. The rate of change of momentum is proportional to the applied force.

Suppose a force F acts on a body of mass m for t seconds and increases its velocity from u to v, then

Momentum of the body in the beginning $= mu$

Momentum of the body after t seconds $= mv$

$\therefore$ Change of momentum in t seconds $= mv - mu$

or Rate of change of momentum $= \dfrac{mv - mu}{t}$

According to the second law of motion, we know that

$$F \propto \frac{mv - mu}{t}$$

or $$F \propto \frac{m(v - u)}{t}$$

or $$F \propto ma \quad \therefore \quad F = kma \qquad \left(\because \frac{v - u}{t} = a\right)$$

Where k is a constant and a is the acceleration produced in the body. Now the unit of force is so chosen that it produces a unit acceleration in a unit mass. Thus if $m = 1$, $a = 1$ and $F = 1$, then

$$1 = k \times 1 \times 1$$

or $$k = 1$$

or $\quad$ Force = Mass × acceleration

A unit force is that much force which produces a unit acceleration in a unit mass.

FM–11. UNITS OF FORCE :

(i) **The Absolute Unit** of force in *S I* system is the *newton* (N).

One **Newton** is defined as that force which when acting on a mass of 1 kg produces an acceleration of 1 ms^{-2}

$$\therefore \quad 1N = 1 \text{ kg } m\, s^{-2}$$

The unit of force in the *C.G.S.* system is called the *dyne* and is defined as that force which when, acting on a mass of 1 g produces an acceleration of 1 cm/s^2

$$\therefore \quad 1 \text{ dyne} = 1 \text{ g } cm/s^2$$

Also $\quad 1\ N = 10^5$ dynes.

(ii) **The gravitational Unit** of force in the *MKS* system is a *kilogram weight* (kg wt) or *kilogram force* (*kgf*).

One *kg wt* or *kgf* is that much force which produces an acceleration equal to the acceleration due to gravity (9·81 m/s^2) in a mass of 1 kg,

$$1\ kgf = 1 \text{ kg} \times 9{\cdot}81\ m\, s^{-2}$$

$$= 9{\cdot}81 \text{ kg } m\, s^{-2} = 9{\cdot}81 \text{ newtons}$$

To convert a physical quantity from *gravitational units to absolute units,* multiply by the value of acceleration due to gravity in the given system. (9·81 $m\, s^{-2}$ in *MKS* system and 981 $cm\, s^{-2}$ in *C.G.S.* system)

Ex. FM–12. A bus moving at a speed of 20 m/s begins to slow at a rate of 3 m/s each second. Find how far it goes before stopping.

Sol. Here, Initial speed of bus, $u = 20\ m\, s^{-1}$

Final speed of bus, $v = 0$

Retardation, $a = -3\ m/s^2$

Let, Distance travelled before stopping, $S = ?$

Using the relation, $v^2 - u^2 = 2\,a\,S$

$$\therefore \quad 0^2 - (20)^2 = 2 \times -3 \times S$$

$$\therefore \quad S = \frac{400}{6} = 66{\cdot}67 \text{ metres (Ans.)}$$

Ex. FM-13. A force of 0·03 N acts on a mass of 400 gram for 10 seconds. Find the velocity and momentum of body after 10 seconds.

Sol. $\quad F = 0{\cdot}03\ N,\ m = 400g = \frac{400}{1000} = 0{\cdot}4 \text{ Kg},\ t = 10s$

We know that, $\quad F = ma \Rightarrow a = \frac{F}{m}$

$$a = \frac{0 \cdot 03}{0 \cdot 4} = 0{\cdot}075 \text{ m/s}^2$$

Also Initial velocity $u = 0$, as body starts from rest.

$$v = u + at$$

$$v = 0 + 0{\cdot}075 \times 10 = 0{\cdot}75 \text{ m/s}$$

Momentum after 10 seconds $= mv = 0{\cdot}4 \times 0{\cdot}75 = 0{\cdot}3$ Kg ms^{-1} **Ans.**

Ex. FM-14. A car of mass 20 quintals is moving with a velocity of 60 km/hr. By appicaltion of brakes it is brought to rest in a distance of 3 km. Find the average force of resistance in Newtons.

Sol. Given $m = 20$ quintals $= 20 \times 100 = 2000$ Kg

$$u = 60\,km\,/\,h = \frac{60 \times 1000}{60 \times 60} = \frac{50}{3}\,m\,/\,s$$

$v = 0$, as car in brought to rest.

$S = 3\text{Km} = 3000m, \quad a = ?$

We know that, $v^2 - u^2 = 2as$

$$\Rightarrow \qquad a = \frac{v^2 - u^2}{2s} = \frac{(0)^2 - \left(\frac{50}{3}\right)^2}{2 \times 3000} = -\frac{5}{108}\,m\,/\,s^2$$

Then retarding force $= F = ma = 2000 \times \frac{5}{108} = 92 \cdot 6N$ **Ans.**

FM–12. INERTIA :

It is defined as the inability of a material body to change by itself, its state of rest or of uniform motion in a straight line.

Inertial Mass. *Inertial mass is the ratio of net force acting on an object to the acceleration it produces.*

If a force F acting on a body produces an acceleration a, then inertial mass m is given as,

$$m_i = \frac{F}{a}, \quad \text{but} \quad a = \frac{dv}{dt}$$

$$\therefore \qquad m_i = \frac{F}{dv\,/\,dt} = \frac{F \times dt}{dv}$$

In general inertial mass is the measure of inertia of the body. A large force is needed to speed up or slow down a body of large mass. Thus if mass of a body is large, we say that the body has large *inertia*. If mass is small, we say that inertia of the body is small.

If same force accelerates two bodies of inertial masses m_1 and m_2 through a_1 and a_2 respectively, then

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Thus, *the ratio of inertial masses of two bodies is equal to the inverse ratio o acceleration produced in the two bodies by the same* force.

FM–13. FORMS OF INERTIA :

Inertia may show itself in any of the following forms ;

1. Inertia of Rest : *The property by virtue of which a material body at res tends to remain at rest is called inertia of rest.*

Examples of Inertia of Rest :

(i) Passengers standing in a bus at rest, experience a jerk in the backwar direction when the bus suddenly starts moving. The reason is that when the bus is a rest, the whole body of a passenger is at rest. But when the bus suddenly starts it motion, the lower part of the body shares the motion of the bus, but the upper part i a tendency to remain at rest gets a jerk in the backward direction.

(ii) The dust particles inside a woollen blanket fall off when it is beaten by stick, because by beating, the blanket is suddenly set in motion, the particles tend t remain at rest and get detached.

(iii) If a stone is thrown on a window pane, the glass is broken into piece where as a high speed bullet makes a clean hole on passing through the glass. It i because the particles of glass near the hole cannot share the very quick motion of th bullet and remain undisturbed.

2. Inertia of Motion. *The property by virtue of which a material body motion tends to remain in its state of uniform motion is called inertia of motion.*

Examples of Inertia of Motion :

(i) The passengers sitting in a moving bus or train experience a sudden jerk the forward direction, when it stops suddenly. In this case the lower part of the bo of the passenger is suddenly brought to rest as it is in contact with the bus while t upper part still continues to move due to inertia of motion.

(ii) A man jumping out of a moving bus falls with his head forward, because jumping his feet suddenly come to rest on touching the ground, whereas the upp part of the body continues to move forward due to inertia of motion.

(iii) Before taking a long jump an athlete runs for some distance in order th the inertia of motion of his body at the time of jumping may help him to ju through a longer distance.

3. Inertia of Direction. *The property by virtue of which a material body te to maintain its direction is called* **inertia of direction.**

Examples of Inertia of direction :

(i) When moving car suddenly takes a turn at a bend, the passengers experien a push in the outward direction. It is due to the reason that the passengers tend maintain their direction along the original motion due to inertia of direction.

(ii) The mud particles get detached tangentially from the fast moving wheels a cycle and move in a straight line.

Imp.

FM-14. RESOLUTION OF FORCES.

The process of splitting a single force into two forces (components) is called resolution of force. The two parts are known as the resolved parts or **Components of the Force.**

Consider a block being pulled by a constant force F, making an angle θ with the horizontal. The force F can be resolved into two components :

(i) The horizontal component (F_z) tending to displace the block.

(ii) The vertical component (F_x) tending to lift the block or to decrease its weight.

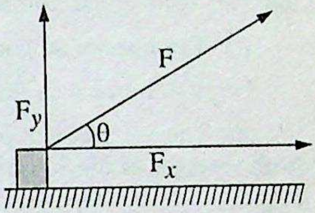


Fig. FM—9

When the force is resolved into two components at right angles to each other, the components are called **rectangular components.**

Resolution of a force into rectangular components :

Let a force F act in a direction OA, as shown in fig FM-10. The rectangular components of force F are represented by the projections OB and OC along X-axis and Y-axis respectively.

$$\therefore \quad F_x = OB \quad \text{and} \quad F_y = OC$$

In the right angled triangle OBA,

$$\frac{OB}{OA} = \cos\theta$$

or
$$\frac{F_x}{F} = \cos\theta$$

$$\because \quad F_x = F\cos\theta$$

Similarly,
$$\frac{AB}{OA} = \sin\theta$$

or
$$\frac{AB}{OA} = \frac{OC}{OA} = \sin\theta \qquad [\because \text{ In rectangle } OBAC,\ AB = OC]$$

or
$$\frac{F_y}{F} = \sin\theta$$

$$\therefore \quad \mathbf{F_y = F\sin\theta.}$$

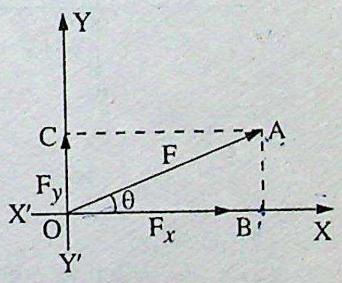


Fig. FM—10

Therefore, rectangular components of force F are $F\cos\theta$ and $F\sin\theta$.

The effect of these two components is the same as that of the force F.

In general *the component of a force along any direction is equal to the product of the force and the cosine of the angle which the force makes with that direction.*

Thus component of F along $OX = F \times$ cosine θ.

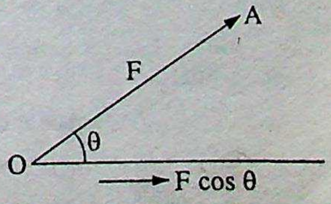


Fig. FM—11

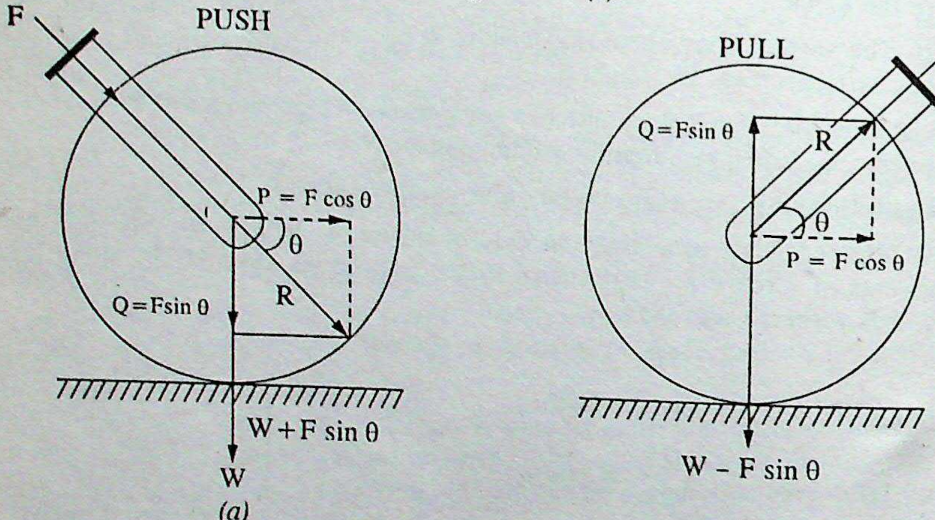
FM–15. IT IS EASIER TO PULL THAN TO PUSH A LAWN ROLLER.

The working of a lawn roller provides an interesting example of resolution of forces. When a lawn roller is **pushed** obliquely, the force of push F can be resolved into two rectangular components :

(i) The horizontal component, $P = F \cos \theta$

helps to move it in the forward direction.

(ii) The vertical component $Q = F \sin \theta$ acts in the downward direction and adds to the weight of the roller as shown in fig. FM-12(a).



(a) *(b)*

Fig. FM—12

Fig. 12.

When the roller is **pulled,** the horizontal component helps to move it in the forward direction as shown in Fig. FM–12 *(b)*. the vertical component $Q = F \sin \theta$ acts in the upward direction and decreases the net weight of the roller.

Thus we find that while pushing, the net weight of the roller increases and the force of friction between the roller and the ground increases, therefore, it appears to be heavier. While pulling, the net weight of the roller decreases and the force of friction decreases, therefore, it appears to be lighter.

Thus it is easier to pull than to push a lawn roller.

Ex. FM-15. The horizontal component of a force making an angle of 30° with the horizontal is 10 N. Find the force and its vertical component.

Sol. Let the force be F and θ be the angle which the force makes with the horizontal.

$\therefore$ Horizontal component, $F_x = F \cos \theta$

Here, $F_x = 10\,N$ and $\theta = 30^\circ$

$$\therefore \quad 10\,N = F \times \cos 30^\circ = F \times \frac{\sqrt{3}}{2}$$

or $$F = \frac{10 \times 2}{\sqrt{3}} N = 6 \cdot 66 \times \sqrt{3}\ N = \mathbf{11 \cdot 14}\ N\ \textbf{Ans.}$$

Now, Vertical component, $F_y = F \sin\theta = \frac{20}{\sqrt{3}} \times \sin 30$

$$= \frac{20}{\sqrt{3}} \times \frac{1}{2} = 3 \cdot 33 \times \sqrt{3}\ N = \mathbf{5 \cdot 735}\ N\ \textbf{Ans.}$$

Ex. FM–16. Determine the horizontal force required to displace a mass of 9 kg suspended by a string, until the string makes an angle of 30° with the vertical.

Sol. The ball is under the action of three forces ;

(i) Weight of the ball, $W = M_g = 9 \times 9{\cdot}81 = 88{\cdot}29\ N.$

(ii) Tension in the string, $T = ?$

(iii) Horizontal Force, $F = ?$

Resolving tension, T into rectangular components, we get,

Horizontal component $= T \cos 60°$

and Vertical component $= T \sin 60°$

Now $T \sin 60° = W = mg = 9 \times 9{\cdot}81\ N.$

$$\therefore \quad T = \frac{9 \times 9 \cdot 81}{\sqrt{3}/2} = \frac{9 \times 9 \cdot 81 \times 2}{\sqrt{3}} N$$

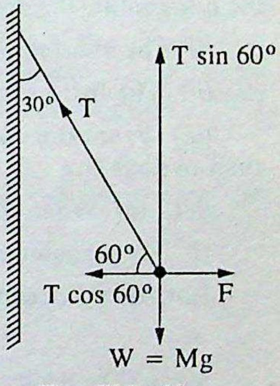


Fig. FM—13

Again Horizontal force, $F = T \cos 60° = \frac{9 \times 9 \cdot 81 \times 2}{\sqrt{3}} \times \frac{1}{2}$

$$= \mathbf{62 \cdot 44}\ N\ \textbf{Ans.}$$

Ex. FM–17. A mass M is suspended by a string and then pulled by a horizontal force F such that the string carrying the mass makes an angle θ with the vertical. Show that :

$$\frac{\mathbf{F}}{\mathbf{Mg}} = \mathbf{\tan\theta}.$$

Sol. Here, the mass is in equilibrium under the action of three forces :

(i) Tension in the string $= T$

(ii) Weight, $W = M_g$

(iii) Horizontal force $= F$

Angle which the string makes with the vertical $= \theta$

Resolving tension T into two components ;

(i) Verical component, $= T \cos\theta$

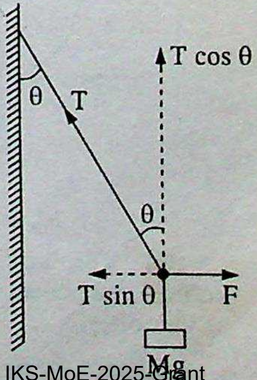


Fig. FM—14

and *(ii)* Horizontal component $= T \sin \theta$

In Equilibrium,

Horizontal force, $F = T \sin \theta$ ...*(i)*

and Vertically downward, weight, $M_g = T \cos \theta$ ...*(ii)*

Dividing *(i)* by *(ii)* we get

$$\frac{F}{Mg} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta$$

Thus $$\frac{F}{Mg} = \tan \theta.$$

Ex. FM–18. The handle of a 100 kg lawn roller makes an angle of 40° with the horizontal. If a force of 60 N is applied to the handle, Calculate :

***(i)* The horizontal and vertical components of this force.**

***(ii)* The force exerted by the roller on the ground.**

***(iii)* From the above calculations justify that it is easier to pull a lawn roller than to push it.**

Sol. *(a)* When the roller is pulled.

(i) The magnitudes of the two components F_x and F_y are calculated as,

Horizontal component, $F_x = F \cos \theta = 60\ N \times \cos 40° = 60 \times 0{\cdot}766$
$= 45{\cdot}96\ N$

Vertical component, $F_y = F \sin \theta = 60\ N \times \sin 40° = 60 \times 0{\cdot}643$
$= 38{\cdot}58\ N.$

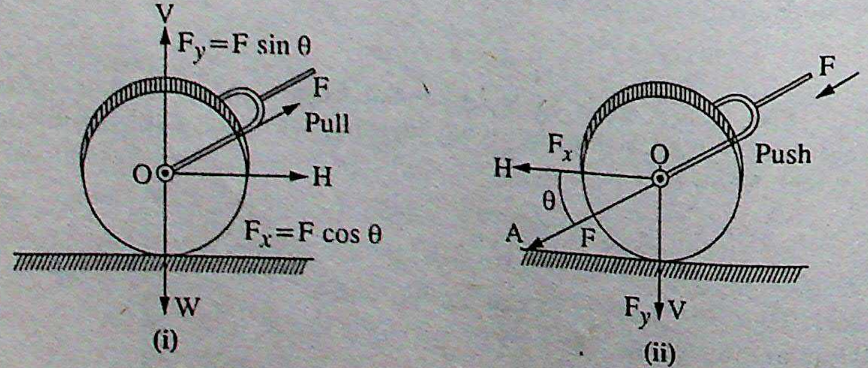


Fig. FM—15.

(ii) The horizontal component causes the motion of the roller, whereas, the vertical component $F_y = 38{\cdot}58\ N$ acting in the upward direction decreases the weight of the roller.

∴ Force exerted by the roller on the ground $= W - F_y$

$= 100 \times 9{\cdot}81 - 38{\cdot}58 = 942{\cdot}42\ N.$ **Ans.**

(b) When the roller is pushed.

(i) The magnitudes of two components F_x and F_y are ;

$$F_x = 60\,N \times \cos 40° = 60 \times 0{\cdot}766 = 45{\cdot}96\,N$$

$$F_y = 60\,N \times \sin 40° = 60 \times 0{\cdot}643 = 38{\cdot}58\,N \text{ (downward)}$$

(ii) Again the horizontal component causes the motion of the roller and the vertically downward component increases the weight of the roller.

∴ Force exerted by roller on the ground $= W + F_y$

$$= 100 \times 9{\cdot}81 + 38{\cdot}58 = 1019{\cdot}58\,N. \text{ Ans.}$$

(iii) The calculations for the force exerted by the roller on the ground show that when the roller is pushed, it appears to be heavier than when it is pulled.

Therefore, it is easier to pull a lawn roller than to push it.

FM–16. PARALLELOGRAM LAW OF VECTORS.

Parallelogram law of vectors states that, "*If two vectors acting simultaneously at a point, can be represented completely in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely in magnitude and direction by the diagonal of the parallelogram passing through that point.*"

Expression for the Resultant Vector :

Let two vectors $\vec{P}$ and $\vec{Q}$ are completely represented by two sides *OA* and *OB* respectively of parallelogram *OACB*. Then according to parallelogram law of vectors, the diagonal *OC* of the parallelogram will give the resultant $\vec{R}$, so that $\vec{R} = \vec{P} + \vec{Q}$

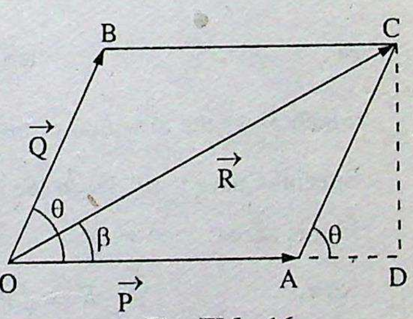


Fig. FM—16

Let θ be the angle between the vectors $\vec{P}$ and $\vec{Q}$. From *C*, draw a perpendicular *CD* on *OA* (produced). In the right–angled Δ *ADC*,

$$\sin\theta = \frac{CD}{AC} \quad \text{or} \quad CD = AC\sin\theta$$

or

$$CD = Q\sin\theta \qquad [\because AC = OB = Q] \qquad \ldots(i)$$

Also,

$$\cos\theta = \frac{AD}{AC} \quad \text{or} \quad AD = AC\cos\theta$$

or

$$AD = Q\cos\theta$$

Now,

$$OD = AO + AD = P + Q\cos\theta \qquad \ldots(ii)$$

In right–angled Δ *ODC*

$$OC^2 = \mathrm{OD}^2 + \mathrm{CD}^2$$

or $$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \qquad \text{(From } (i) \text{ and } (ii))$$

or $$R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$
$$= P^2 + Q^2\cos^2\theta + Q^2\sin^2\theta + 2PQ\cos\theta$$
$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

∴ $$R^2 = P^2 + Q^2 + 2PQ\cos\theta \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

or $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots(iii)$$

which is the required expression for the magnitude of the resultant of two vectors $\vec{P}$ and $\vec{Q}$ inclined to each other at an angle θ.

Direction of the resultant vector :

Let β be the angle which the resultant $\vec{R}$ makes with $\vec{P}$.

Then, $$\tan\beta = \frac{CD}{OD}$$

or $$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

or $$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

which gives the direction of the resultant vector.

Special Cases : *(i) When the given vectors* $\vec{P}$ *and* $\vec{Q}$ *act in the same direction.*

In this case, $\theta = 0°$

Now, $$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ}$$
$$= \sqrt{P^2 + Q^2 + 2PQ} \qquad [\because \cos 0^\circ = 1]$$
$$= \sqrt{(P+Q)^2} = P + Q$$

So, the magnitude of the resultant vector is equal to the sum of the magnitudes of the given vectors.

Also, $$\tan\beta = \frac{Q\sin 0^\circ}{P + Q\cos 0^\circ}$$

or $$\tan\beta = 0$$

∴ $$\beta = 0^\circ \qquad [\because \sin 0^\circ = 0]$$

Thus the resultant vector acts in the direction of the given vectors.

(ii) When the given vectors $\vec{P}$ *and* $\vec{Q}$ *act at right angles to each other.*

In this case, $\theta = 90°$

$\therefore$ $$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

or $$R = \sqrt{P^2 + Q^2} \qquad [\because \cos 90° = 0]$$

Also, $$\tan\beta = \frac{Q\sin 90^\circ}{P + Q\cos 90^\circ}$$

or $$\tan\beta = \frac{Q}{P} \qquad [\because \sin 90° = 1]$$

or $$\beta = \tan^{-1}\left(\frac{Q}{P}\right)$$

(iii) When the given vectors $\vec{P}$ and $\vec{Q}$ act in opposite directions.

In this case, $\theta = 180°$

$\therefore$ $$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ}$$

$$= \sqrt{P^2 + Q^2 - 2PQ} \qquad [\because \cos 180° = -1]$$

$$= \sqrt{(P-Q)^2}$$

$\therefore$ $$R = \pm (P - Q) = P - Q$$

So, the magnitude of the resultant vector is equal to the difference of the magnitudes of the given vectors.

Also, $$\tan\beta = \frac{Q\sin 180^\circ}{P + Q\cos 180^\circ} = 0 \qquad (\because \sin 180° = 0)$$

or $$\tan\beta = 0$$

$\therefore$ $$\beta = 0° \quad \text{or} \quad 180°$$

When $|\vec{P}| > |\vec{Q}|$, then $\beta = 0°$

When $|\vec{P}| < |\vec{Q}|$, then $\beta = 180°$

To sum up, the resultant vector acts in the direction of the greater of the two vectors.

Ex. FM–19. An airplane is travelling Eastward at airspeed of 500 km/hr. But a 90 km/hr wind is blowing Southward. What are the direction and speed of the plane relative to ground ?

Sol. Here, Velocity of plane, $U = 500$ *km/hr*

Velocity of wind, $V = 90$ *km/hr.*

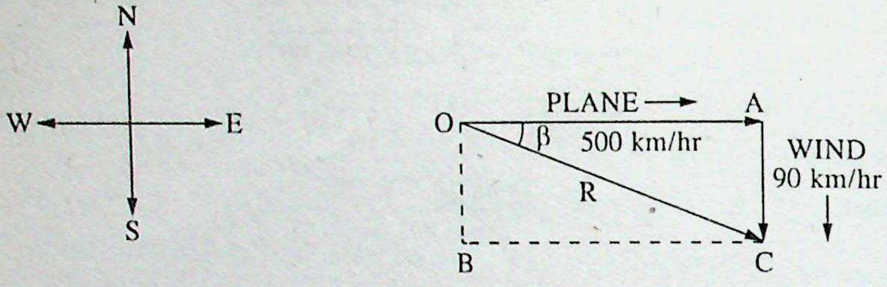


Fig. FM–17

Since the two are acting at right angles to each other, therefore, $\theta = 90°$.

Resultant velocity R is given by the relation,

$$R = \sqrt{U^2 + V^2}$$

$$\therefore \quad R = \sqrt{(500)^2 + (90)^2} = \sqrt{258100} = 508{\cdot}3 \ km/hr.$$

$\therefore$ Speed of plane $= 508{\cdot}3$ *km/hr* **(Ans.)**

Let the plane makes an angle β with the East

$$\therefore \quad \tan\beta = \frac{90}{500} = \frac{0\cdot 9}{5} = 0\cdot 1800$$

$$\therefore \quad \beta = 10°\ 12'.$$

Thus the plane moves towards South–East making an angle of 10° 12′ with the East.

FM–17. RESULTANT FORCE.

If a number of forces are acting simultaneously on a body, then a single force which can produce the some effect in the body as is done by all the forces together is called **resultant force.** It is represented by R.

Composition of forces. *The process of finding the resultant of a number of forces acting on a body is called* **composition of forces.**

Parallelogram method. Composition of forces by this method is based upon parallelogram law of forces.

Parallelogram law of forces. *If two forces acting simultaneously on a body can be represented completely in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely in magnitude and direction by the diagonal of the parallelogram passing through the same point.*

Let P and Q the two forces acting simultaneously on a particle be represented in magnitude and direction by sides OA *and* OB of a parallelogram inclined at an angle θ. Complete the parallelogram $OACB$ and join OC. Then the resultant force R is represented in magnitude and direction by the diagonal OC of the parallelogram.

To find the magnitude of the resultant R, draw CD perpendicular on OA produced. Side AC of the parallelogram completely represents the force Q. (fig. FM–18.)

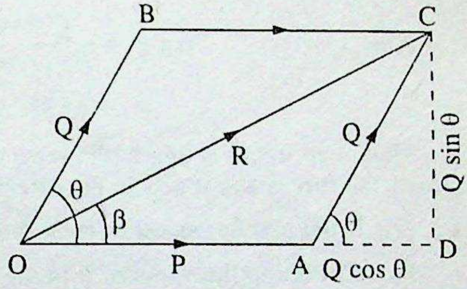


Fig. FM—18

$\therefore$ In triangle ACD, we have

$$CD = AC \sin\theta = Q \sin\theta$$

and $$AD = AC \cos\theta = Q \cos\theta$$

In the right angled triangle OCD

$$\begin{aligned} OC^2 &= OD^2 + CD^2 \\ &= (OA + AD)^2 + CD^2 \\ &= OA^2 + AD^2 + 2\,OA.AD + CD^2 \\ &= P^2 + Q^2\cos^2\theta + 2\,PQ\cos\theta + Q^2\sin^2\theta \\ &= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2\,PQ\cos\theta \end{aligned}$$

or $$R^2 = P^2 + Q^2 + 2\,PQ\cos\theta$$

$\therefore$ The resultant force, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Direction of resultant. If β is the angle which the resultant R makes with the direction of the force P, then

$$\tan\beta = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

or $$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

The composition of two forces by the parallelogram law is illustrated by the following example.

Suppose two persons walking along the bank of river pull a boat by applying forces P and Q acting along OA and OB respectively. The boat will neither move along OA, nor along OB but will move along OC in the direction of the resultant force R as shown in fig. FM–19.

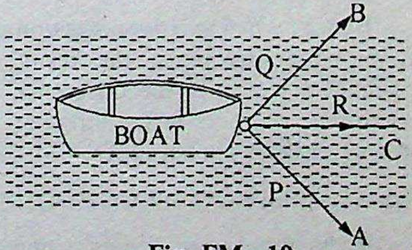


Fig. FM—19

Different Cases. *(i) When two forces are acting in the same direction i.e.,* $\theta = 0°$.

Then, substituting the value of θ in the expression for resultant.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} \qquad [\cos 0^\circ = 1]$$

$\therefore$ $$\mathbf{R = P + Q.}$$

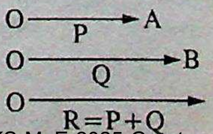


Fig. FM—20

Also, $$\tan\beta = \frac{Q\sin 0^\circ}{P+Q\cos 0^\circ} = 0$$

$$\therefore \quad \beta = 0^\circ$$

Thus, two forces acting in the same direction, have resultant equal to the algebraic sum of the forces and it acts in the direction of the forces.

(ii) When two forces act at right angles to each other i.e., $\theta = 90^\circ$

By substituting the value of θ in the expression for resultant,

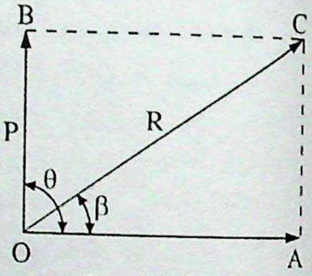


Fig. FM—21

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$\therefore \quad \mathbf{R} = \sqrt{P^2 + Q^2} \qquad [\because \cos 90^\circ = 0]$$

$$\tan\beta = \frac{CA}{OA} = \frac{Q}{P}$$

(iii) When two forces act in the opposite direction i.e., $\theta = 180^\circ$.

By substituting the value of θ in the expression for resultant.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ}$$

$$= \sqrt{P^2 + Q^2 + 2PQ(-1)}$$

$$\therefore \quad \mathbf{R = P - Q}$$

also $$\tan\beta = \frac{Q\sin 180^\circ}{P+Q\cos 180^\circ} = 0$$

$$\therefore \quad \beta = 0$$

Hence, when two forces act on a body in opposite directions, the resultant the resultant is equal to the difference of the two forces and acts in the direction of the greater force as shown in fig. FM-22.

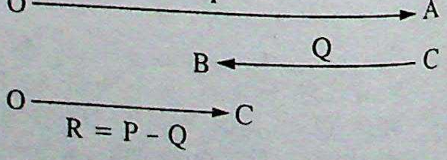


Fig. FM—22

FM–18. METHOD OF COMPONENTS.

If the number of forces acting on a body is more than two, the parallelogram method becomes difficult. In such cases method of components is employed and is suitable for any number of forces.

To find the resultant by this method, consider three forces F_1, F_2 and F_3 acting simultaneously on a body making angles θ_1, θ_2 and θ_3 respectively with the positive direction of X-axis.

Let each force be resolved into rectangular components along X-axis and Y-axis.

$\therefore$ Components along X-axis are $F_1\cos\theta_1$, $F_2\cos\theta_2$ and $F_3\cos\theta_3$. Similarly components along Y-axis are $F_1\sin\theta_1$, $F_2\sin\theta_2$ and $F_3\sin\theta_3$.

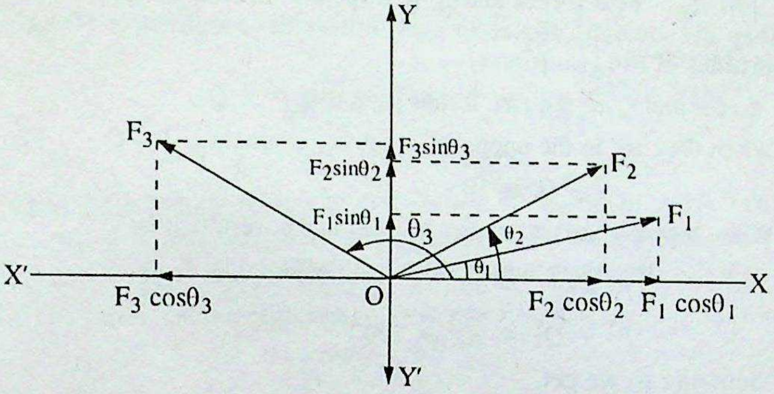


Fig. FM—23.

Find F_x the algebraic sum of all the components along X-axis.

$$\therefore \quad F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + \ldots\ldots$$

Also, *find F_y the algebraic sum of all the components along Y-axis.*

$$\therefore \quad F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + \ldots\ldots$$

Thus, the forces F_1, F_2, F_3 etc. get replaced by two forces F_x and F_y at right angles to each other as shown in fig FM-24.

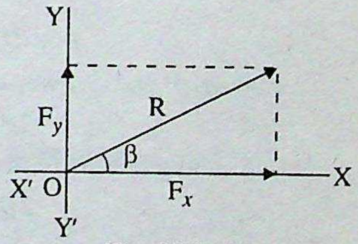


Fig. FM—24

Now the resultant R of F_x and F_y is given as,

$$R = \sqrt{F_x^2 + F_y^2} \quad \ldots(i)$$

The angle β which the resultant makes with the X-axis is given by the relation,

$$\tan\beta = \frac{F_y}{F_x} \quad \ldots(ii)$$

Hence the magnitude and direction of resultant can be calculated.

Ex. FM-20. Two equal forces have a resultant equal to either of them. At what angle are they inclined ?

Sol. Let P and Q be the two forces, such that $P = Q$. If R is the resultant,

then $\quad P = Q = R$

Let θ be the angle between the forces P and Q.

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\text{or} \quad R^2 = R^2 + R^2 + 2R^2\cos\theta$$

$$\text{or} \quad 2\cos\theta = -1$$

$$\text{and} \quad \cos\theta = -\frac{1}{2} = -\cos 60° = \cos(180° - 60°) = \cos 120°$$

$$\therefore \quad \theta = 120° \text{ Ans.}$$

Ex. FM-21. Two forces acting in opposite directions have a resultant of 10 kgf. If they act at right angles to each other, the resultant is 50 kgf. Calculate the magnitudes of the two forces.

Sol. Let P and Q be the two forces such that $P > Q$

(i) When they act in the opposite direction, resultant is $P - Q$

$$\therefore \quad P - Q = 10 \qquad \ldots(i)$$

(ii) When they act at right angles to each other, resultant is

$$\sqrt{P^2 + Q^2} = 50$$

$$\therefore \quad P^2 + Q^2 = 50^2 = 2500 \qquad \ldots(ii)$$

∴ Squaring *(i)* we get,

$$P^2 + Q^2 - 2PQ = 100 \qquad \ldots(iii)$$

Subtracting *(iii)* from *(ii)*, we get,

$$2PQ = 2500 - 100 = 2400$$

or $$PQ = 1200$$

Now $$P + Q = \sqrt{(P-Q)^2 + 4PQ}$$

or $$P + Q = \sqrt{(10)^2 + 4 \times 1200} = \sqrt{4900}$$

$$\therefore \quad P + Q = 70 \qquad \ldots(iv)$$

Adding *(i)* and *(iv)* we, get,

$$2P = 80 \qquad \text{or} \qquad P = 40\ kgf$$

and subtracting *(i)* from *(iv)*, we get,

$$2Q = 60 \qquad \text{or} \qquad Q = 30\ kgf$$

Hence, $$\left.\begin{array}{l} P = 40\ kgf \\ Q = 30\ kgf \end{array}\right\} \ \textbf{(Ans.)}$$

Ex. FM-22. Two forces of 60 N and 40 N are inclined to each other at an angle of 60°. Find their resultant.

Sol. Here, force, $F_1 = 60\ N$

and force, $F_2 = 40\ N$

Angle between the forces, $\theta = 60°$

Resultant, $R = ?$

Using the expression for resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{(60)^2 + (40)^2 + 2 \times 60 \times 40 \times \cos 60^\circ}$$

$$= \sqrt{3600 + 1600 + 2400} = \sqrt{7600}$$

$$= \mathbf{86{\cdot}1\ N\ (Ans.)}$$

Let the resultant makes an angle β with the force of 60 *N*.

$$\therefore \quad \tan\beta = \frac{40\sin 60^\circ}{60+40\cos 60^\circ} = \frac{40\times\cdot 8660}{60+40\times 0\cdot 5} = 0\cdot 4330$$

$$\therefore \quad \beta = 23^\circ - 24' \text{ (Ans.)}$$

Ex. FM–23. If four forces of 5 N acting 30° north of west, 7 N acting 30° west of north, 8 N due west and 4 N due south act at a point, determine their horizontal and vertical components and hence their resultant.

Sol. The forces can be represented as in fig. FM–25.

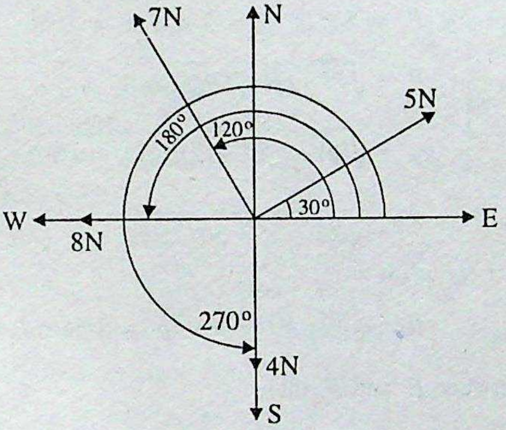


Fig. FM—25

If F_x is the sum of the components of all the force along X–axis and F_y is the sum of all the components along Y–axis, then

$$\text{Horizontal component, } F_x = 5\,N\times\cos 30^\circ + 7\,N\times\cos 120^\circ + 8\,N \times\cos 180^\circ + 4\,N\times\cos 270^\circ$$

$$= 5\times 0\cdot 8660 + 7\times -0\cdot 5 + 8\times -1 + 4\times 0$$

$$= 4\cdot 33 - 3\cdot 5 - 8$$

$$= -7\cdot 17\,N \text{ or } 7\cdot 17\,N \text{ due west } \textbf{(Ans.)}$$

$$\text{Vertical component, Fy} = 5\times\sin 30^\circ + 7\times\sin 120^\circ + 8\times\sin 180^\circ + 4\times\sin 270^\circ$$

$$= 5\times 0 + 7\times 0\cdot 8660 + 8\times 0 + 4\times -1$$

$$= 2\cdot 5 + 6\cdot 062 + 0 - 4$$

$$= 4\cdot 562\text{ N} \quad\text{or}\quad 4\cdot 562\text{ N due north } \textbf{(Ans.)}$$

$$\text{Resultant,} \quad R = \sqrt{F_x^2+F_y^2} = \sqrt{(7\cdot 17)^2+(4\cdot 562)^2}$$

$$= \sqrt{51\cdot 21+20\cdot 81} = \sqrt{72\cdot 02}$$

$$= \mathbf{8\cdot 48}\ N\ \textbf{(Ans.)}$$

If the resultant makes an angle β with the horizontal, then

$$\tan\beta = \frac{4\cdot 562}{-7\cdot 17} = -0\cdot 6363$$

∴ $$\beta = \tan^{-1}(-0\cdot 6363) = -147^\circ 32'$$

or $$\beta = 147^\circ 32' \text{ or } 32^\circ 28' \text{ west of north.}$$

Hence resultant force is 8·48 N and acts at an angle 32° 28′ west of north.

Ex. FM-24. Two equal velocities each of 5 m/s are inclined to each other at 60°. Find the magnitude and direction of their resultant.

Sol. Given, $$P = 5 \text{ m/s}, Q = 5 \text{ m/s}, \theta = 60^\circ$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\,\text{Cos}\theta}$$

$$= \sqrt{(5)^2 + (5)^2 + (2)(5)(5)\cos 60^\circ}$$

$$= \sqrt{25 + 25 + 50\,\text{Cos}\,60^\circ} \qquad \left[\text{Cos}\,60^\circ = \frac{1}{2}\right]$$

$$R = \sqrt{50 + 50\cdot\frac{1}{2}} = \sqrt{75} = 8\cdot 66 \text{ m/s}$$

It β is angle between R and P, then,

$$\tan\beta = \frac{Q\text{Sin}\theta}{P + Q\text{Cos}\theta} = \frac{5\text{Sin}\,60^\circ}{5 + 5\text{Cos}\,60^\circ} = \frac{5\times 0\cdot 866}{5 + 5\times\frac{1}{2}} = 0\cdot 5773$$

$$\tan\beta = 0\cdot 5773$$

⇒ $$\beta = \tan^{-1}(0\cdot 5773) = 30^\circ$$

Ex. FM-25. Resultant of two equal forces acting at right angle to each other is 1414 N. Find the magnitude of each force.

Sol. Given, $$P = Q, \quad \text{and } R = 1414\ N,\ \theta = 90^\circ$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\,\text{Cos}\theta}$$

$$1414 = \sqrt{P^2 + P^2 + 2P\cdot P\,\text{Cos}\,90^\circ}$$

$$1414 = \sqrt{2P^2 + 0}$$

⇒ $$1414 = \sqrt{2}\cdot P$$

$$P = \frac{1414}{\sqrt{2}} = 1000N$$

∴ Hence, $$P = Q = 1000\ N \text{ each } \textbf{Ans.}$$

FM–19. LAMI'S THEOREM.

It states that *when three forces acting at a point are in equilibrium, each of the forces is proportional to the sine of the angle enclosed between the other two forces.*

Proof. : Suppose P, Q and R are three forces in equilibrium acting at a point O. These forces can be represented by a triangle ABC, the side AB of which is parallel to R, BC parallel to P and CA parallel to Q.

$$\therefore \quad \frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB}$$

or $$\frac{P}{a} = \frac{Q}{b} = \frac{R}{c} \quad \ldots(i)$$

Where, a, b and c are the sides of the triangle opposite to the angles A, B and C respectively.

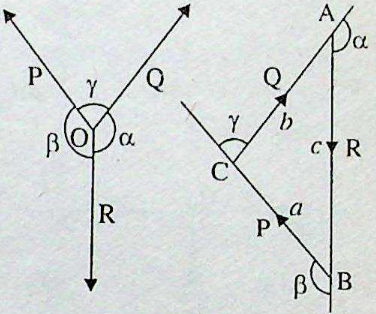


Fig. FM—26

In any triangle ABC, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(Sine formula)}$$

Now $$\angle A = 180° - \alpha, \quad \angle B = 180° - \beta \quad \text{and} \quad \angle C = 180° - \gamma$$

$$\therefore \quad \sin A = \sin(180° - \alpha) = \sin \alpha$$

$$\sin B = \sin(180° - \beta) = \sin \beta$$

$$\sin C = \sin(180° - \gamma) = \sin \gamma$$

$$\therefore \quad \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \quad \ldots(ii)$$

From *(i)* and *(ii)*, we get,

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Thus each force is proportional to the angle between the other two forces. This proves Lami's theorem.

Ex. FM–26. A body of mass 1 kg is suspended by two strings AC and BC of lengths 5 cm and 12 cm respectively from two hooks separated by 13 cm. Calculate the tension in the strings.

Sol. In ABC,

$$AB^2 = AC^2 + BC^2$$

$$\therefore \quad (13)^2 = (5)^2 + (12)^2 = 25 + 144$$

or $$169 = 169$$

$\therefore$ $\Delta\, ABC$ is a right angled Δ, with right angle at C.

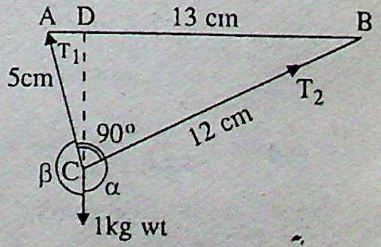


Fig. FM—27

Let T_1 and T_2 be the tensions in the strings AC and BC.

Since three forces T_1, T_2 and 1 *kg Wt*. are acting at *C* and are in equilibrium. Let the angle between T_1 and 1 *kg wt* be β and between T_2 and 1 *kg wt*. be α. Therefore, by Lami's Theorem,

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{1\ kg\ wt}{\sin 90^\circ}$$

$$\therefore \qquad T_1 = \frac{1\ kg\ wt}{\sin 90^\circ} \times \sin\alpha$$

$$= \frac{1}{1} \times \sin(180^\circ - \angle BCD) = 1 \times \sin(180^\circ - \angle A)$$

$$= 1 \times \sin\angle A = 1 \times \frac{12}{13} = 0\cdot 923\ kg\ wt.$$

Also,

$$T_2 = \frac{1\ kg\ wt}{\sin 90^\circ} \times \sin\beta$$

$$= \frac{1}{1} \times \sin(180^\circ - \angle ACD) = 1 \times \sin(180^\circ - \angle B)$$

$$= 1 \times \sin\angle B = 1 \times \frac{5}{12} = 0\cdot 4166\ kg\ wt$$

∴ Tension in string $AC = 0{\cdot}923\ kg\ wt$

and tension in string $BC = 0{\cdot}4166\ kg\ wt$ **(Ans.)**

Ex. FM–27. Find the tension in each cord for the system described in the figure shown.

Sol. Angle between cords. AC and $BC = 180 - (40 + 50) = 90°$

Now, $\angle\alpha = 180 - \angle A = 180° - 40° = 140°$

and $\angle\beta = 180 - \angle B = 180 - 50 = 130°$

By Lami's theorem, we have,

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{T_3}{\sin 90^\circ}$$

Now, $T_3 = 5\text{ kg} = 5 \times 9{\cdot}81\ N = 49{\cdot}05\ N$

$$\therefore \qquad T_1 = \frac{T_3}{\sin 90^\circ} \times \sin\alpha = \frac{49\cdot 05}{1} \times \sin 140^\circ$$

$$= 49{\cdot}05 \times 0{\cdot}6428 = 31{\cdot}53\ N \textbf{ Ans.}$$

and

$$T_2 = \frac{T_3}{\sin 90^\circ} \times \sin\beta = \frac{49\cdot 05}{1} \times \sin 130^\circ$$

$$= 49{\cdot}05 \times {\cdot}7660 = 37{\cdot}57\ N. \textbf{ Ans.}$$

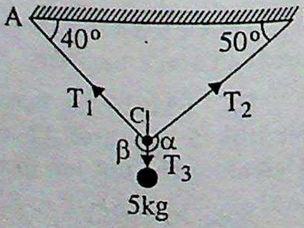


Fig. FM—28

Ex. FM–28. Find the horizontal force required to displace a mass of 30 kg suspended by means of a string from a hook in the wall so that the string makes an angle of 30° with the vertical. Also find the tension in the string.

Sol. Let F be the horizontal force required to displace the mass of 30 *kg*, so that the string makes an angle of 30° with the vertical.

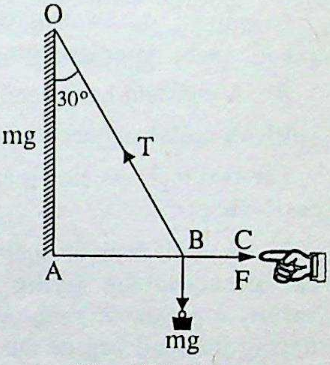


Fig. FM—29

The mass is in equilibrium under the action of three forces ;

(i) Force F acting horizontally.

(ii) The weight mg acting vertically downwards.

(iii) The tension T, acting along the string.

The angle between tension T and force

$$F = 180 - 60 = 120°$$

Angle between weight and tension

$$T = 180 - 30 = 150°$$

and angle between weight and force = 90°

∴ By Lami's theorem.

$$\frac{F}{\sin 150^\circ} = \frac{F}{\sin 90^\circ} = \frac{mg}{\sin 120^\circ}$$

∴ $$\text{Force, } F = \frac{mg}{\sin 120^\circ} \times \sin 150^\circ = \frac{30 \times 9 \cdot 81}{\sqrt{3}/2} \times \frac{1}{2}$$

$$= 277 \cdot 4 \ N \text{ (Ans.)}$$

FM–20. PROJECTILE.

An object thrown in the air with some initial velocity and then allowed to move under the effect of gravity is called a projectile.

When a ball is thrown in the air with some initial velocity along a straight line, it moves in a curved path.

The motion of the ball can be understood if we make the following assumptions.

(i) The acceleration due to gravity (g) is constant throughout the motion and is directed downwards.

(ii) The effect of air resistance is negligible, and

(iii) The rotation of earth does not effect the motion.

Under the above assumptions we find that the path of projectile (*trejectory*) is a *parabola*.

Trajectory. *The path described by a projectile is called its trajectory.*

A projectile may be thrown by giving two types of initial velocities :

(i) **Horizontal direction.** In this case projection is called *horizontal projection.*

(ii) **Inclined with the horizontal direction.** In this case projection is called *inclined projection.*

FM-21. PARABOLIC MOTION.

When a body simultaneously moves under the action of a uniform horizontal motion and a uniformly increasing or decreasing (accelerated) motion at right angles to it, the body describes a **parabolic path.** *the motion of such a body is called* **parabolic motion.**

Example. 1. A ball thrown horizontally from the top of a tower follows a parabolic path. After being thrown, the ball has simultaneously two motions :

(i) A constant horizontal motion given by horizontal throw.

(ii) A uniformly accelerated downward motion due to gravity.

The two motions acting at right angles to each other make the ball to move along a parabolic path.

2. A mail bag or a bomb dropped from an aeroplane flying horizontally describes a parabolic path. At the time of dropping the mail bag or the bomb moves in the horizontal direction with same velocity as that of the plane. Its velocity in the downward direction at this instant is zero, but goes on increasing due to the pull of gravity.

The horizontal uniform motion combines with vertically downward accelerated motion to make the mail bag move along a parabolic path (see fig. FM-30).

Thus during war an aeroplane drops the bomb a little earlier than reaching the top of the target so that it hits the target after following parabolic path.

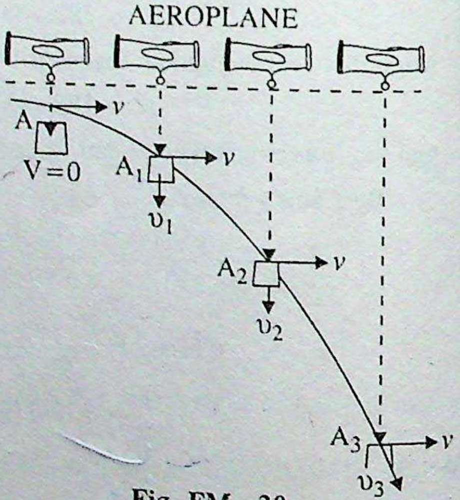


Fig. FM—30

3. The path of a bullet fired from a gun is parabolic (see fig. FM-31.)

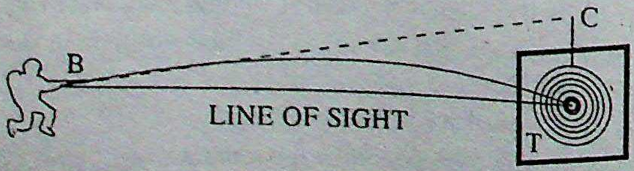


Fig. FM—31.

Thus, while shooting a target, a person must aim at a point higher than the target.

4. Water coming out of a horizontal pipe, of a tube-well falls on the ground after following a parabolic path.

5. A foot-ball kicked by a player falls on the ground after following parabolic path.

FM–22. GRAPHICAL DESCRIPTION OF PARABOLIC MOTION.

When a body is thrown horizontally from the top of a tower with uniform velocity v. The body will be under the action of two velocities.

The horizontal velocity produces its own effect in the horizontal direction, while the vertical velocity due to gravity produces downward motion. In other words, each velocity produces its own effect and resultant path traced by the body is due to their combined effect.

Suppose the body starts from point O in the horizontal direction with a uniform velocity v m/s. (See fig. FM–32.)

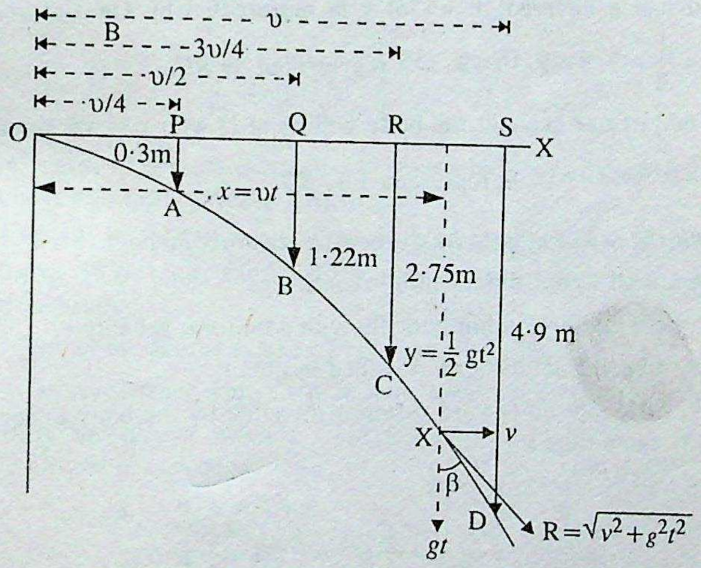


Fig. FM–32.

Position of body after $\frac{1}{4}$ th of a second :

Horizontal distance travelled by the body, $x = v \times \frac{1}{4} = v/4m$.

This distance is represented by OP.

Vertically downward distance travelled by the body,

$$y = ut + \frac{1}{2}gt^2 = 0 \times \frac{1}{4} + \frac{1}{2} \times 9 \cdot 8 \times \left(\frac{1}{4}\right)^2 = 0 \cdot 3\, m$$

This distance is represented by PA

Thus after $\frac{1}{4}$ second the body will be at A. [At point O, $u = 0$]

Position of body at the end of $\frac{1}{2}$ second :

Horizontal distance travelled, $x = \frac{1}{2} \times v = v/2\ m$

Vertical distance travelled, $y = 0 + \frac{1}{2} \times 9 \cdot 8 \times \left(\frac{1}{2}\right)^2 = 1 \cdot 225\ m$

The horizontal distance is represented by OQ and vertical distance by QB and the body is at point is B.

In a similar manner, the body will be at C at the end of 3/4th of a second. The horizontal distance covered, $x = 3/4\ v\ m$ represented by OR and vertical distance covered, $y = \frac{1}{2} \times 9 \cdot 8 \times 9/16 = 2 \cdot 755$ represented by RC.

At the end of one second, the body will be at D with $x = vm$ represented by OS and $y = \frac{1}{2} \times 9 \cdot 8 \times 1 = 4 \cdot 9\ m$ represented by SD.

The path OABCD followed by the body is a parabolic path.

Mathematical Proof for parabolic motion.

Let a body be thrown horizontally with a uniform velocity v.

Let x = horizontal distance travelled in time t.

and y = vertically downward distance travelled by the body under the action of gravity in the same time t.

Then,
$$x = vt \qquad \ldots(i)$$

and,
$$y = ut + \frac{1}{2}gt^2 = 0 \times t + \frac{1}{2}gt^2$$

[$u = 0$, for vertical velocity at O]

$\therefore$
$$y = \frac{1}{2}gt^2 \qquad \ldots(ii)$$

From equation *(i)*, $t = x/v$

Substituting the value of t in *(ii)*, we get,

$$y = \frac{1}{2}g\left(\frac{x}{v}\right)^2 = \frac{g}{2v^2}x^2$$

or
$$x^2 = \frac{2v^2}{g}y, \quad x^2 = ky. \qquad \ldots(iii)$$

Where, $k = \frac{2v^2}{g}$, as v and g are constant, equation *(iii)* is the equation of parabola. *Thus, the path followed by the body projected horizontally is a parabola.*

Resultant velocity. At any instant t, the resultant velocity has two components.

Let the body be at X, $\therefore$ At X,

(i) Horizontal velocity $= v$.

(ii) Vertical velocity $= u + gt = 0 + gt = gt$.

$$\therefore \quad \text{Resultant velocity} = \sqrt{v^2 + g^2t^2}$$

The direction of the resultant is along the tangent to the path at every instant. If β is the angle with the vertical at any instant, then,

$$\tan\beta = \frac{v}{gt}$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{v}{gt}\right)$$

As t increases, β decreases and approaches zero when t becomes very large. The body then begins to fall vertically downwards.

Ex. FM–29. A shot is fired horizontally with a velocity of 100 m/s from the top of a tower 50 m high. Find the distance from the foot of the tower where it strikes the ground.

Sol. Suppose the shot is fired from O. (see Fig. FM–33.)

Let, x be the distance from the foot of the tower, where the shot strikes the target.

Vertically downward motion. Let the shot reaches the ground in t seconds.

Now, $u = 0, h = 80\ m$

$g = 9{\cdot}8\ m\ s^{-2}$

As $h = ut + \frac{1}{2}gt^2$

$\therefore \quad 80 = 0 \times t + \frac{1}{2} \times 9{\cdot}8 \times t^2$

or, $t^2 = \frac{160}{9{\cdot}8}$

$\therefore \quad t = 5{\cdot}7$ seconds.

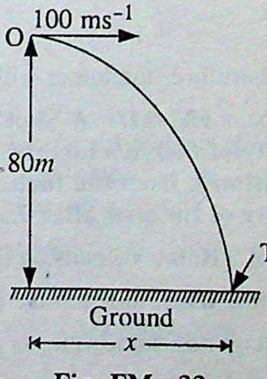


Fig. FM—33

Horizontal motion. During these 5·7 seconds, the shot will continue to move in the horizontal direction with a velocity of 100 m/s.

$\therefore$ Distance travelled by the shot in the horizontal direction.

$$x = v \times t = 100 \times 5{\cdot}7 = 570\ m \ \textbf{(Ans.)}$$

Thus, the shot strikes the ground at a distance of 570 m from the foot of the tower.

Ex. FM–30. A bomb is dropped from an aeroplane when it is directly above the target a height of 1500 m. The aeroplane is going horizontally with a speed of 500 km at hr. By how much distance will the bomb miss the target ? Given ($g = 9{\cdot}8\ m\ s^{-2}$)

Sol. When the bomb is dropped, its horizontal velocity is the same as that of the aeroplane *i.e.*, 500 $km/hr = \frac{1250}{9}\ m\ s^{-1}$ and its downward velocity is zero,

Due to downward uniform acceleration, $g = 9{\cdot}8\ m\ s^{-1}$, the bomb follows a parabolic path (Fig. FM–34) given by the equation.

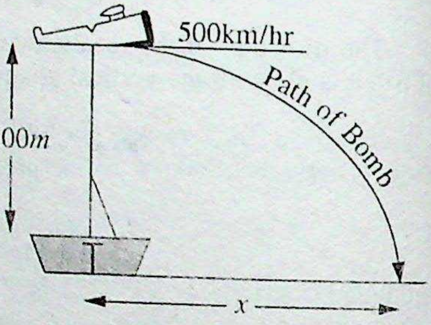


Fig. FM—34

$$x^2 = \frac{2\,v^2}{g} \times y$$

Here, $v = 500\ km/hr = \frac{1250}{9} m\ s^{-1}$

$g = 9{\cdot}8\ m\ s^{-2}$ and $y = 15{\cdot}00\ m$

Let, x be the distance by which the bomb misses the target.

$$\therefore \qquad x^2 = \frac{2 \times \left(\frac{1250}{9}\right)^2}{9{\cdot}8} \times 1500$$

or

$$x = \sqrt{\frac{2 \times 1500}{9{\cdot}8}} \times \frac{1250}{9} = 2430{\cdot}5\ m = 2{\cdot}431\ km$$

Therefore, the bomb will strike the ground 2·431 *km* away from the target.

Ex. FM–31. A shot fired horizontally from the top of a tower with a velocity of 500 m/s hits the ground in 3 seconds. What is the height of tower and the distance from the foot of the tower where it strikes the ground ? What is the velocity of the shot after 2 seconds ? ($g = 9{\cdot}80$ metres/sec^2.)

Sol. Here, Velocity in the horizontal direction, $u = 500\ m/s$

Time taken to reach the ground, $t = 3s$

Height of the tower y is given by the relation,

$$y = \frac{1}{2} g t^2 = \frac{1}{2} \times 9{\cdot}8 \times 9 = 44{\cdot}1\ m \text{ (Ans.)}$$

Horizontal range, x = the distance from the foot of the tower where the shot strikes

$$= ut = 500 \times 3 = 1500\ m$$

Velocity in the vertical direction after 2 seconds

$$v = u + gt = 0 + 9{\cdot}8 \times 2 = 19{\cdot}6\ m/s$$

$$\therefore \quad \text{Resultant velocity} = \sqrt{u^2 + g^2t^2}$$

$$= \sqrt{500^2 + 19 \cdot 6^2} = 500{\cdot}4 \ m/s \ \textbf{(Ans.)}$$

Let the shot makes an angle β with the vertical,

$$\therefore \quad \tan\beta = \frac{19\cdot 6}{500} = 0\cdot 0392$$

$$\text{or} \quad \beta = 2^\circ \ 15' \ \textbf{(Ans.)}$$

Ex. FM–32. A body is thrown horizontally from the top of a tower and strikes the ground after 5 seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was projected (g = 9·8 m/s)

Sol. Time taken by the body to strike the ground, $t = 5$ secs.

$$\therefore \quad \text{Height of the tower, } y = \frac{1}{2}gt^2 = \frac{1}{2} \times 9\cdot 8 \times 25 = 122\cdot 5 \ m$$

As the body starts from rest, the vertical component of the velocity v with which it strikes the ground is given by

$$v = gt = 9{\cdot}8 \times 5 = 49{\cdot}0 \ m/s$$

Since the body strikes the ground at an angle of 45°, therefore, the horizontal component u is given by,

$$\tan 45^\circ = \frac{v}{u} = 1$$

$$\text{or} \quad v = u$$

Hence, horizontal velocity, $u = 49{\cdot}0 \ m\ s^{-1}$ **Ans.**

Ex. FM–33. A mail bag is dropped into a post office from an aeroplane flying horizontally with a velocity of 60 km/hr at a height of 490 m above the ground. How far must the aeroplane be from the post office at the time of dropping the mail bag so that it may directly fall into post office.

Sol. Let x be the distance of the aeroplane from the post office at the time of dropping the mail bag.

Again let the mail bag falls through height of 490 m in t seconds. Now, during vertically downward motion, $u = 0$, $h = 490 \ m$ and $g = 9{\cdot}8 \ m\ s^{-2}$.

$$\therefore \quad h = ut + \frac{1}{2}gt^2$$

$$\text{or} \quad 490 = 0 \times t + \frac{1}{2} \times 9\cdot 8 \times t^2$$

$$\text{or} \quad t^2 = \frac{2 \times 490}{9\cdot 8} = \frac{490}{4\cdot 9} = 100$$

$$\text{Horizontal velocity}, v = 60 \ km/hr = \frac{60 \times 1000}{3600} = \frac{50}{3} \ m\ s^{-1}$$

Thus the bag will move in the horizontal direction for 10 s with a velocity of 50/3 $m\ s^{-1}$

$\therefore$ Distance of aeroplane from post office should be

$$= v \times t = \frac{50}{3} \times 10 = 166 \cdot 67\ m$$

FM–23. PROJECTILE MOTION (FIRED AT AN ANGLE WITH THE HORIZONTAL) :

A projectile thrown with velocity u making an angle θ with the horizontal follows a *parabolic path*.

Examples :

- *(i)* Throw of cricket ball.
- *(ii)* Firing of bullet from a gun.
- *(iii)* Foot–ball kicked by a player.
- *(iv)* Throw of javlin by an a athlete.

Path Described by Projectile :

A body projected from O with a velocity u making an angle θ with the horizontal is subjected simultaneously to a uniform horizontal motion and a uniform vertical acceleration. Under the action of these motions, the body describes a path (trajectory) OAB and hits at a point B in the same horizontal level as that of O.

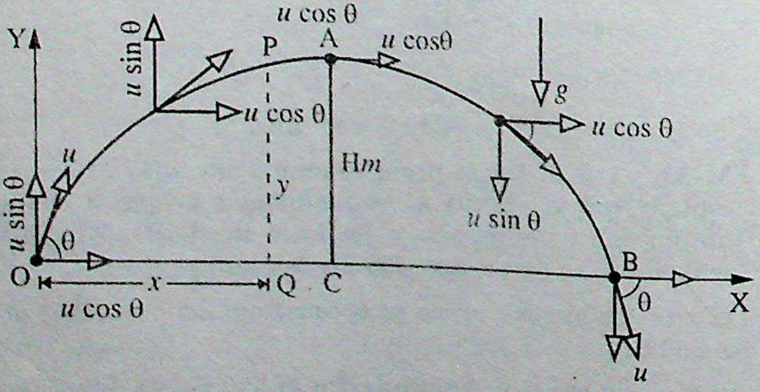


Fig. FM—35.

To study the motion of the projectile, resolve the initial velocity into two rectangular components along the horizontal and the vertical directions.

$\therefore$ Horizontal component along $OX = u \cos \theta$

Vertical component along $OY = u \sin \theta$

The horizontal component remains constant as there is no acceleration acting in the horizontal direction. The force of gravity acts in the vertically downward direction. The two components being at right angles, therefore, do not interfere with the motion due to each other.

The magnitude of vertical component goes on decreasing the becomes zero when the projectile reaches the maximum height at A. After this the vertical component increases in the downward direction till the projectile reaches at B, a point in the same horizontal level as O. Thus the projectile describes the path OAB, a parabolic path.

Equation of the Path. Let at any time t the projectile be at P after travelling horizontal distance x and vertical distance y, then

$$x = u \cos \theta \times t \qquad \ldots(i)$$

and

$$y = u \sin \theta \times t - \frac{1}{2} g t^2 \qquad \ldots(ii)$$

From *(i)*,

$$t = \frac{x}{u \cos \theta}$$

Substituting the value of t in *(ii)*, we get,

$$y = u \sin \theta \times \frac{x}{u \cos \theta} - \frac{1}{2} g \left(\frac{x}{u \cos \theta} \right)^2$$

$$\therefore \quad y = \tan \theta \cdot x - \frac{g}{2u^2 \cos^2 \theta} \cdot x^2 \qquad \ldots(iii)$$

$$y = k_1 x + k_2 x^2.$$

Equation *(iii)* being a first degree equation in y and second degree equation in x which is the equation of a parabola.

Thus the trajectory of the projectile is a Parabola.

Maximum height reached or Vertical range : *It is the maximum height to which a projectile rises above the horizontal plane of projection.* It is denoted by H_m.

Expression for H_m. At the highest point the vertical component of velocity ($u \sin \theta$) becomes zero. Therefore, maximum height reached is given by the relation,

$$0^2 - (u \sin \theta)^2 = -2g H_m$$

$$\therefore \quad H_m = \frac{u^2 \sin^2 \theta}{2g}$$

Total time of Flight. *It is the total time taken by the projectile to return to the same level from which it was projected.*

Or

Total time of fight is the time for which the projectile remains in the air. It is denoted by T. It is equal to twice the time taken by the projectile to reach the maximum height because time of ascent (rise) is equal to the time of descent (fall). If t is the time of ascent, then time of flight, $T = 2t$.

Expression for T. At the highest point vertical velocity of projectile is zero. Hence the time of ascent is given by the relation ;

$$0 = u \sin \theta - gt$$

or
$$t = \frac{u \sin \theta}{g}$$

$\therefore$
$$T = 2t = \frac{2u \sin \theta}{g}$$

Horizontal range : *It is the total horizontal distance from the point of projection to the point where the projectile comes back to the plane of projection.* It is denoted by R.

Expression for R. Horizontal range is the distance covered by the projectile in total time of fight T, moving with uniform horizontal velocity.

$\therefore$ Horizontal range, $R = u \cos \theta \times$ time of flight (T)

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

$$= \frac{u^2 \times 2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin 2\theta}{g}$$

Angle of Projection for Maximum Horizontal Range. For a given velocity, range (R) is maximum if the value of $\sin 2\theta$ is maximum *i.e.*, 1.

$\therefore$ For R to be maximum, $\sin 2\theta = 1$

or $\sin 2\theta \ \sin 90°$ or $2\theta = 90°$.

Thus for a given velocity, the angle of projection for maximum range should be $45°$ or $\frac{\pi}{4}$

$\therefore$
$$R_{max} = \frac{u^2}{g}$$

Ex. FM–34. A body is projected from a point on the surface of earth with a velocity u making an angle θ with the horizontal. For what other angles with the horizontal will the range be the same and why ?

Ans. Other angles for the same range :

From the relation for the horizontal range *i.e.*, $R = \frac{u^2 \sin 2\theta}{g}$, it follows that for a given velocity of projection the horizontal range is maximum,

if $\sin 2\theta = 1$ or $2\theta = 90°$.

or $\theta = 45°$

Again, since $\sin \theta = \sin (180 - \theta)$

$\therefore$
$$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin (180 - 2\theta)}{g}$$

This shows that the projectile will have the same horizontal range for angles of projection θ and (90 – θ). To illustrate, various trajectories for a projectile thrown with a certain initial velocity and at various angles of projection are drawn (see fig. FM–36). The range is maximum for θ = 45°. A point along the X–axis can be reached at any two complementary plementary values of angle θ such as 75° and 15° ; 30° and 60°. The two paths for the same range have different maximum heights and different times of flight for these two values of angle θ. These are called *high* and *low* trajectories. In such cases, the horizontal range is not maximum. For maximum horizontal range, θ should be 45°, and in this case the two paths will coincide.

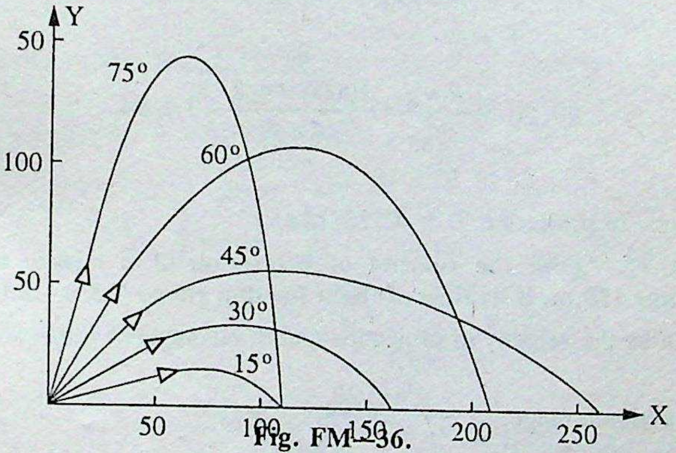


Fig. FM–36.

Ex. FM–35. A stone is dropped from a rising balloon at a height of 76 m above the ground. The stone reaches the ground in 6 second. What was the velocity of the balloon when the stone was dropped ? *(Pb. Nov. 1986)*

Sol. Here, the angle of projection with horizontal is 90° *i.e.*, the stone is thrown vertically upward by the balloon.

Let, the velocity of balloon = v m/s (upward).

∴ Velocity of stone, = v m/s

Height of balloon, $h = 76\ m$

Time taken to reach the ground, $t = 6\ s$

$$\therefore \quad h = vt - \frac{1}{2}gt^2$$

$$\text{or} \quad 76 = v \times 6 - \frac{1}{2} \times 9 \cdot 8 \times 36$$

$$\text{or} \quad 6v = 252 \cdot 4$$

$$\therefore \quad v = 42 \cdot 06\ m\ s^{-1}$$

Hence velocity of balloon at the time of dropping the stone

= $42 \cdot 06\ m\ s^{-1}$ **Ans.**

Ex. FM–36. A ship–gun is used to fire at a target 10000 m away. If the muzzle velocity is 784 m s^{-1}, find the angle of elevator neglecting air resistance.

Sol. Horizontal range is given by the relation,

$$R = \frac{u^2 \sin 2\theta}{g}$$

Here,

$$R = 10000\ m$$
$$u = 784\ m\ s^{-1}$$
$$g = 9\cdot8\ m\ s^{-2}$$
$$\theta = ?$$

$$\therefore \quad \text{Sin } 2\theta = \frac{R \times g}{u^2} = \frac{10000 \times 9\cdot8}{784 \times 784} = 0\cdot1594$$

$$\therefore \quad 2\theta = 9° - 10'$$

Hence, angle of projection, $\theta = 4° \ 35'$ **(Ans.)**

Ex. FM–37. Find the velocity of projection of a missile which has a horizontal range 150 m, if its time of flight for that range is 3 seconds.

Sol. Let u be the velocity of projection and θ, the angle of projection.

$$\therefore \quad \text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g}$$

also

$$R = u \cos\theta \times t \qquad \ldots(i)$$

Time of flight,

$$T = \frac{2u \sin\theta}{g} \qquad \ldots(ii)$$

Here,

$$R = 150\ m,\ T = 3s$$

$$\therefore \quad 150 = u\cos\theta \times 3$$

or

$$u\cos\theta = 50 \qquad \ldots(iii)$$

Also

$$3 = \frac{2u\sin\theta}{g}$$

or

$$u\sin\theta = \frac{3 \times 9\cdot8}{2} = 14\cdot7 \qquad \ldots(iv)$$

Squaring and adding *(iii)* and *(iv)*, we have,

$$u^2(\cos^2\theta + \sin^2\theta) = \sqrt{(50)^2 + (14\cdot7)^2}$$

or

$$u = \sqrt{2500 + 216\cdot09} = 52\cdot116\ m\ s^{-1}$$ **(Ans.)**

Dividing *(iv)* by *(iii)*, we get,

$$\frac{u\sin\theta}{u\cos\theta} = \frac{14\cdot7}{50}$$

or $$\tan\theta = 0{\cdot}2940$$

∴ $$\theta = 16°\ 24'.$$

Hence, the missile should be projected with a velocity of $52{\cdot}116\ m\ s^{-1}$ making an angle of 16° 24′ with the horizontal.

Ex. FM–38. A long jumper makes a record jump of 8·5 m. The jumper leaves the ground making an angle of 20° with the horizontal. Find :

(*a*) The velocity at which he leaves the ground

(*b*) The maximum height reached by the jumper.

Sol. Let v be the velocity with which the jumper leaves the ground.

∴ Horizontal range, $$R = \frac{v^2 \sin 2\theta}{g}$$

Here, $R = 8{\cdot}5\ m,\ \theta = 20°$ and $g = 9{\cdot}8\ m\ s^{-2}$

∴ $$v = \sqrt{\frac{Rg}{\sin 2\theta}} = \sqrt{\frac{(8\cdot 5\ m)(9\cdot 8\ m\ s^{-2})}{\sin 40^\circ}} = 11\ m\ s^{-1}\ \textbf{(Ans.)}$$

(*b*) Maximum Height, $$H_m = \frac{v^2 \sin^2\theta}{2g} = \frac{(11\ m/s)^2 \times (\sin 20^\circ)^2}{2\times 9\cdot 8\ m/s^2}$$

$$= \frac{121\ m^2 s^{-2} \times (0\cdot 3420)^2}{19\cdot 6\ m\ s^{-2}} = 0{\cdot}78\ m\ \textbf{(Ans.)}$$

Ex. FM–39. A body is projected with a velocity of 30 m/s at an angle of 60° to the horizontal. How high does it rise ? How long will it be in air ? How far from the point of projection will it strike the ground ?

Sol. Here, $u = 30\ m/s$ and $\theta = 60°$.

∴ Max. height, $$H_m = \frac{u^2 \sin^2\theta}{2g} = \frac{30\times 30\times 3}{2\times 9\cdot 81\times 4} = 34{\cdot}4\ m\ \textbf{(Ans.)}$$

∴ Time of flight, $$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times 30\times \sqrt{3}}{9\cdot 81\times 2} = \frac{10\sqrt{3}}{3\cdot 27} = 5{\cdot}26\ s\ \textbf{(Ans.)}$$

∴ Horizontal Range, $$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{30\times 30\times \sin 120^\circ}{9\cdot 81} = 79{\cdot}5\ m\ \textbf{(Ans.)}$$

FM–24. CIRCULAR MOTION.

The motion of a body in a circle with a fixed radius and at a constant speed is called uniform **circular motion.**

Examples. 1. The motion of a stone tied to a string when whirled in the air is a circular motion.

2. The motion of a point on the rim of a cycle wheel about its axle is an example of circular motion.

(b) **Linear velocity.** *The distance travelled by a body along a straight path per unit time is called* **linear velocity.** It is denoted by v.

Angular velocity. *The angle described by a rotating body per unit time is called* **angular velocity.** It is denoted by Greek letter ω.

Consider a particle moving in a circular path of radius r about an axis through O. Let the particle moves from A to B in a time t and the radius OA describes the angle $AOB = \theta$. Then the angle described by radius in unit time will be $\frac{\theta}{t}$

$$\therefore \quad \text{Angular velocity, } \omega = \frac{\theta}{t}$$

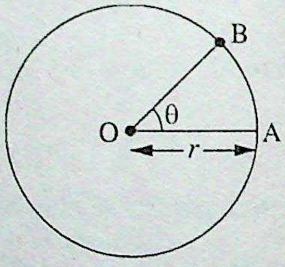


Fig. FM—37

If the angle θ is measured in radians and the time t is measured in seconds, then the units of angular velocity are radians/sec.

Relation between linear velocity and angular velocity. To find the relation between linear velocity and angular velocity, suppose a particle travels from A to B along the circular path in a time t with a linear velocity v, then

$$\text{The distance} \quad AB = v \times t \qquad \ldots(i)$$

$$\text{But} \quad AB = r\theta \qquad \ldots(ii) \left(\because \theta = \frac{l}{r}\right)$$

Where θ is the angular displacement measured in radians.

From *(i)* and *(ii)*, we have,

$$vt = r\theta$$

$$\text{or} \quad v = r\frac{\theta}{t} = r\omega$$

$$\therefore \quad v = r\omega.$$

FM–25. ANGULAR VELOCITY IN TERMS OF TIME PERIOD.

The time taken to complete one revolution is called the **time period.** If T is the time period, then angle described in time, $T = 2\pi$ radians

and angle described in unit time $= \frac{2\pi}{T}$ radians.

But angle described in unit time is angular velocity ω.

$$\therefore \quad \omega = \frac{2\pi}{T} \text{ rad/s.}$$

Angular velocity in terms of frequency. *The number of revolutions made by a rotating body in one second is called* **Frequency.**

If n is the frequency of rotation, then

angle described in 1 revolution $= 2\pi$ radians

and angle described in n revolutions $= 2\pi\, n$ radians

But n revolutions are made in 1 s.

$$\therefore \quad \omega = 2\pi\, n.$$

Angular velocity in terms of r.p.m. Let, N be the *r.p.m.* (revolutions per minute of the rotating body, then,

Revolutions made in 1 minute (60 s) $= N$

$\therefore$ Revolutions made in 1 s, $(n) = \dfrac{N}{60}$

Now $\omega = 2\pi\, n$

or $\omega = 2\pi \dfrac{N}{60}$ rad/s

Hence, Angular velocity $= 2\pi \dfrac{RPM}{60}$ rad/s.

Ex. FM–40. A fly wheel of radius 42 cms. is revolving with a speed of 120 r.p.m. Find its angular and linear velocities.

Sol. *RPM* of wheel, $N = 120$

$\therefore$ No. of revolutions/sec, $n = \dfrac{120}{60} = 2$

Now, Angular velocity, $\omega = 2\pi\, n$

or $\omega = 2 \times \dfrac{22}{7} \times 2 = \dfrac{88}{7} = 12{\cdot}6$ radians/sec.

Radius, $r = 42 \text{ cm} = 0{\cdot}42\ m$

$\therefore$ Linear velocity, $v = r\,\omega = 0{\cdot}42 \times \dfrac{88}{7} = 5{\cdot}28\ m/s$ **(Ans.)**

FM–26. ANGULAR ACCELERATION.

Angular acceleration is defined as the rate of change of angular velocity with time. It is denoted by α. If the angular velocity changes from ω_1 to ω_2 in time t, then,

$$\text{Angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t}$$

or
$$\alpha = \frac{2\pi n_2 - 2\pi n_1}{t}$$

Where n_1 and n_2 are the frequencies in each case.

If, however, the angular velocity changes by a very small value $d\omega$ in a very small interval of time dt, then

$$\text{Angular acceleration, } \alpha = \frac{d\omega}{dt}$$

Angular velocity and angular acceleration like the linear velocity and linear acceleration are vector quantities, because they have magnitude as well as a direction or a sense of rotation *clockwise* or *anti-clockwise*. The angular acceleration is expressed in *radians per sec*^{2}.

Relation between linear acceleration and angular acceleration.

Let the angular velocity of particle be ω_1 when it is at P and ω_2 when it is at Q. (See Fig. FM–38). If the angular velocity changes at a uniform rate and t is the time taken by the particle to travel from P to Q, then

$$\text{Angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t}$$

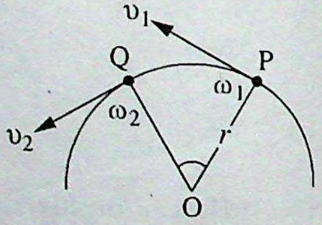


Fig. FM—38

If v_1 is the linear velocity of the particle at P, v_2 at Q and r is the radius, then,

$$\omega_1 = \frac{v_1}{r} \quad \text{and} \quad \omega_2 = \frac{v_2}{r}$$

$$\therefore \quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{v_2 - v_1}{rt}$$

But $\frac{v_2 - v_1}{t} = a$, the linear acceleration of the particle while travelling from P to Q.

$$\therefore \quad \alpha = \frac{a}{r}$$

or Linear acceleration, $a = r\alpha$.

$\therefore$ Linear acceleration = Angular acceleration (α) × radius of circular path (r)

Ex. FM-41. A wheel 0·6 m in radius is moving with a speed of 10 m/s. Find its angular speed.

Sol. Given, $r = 0{\cdot}6$ m, $v = 10$ m/s, $w = ?$

We know that $v = wr$

$$\Rightarrow \quad w = v/r = \frac{10}{0{\cdot}6} = 16{\cdot}66 \text{ rad/s. } \textbf{Ans.}$$

Ex. FM-42. Moon is 384000 km from earth and makes one revolution in 27·3 days. Find its angular and linear velocity in its orbit.

Sol. Time of one revolution = 27·3 days = 27·3 × 24 × 60 × 60s

Angle covered in one revolution = 27 radiaus.

$$\therefore \quad w = \frac{\text{angle}}{\text{Time}} = \frac{2\pi}{27\cdot3 \times 24 \times 60 \times 60} = 2\cdot664 \times 10^{-6} \text{ rad/s}$$

Also

$$v = wr = 2\cdot664 \times 10^{-6} \times 384000 \times 1000$$

$$\therefore \quad v = 1023 \; m/s$$

Ex. FM-43. A wheel having diametre 2m starts from rest and accelerates uniformly to an angular velocity of 90 revolutions per minute in *5 s*. Calculate its angular acceleration.

Sol. Given diameter of wheel = 2m, So radius $= \frac{2}{2} = 1\text{m}$.

Initial angular velocity of wheel, $w_1 = 0$ rad/s

time = 5s

Final no. of revolutions/sec, $n_2 = \frac{90}{60} = \frac{3}{2}$

$$w_2 = 2\pi n_2 = 2 \times 3\cdot142 \times \frac{3}{2} = 9\cdot426 \text{ rad/s}$$

$$\text{Angular acceleration} = \alpha = \frac{w_2 - w_1}{t} = \frac{9\cdot426 - 0}{5}$$

$$\therefore \quad \alpha = 1\cdot88 \text{ rad./s}^2 \textbf{ Ans.}$$

Ex. FM–44. A particle moving in a circle of radius 0·20 m has a speed increased from 300 revolutions per minute to 720 revolutions per minute in 22 seconds. Calculate,

***(i)* angular acceleration and *(ii)* linear acceleration.**

Sol. Here, Radius of the circle, $r = 0\cdot20 \; m$

Initial speed, $N_1 = 300 \; r.p.m.$

$$\therefore \quad \omega_1 = 2\pi \frac{N_1}{60} \; rad./s$$

Final speed, $N_2 = 720 \; r.p.m.$

$$\therefore \quad \omega_2 = 2\pi \frac{N_2}{60}$$

Time, $t = 22$ seconds

$$\therefore \text{ Angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi\frac{N_2}{60} - 2\pi\frac{N_1}{60}}{22} = \frac{2\pi}{60}\frac{(N_2 - N_1)}{22}$$

$$= \frac{2 \times 22}{60 \times 7} \times \frac{720 - 300}{22} = \frac{420}{210} = 2 \text{ rad. /s}^2 \text{ (An}$$

FM-27. CENTRIPETAL FORCE.

The force acting along the radius towards the centre of a circle to keep a body moving with uniform speed in a circular path is called **centripetal force.** It is denoted by F_c.

According to Newton's first law of motion, a body can not change its direction by itself unless an external force acts upon it. But when body is moving in a circular path, some force must be acting upon it to *change its direction of motion at every step and no change in its uniform speed.*

This is possible only if the force acts perpendicular to the direction of motion of the body at all points so that there is no component of the force in the direction of motion to change its speed. The radius being perpendicular to the circle at all points, *therefore, the force acts along the radius towards the centre.*

Thus in order to move a body in a circular path with a constant speed, a force must act along the radius towards the centre. This force is called centripetal force and constantly accelerates the body towards the centre of the circle.

Examples of Centripetal Force :

(i) A stone tied at one end of a string whose other end is held in hand, when whirled round in the air, the centripetal force is supplied the tension in the string.

(ii) The moon revolves round the earth, the centripetal force provided by the gravitational attraction between them.

If the *centripetal force* some how *vanishes* at any point in its circular path, the body will *fly off tangantially* to the circle at that point, because it will no longer be compelled to move in the circular path.

FM-28. CENTRIFUGAL FORCE.

A body moving in a circle with a uniform speed experiences a force which tends to take it away from the centre of the circle. This force is called centrifugal force.

According to Newton's, third law of motion, *to every action there is an equal and opposite reaction.* Thus, a force equal to centripetal force must act on a body, moving in a circle, in the outward direction. This is called **centrifugal reaction.**

The presence of centrifugal force in circular motion is explained from the fact that under the action of centripetal force, the body is accelerated towards the centre. Therefore, it should come towards the centre, but it does not happen. Thus a force (*centrifugal reaction*) equal and opposite to centripetal force must be acting on the body.

Examples : *(i)* In the case of a stone, whirled round at the end of a string whose other end is held in the hand (see fig. FM-39), the hand exerts on the stone the necessary centripetal force along the string towards the centre. The stone exerts the *centrifugal force* on the hand away from the centre and also along the string. If the string suddenly breaks, the stone would

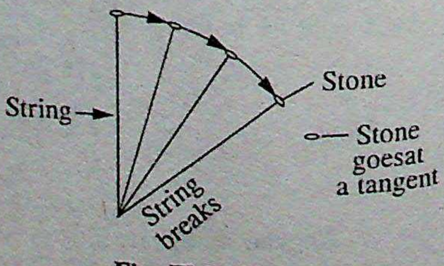


Fig. FM—39

fly off tangentially to the circular path and away from the centre. As soon as the string breaks, the centripetal force ceases to act and the centrifugal force also vanishes.

(ii) Due to centrifugal force, when a bus takes a turn the passengers inside the bus seem to be thrown outwards.

(iii) **Sparks from grinding wheel.** When you sharpen a knife on a grinding wheel, the spark particles fly at tangent to the wheel. It is because of the centrifugal force acting on the spark particles.

Centripetal acceleration. *A body moving in a circle with a constant speed is always accelerated towards the centre of the circle. This acceleration is called centripetal acceleration.*

In case of circular motion, although the body moves with constant speed, the direction of velocity changes with time, therefore, the body is under acceleration. The acceleration is perpendicular to the circular path and always directed towards the centre of the circle.

FM–29. EXPRESSION FOR CENTRIPETAL FORCE :

Consider a particle of mass m moving in a circle of radius r with a uniform speed v. Suppose the particle at any instant be at A and its velocity v is along AP the tangent to the circle at A. After a small time t, the particle reaches B where its velocity v is represented by tangent at B along BQ. The linear distance AB, travelled by the particle in *very small time t* is given by,

$$\text{Length of arc } AB = vt$$

Since time t is very small, therefore AB is practically a straight line, To determine the change in velocity consider a point O' outside the circle as shown in fig FM–40. Draw a line $O'a$ parallel and equal to vector AP, representing the velocity v at A. Similarly, draw a line $O'\,b$ parallel and equal to the vector BM, representing the velocity v at B.

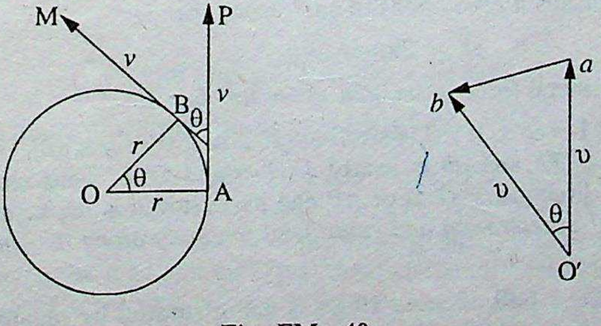


Fig. FM—40.

$$\therefore \qquad O'\,a = O'\,b = v \text{ (magnitude alone)}$$

Now, $\qquad ab$ = Change in velocity.

This change is only due to the change in direction and has taken place in time t.

$\therefore$ Acceleration of the particle = Rate of change of angular velocity

$\therefore$ Centripetal acceleration $= \dfrac{ab}{t}$ ...(i)

AB being a straight line, therefore, AOB is a triangle $\angle AOB = \angle aO'b = \theta$. Hence triangles AOB and $O'ab$ are similar.

$$\therefore \quad \frac{ab}{AB} = \frac{O'a}{OA}$$

$$\text{or} \quad \frac{ab}{vt} = \frac{v}{r} \qquad (\because AB = vt)$$

$$\text{or} \quad \frac{ab}{t} = \frac{v^2}{r}$$

But $\dfrac{ab}{t}$ is the centripetal acceleration (a_c)

$\therefore$ Centripetal acceleration $= \dfrac{v^2}{r}$ or $a_c = \dfrac{v^2}{r}$

Centripetal force (F_c) : If m is the mass of the particle, then

Centripetal Force = Mass × Centripetal acceleration

$$= m\,a_c = m\frac{v^2}{r}$$

$\therefore$ Centripetal force, $F_c = m\dfrac{v^2}{r}$

Another expression. The expression for centripetal force can also be given in terms of angular velocity ω.

Centripetal force, $F_c = \dfrac{mv^2}{r} = m\dfrac{(r\omega)^2}{r} = mr\,\omega^2$

If the body makes n revolutions in one second then,

$$\omega = 2\pi n$$

$\therefore$ Centripetal force $= mr\,(2\pi\,n)^2 = 4\pi^2\,m\,r\,n^2$

Direction of Force : If AB is small angle θ is also very small. Therefore $O'a$ is almost parallel to $O'b$ and ab is nearly $\perp O'a$ and $O'b$. Thus change in velocity takes place in a direction $\perp O'a$ or AP and hence towards the centre of the circle. Hence *centripetal acceleration and centripetal force act along the radius towards the centres of the circle.*

Ex. FM–45. A ball of mass 10 kg is being rotated by means of a cord 4 m long. If its velocity is 20 m/s, find the force that tends to break the cord.

Sol. Here, Mass of ball, $m = 10\ kg$

Velocity of ball, $v = 20\ m/s$

Radius, $r = 4\ m$

The centripetal force F_c is given by,

$$F_c = \frac{mv^2}{r} = \frac{10kg \times 20m/s \times 20m/s}{4\,m}$$

$$= 1000\,N$$

This centripetal force acts towards the centre and keeps the ball moving in a circle. But according to Newton's third law of motion, the ball exerts an equal and opposite force (centrifugal reaction) on the cord which tends to break the cord.

∴ Force which tends to break the cord = $1000\,N$

Ex. FM–46. A stone of mass 1·0 kg, tied at the end of a rope of length 2·0 m, is making vertical revolutions. Find the maximum velocity of the stone at the highest point of the vertical circle so that the rope may not loose. (given = 9·8 m/s^2)

Sol. Here, mass of the stone, $m = 10\ kg$.

Radius of circle, $r = 2{\cdot}0\ m$.

Let, v be the maximum velocity of the stone at the highest point of the vertical circle so that the rope does not loose (See fig. FM–41).

∴ At the highest point two forces act on the stone *i.e.*,

(i) Centrifugal force $= \frac{mv^2}{r}$ (up wards)

(ii) Weight of the stone = mg (downwards)

In order that the rope may not loose at the highest point,

Centrifugal force = weight of stone

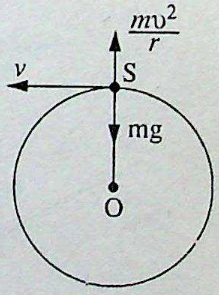


Fig. FM—41

$$\text{or} \qquad \frac{mv^2}{r} = mg$$

$$\text{or} \qquad v^2 = rg$$

$$\text{or} \qquad v = \sqrt{rg} = \sqrt{2\cdot0 \times 9\cdot8} = 4{\cdot}43\ m\ s^{-1}\ \textbf{(Ans.)}$$

FM–30. BANKING OF ROADS AND TRACKS.

When four wheeled vehicles like cars and buses move along curved roads at a great speed, they are moving on a circular track. Therefore, centripetal force is required in order to move the vehicle in the circular path. This force is provided by the friction between the road and the tyres. If this force of friction is insufficient, the necessary centripetal force may not act to keep the vehicle moving in a circular path. This results in skidding of the vehicle *i.e.,* the vehicle leaves the circular path.

In order to provide the necessary centripetal force to keep the vehicle moving on sharp bends of the road, road curves are banked *i.e., the outer edge of the road is slightly raised above the level of the inner edge. This is called banking of roads.*

Consider a car of mass m, moving with speed v in a circular track of radius r.

Then, in order to keep the car moving in a circular path, centripetal force required is given by,

$$F_c = \frac{mv^2}{r}$$

If the friction between road and tyre is less than F_c, then skidding results. In order to ensure that required centripetal force (mv^2/r) is provided, the road is banked *i.e.*, the road is inclined at the bends with slope towards the centre of the bend. Let the angle of inclination required be θ. The various forces acting on the car are :–

(i) The weight mg of the car acting vertically downwards.

(ii) The reaction R acting normal to the track.

Now reaction R can be resolved into two components (See fig, FM–42) balanced by the weight of the car *i.e.*,

$$R \cos \theta = mg \qquad \ldots(i)$$

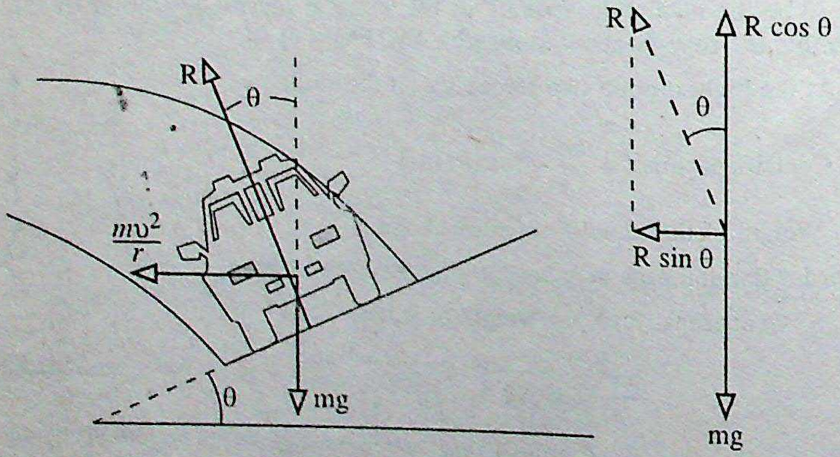


Fig. FM—42.

(b) Horizontal component R sin θ acting inward *i.e.*, toward the centre of the curve which provides the necessary centripetal force *i.e.*,

$$R \sin \theta = \frac{mv^2}{r} \qquad \ldots(ii)$$

Dividing *(ii)* by *(i)*, we have,

$$\frac{R \sin \theta}{R \cos \theta} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

$$\therefore \tan \theta = \frac{v^2}{rg}$$

or $$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \qquad \ldots(iii)$$

The equation *(iii)* gives the angle at which the road should be banked for speed v and radius r. The angle θ is independent of the mass of the vehicle. Thus the proper road design *i.e.*, angle θ depends upon the value of r and the expected speed of the vehicle. In general θ is adjusted for average speeds.

Ex. FM–47. A road is banked at 5° to the horizontal at a bend of radius 30 m. Calculate the maximum speed which a vehicle can have while rounding the bend without skidding ($g = 9{\cdot}8\ m/s^2$).

Sol. Let v be required maximum speed. Therefore, angle of inclination θ is given by.

$$\tan\theta = \frac{v^2}{rg}$$

Here, $\theta = 5°$

Radius of bend, $r = 30\ m$

Acceleration due to gravity, $g = 9{\cdot}8\ m/s^2$

Now, $$\tan\theta = \frac{v^2}{rg}$$

Hence, $$\tan 5° = \frac{v^2}{30 \times 9\cdot 8}$$

$\therefore$ $v^2 = 30 \times 9{\cdot}8 \times 0{\cdot}0872$ $[\because \tan 5° = 0{\cdot}0875]$

or $$v = \sqrt{30 \times 9\cdot 8 \times 0\cdot 0875}$$

$= 5{\cdot}07\ m/s$

$=$ **18·25 km/hr (Ans.)**

Ex. FM–48. What is the maximum speed at which a car can round a curve of 25 m radius on a level road if co–efficient of static friction between tyres and road is 0·80 ?

Sol. Here, Radius of curve, $r = 25\ m$

Co–efficient of friction, $\mu = 0{\cdot}80$

Let v be the maximum speed with which the car can move round the curve.

If m is the mass of the car, then centripetal force required $= \dfrac{mv^2}{r}$

This is provided by the force of friction which acts in the inward direction.

Now, Force of friction $= \mu\ mg$.

In order that the car moves round the curve without skidding, Centripetal force = Force of friction.

$$\therefore \quad \frac{mv^2}{r} = \mu mg$$

or $$v = \sqrt{\mu \, rg} = \sqrt{0 \cdot 8 \times 25 \times 9 \cdot 8} = 14 \, m \, s^{-1}$$

$$= 50 \cdot 4 \; km/hr \textbf{ (Ans.)}$$

FM–31. BENDING OF A CYCLIST.

A cyclist while going along a curved path requires centripetal force to do so. Centripetal force is necessary to keep moving along a circular path. The centripetal force is provided by the friction between road and tyres. If the friction does not provide the required amount of centripetal force, the cyclist bends inwards (*i.e.*, towards the centre of the curve) to provide the necessary centripetal force.

Consider a cyclist of mass *m* moving along a curved path of radius *r*, with speed *v*. Let the cyclist bend inwards making an angle θ with the vertical to provide the necessary centripetal force as shown in fig. FM–43.

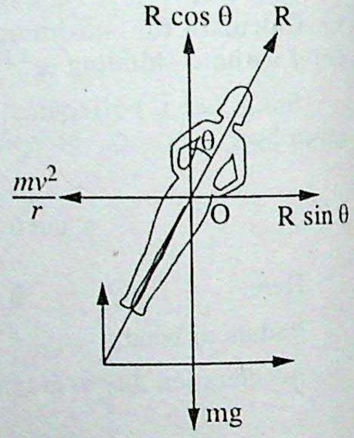


Fig. FM—43

The various forces acting on the system are :

(i) The weight *mg* acting vertically downwards.

(ii) Normal reaction R of the ground making an angle θ with the vertical.

The necessary centripetal force $\frac{mv^2}{r}$ is provided by horizontal component $R \sin \theta$ of the normal reaction R. The vertical component $R \cos \theta$ of the normal reaction balances the weight *mg* of the system. Therefore, in equilibrium position,

$$R \cos \theta = mg \qquad \ldots(i)$$

$$R \sin \theta = \frac{mv^2}{r} \qquad \ldots(ii)$$

Dividing *(ii)* by *(i)*, we get,

$$\tan \theta = \frac{v^2}{rg}$$

$$\therefore \quad \theta = \tan^{-1} \frac{(v^2)}{rg}$$

Thus greater the speed and smaller the radius of the curve, the more the cyclist will have to bend inwards from the vertical *i.e.*, θ must be large.

Ex. FM–49. A cyclist is moving with a velocity of 15 m/s. If he turns towards a sharp circular bend of radius 40 m then by how much angle from the vertical has he to shift himself along with cycle to pass the bend safely ? (Given $g = 10\ m/s^2$)

Sol. Here, velocity of the cyclist, $v = 15\ m/s$

The radius of curve $(r) = 40\ m$

Acceleration due to gravity, $g = 10\ m/s^2$

Let θ be the angle through which the cyclist must shift from the vertical to pass the bend safely.

$$\therefore \qquad \tan\theta = \frac{v^2}{rg} = \frac{15\ m\ s^{-1} \times 15\ m\ s^{-1}}{40\ m \times 10\ m\ s^{-2}}$$

$$= 0{\cdot}5625$$

Hence $\theta = 29°\ 20'$ **Ans.**

FM–32. APPLICATION OF CENTRIFUGAL FORCE.

The following are the applications of centrifugal force :

(i) **The centrifuges.** These are simple devices used for separating the substances of different densities suspended in a liquid, by rapidly rotating the liquid. When a liquid is rapidly rotated, it is acted upon by a centrifugal force equal to $= mr\,\omega^2$. As ω is the same for all particles, therefore, the force is directly proportional to the mass of the particle. The heavier particles, under the action of this force are driven away from the axis of rotation and lighter particles drawn inward towards the axis of rotation.

(ii) **Cream separator.** It is a device used to separate the lighter contents of milk *i.e.*, cream from milk. When milk is rotated in a cylinder at very large speed, the heavier particles move toward the wall of the cylinder where as the lighter particles *i.e.*, cream collects near the axis of the cylinder from where it can be removed very easily.

(iii) **Centrifugal drying machine.** With the help of such machines the clothes can be dried very quickly. The clothes are placed in a large cylindrical vessel having a large number of fine holes along the side walls. When the wet clothes are rotated at a very high speed, the water particles fly tangentially toward the outer walls and as such escape through the holes and clothes get dried.

(iv) In a similar manner, the crystals of sugar are separated from and heavier fluid *i.e.*, molasses and honey is separated from bees wax.

(v) Butter contents can be separated from milk.

(vi) Corpuscles can be separated from the plasma of blood for the preperation of serum.

QUESTIONS FOR SELF TEST

1. Explain the term force with suitable examples. What are *S.I.* : units of force.
2. Define the terms :–

   Momentum, Impulse, newton, scalar quantity, vector quantity, Inertia.
3. Find an expression for the measurement of force from Newton's second law of motion.
4. Explain how two vectors acting at a point can be compounded.
5. What is meant by components of a vector ? Resolve a given vector into its rectangular components.
6. Obtain a general mathematical expression for the magnitude and direction of the resultant of two forces inclined at an angle θ by,
   - *(i)* Parallelogram method,
   - *(ii)* Method of components,
7. State and explain Lami's theorem.
8. A body is projected in a horizontal direction with a uniform velocity from the top of a tower. Show mathematically that the path is a parabola.
9. A stone is projected with a velocity u in a direction making an angle θ with the horizontal. Show that the path of the projectile is a parabola.

   Find the expressions for :–
   - *(i)* the maximum height reached,
   - *(ii)* total time of flight,
   - *(iii)* the horizontal range.
10. Define and explain centripetal force and centrifugal force.

    Derive an expression for the centripetal force acting on a body moving in a circle.
11. Explain the following :–
    - *(i)* Banking of roads.
    - *(ii)* Bending of a cyclist.
12. Given some applications of centrifugal force.

PROBLEMS

1. What will be the components of a force of 100 N along directions making angles of 60° and 90° with the direction of the force. **(Ans.** 50 N, 0 N)
2. A man using a 70kg roller on a level lawn, exerts a force of 196 N at 45° to the ground. Calculate the vertical force of the roller on the ground *(i)* if he pulls, *(ii)* if he pushes. **(Ans.** *(i)* 538·8 N, *(ii)* 823·2 N)

3. Two forces OA and OB of $5N$ and $8N$ respectively pull on a body at a point O, the angle AOB being 60°. Find the magnitude and direction of a third force which will balance these two. **(Ans.** 11·35 N at 22° 24′ to 8 N force)

4. An aeroplane is flying in a horizontal direction with a velocity of 720 *km/ht* at a height of 1960 *m*. How far from a given target should it release a bomb so as to hit the target ? ($g = 9{\cdot}8\ ms^{-2}$) **(Ans.** 4000 *m*)

5. An athlete throws a javelin to a maximum distance of 80 metres. How long is it in the air and to what height does it rise ? Neglect the height of the athlete. ($g = 9{\cdot}8\ m/s^2$) **(Ans.** 4·04 *s* ; 20 *m*)

6. A roadway bridge over a canal is in the form of an arc of radius 39·2 *m*. What is the maximum speed with which the car can move without leaving the ground at at the highest point ? **(Ans.** 19·62 $m\ s^{-1}$)

7. Find the angle through which a cyclist bends when he covers a circular path 34·3 metres in circumference in $\sqrt{22}$ seconds. (given, $g = 9{\cdot}8\ m/s^2$). **(Ans.** 45°)

8. At what maximum speed a racing car can transverse an unbanked curve of 30 *m* radius ? The co–efficient of friction between tyres and road is 0·6. **(Ans.** 47·8 *km/h*)

9. At what angle must a track with a bend of 250 metres be banked for safe running of the trains at a speed of 72 *km/hr* ? **(Ans.** 9° 16′)

10. Find the minimum velocity with which the horizontal range of a projectile is 39·2 metres. **(Ans.** 19·6 *m/s*)

11. While launching a rocket of mass 2×10^{14} kg, a force of $5 \times 10^5\ N$ is applied for 20 seconds. What is the velocity attained by the rocket at the end of this interval **(Ans.** 500 *m/s*)

12. A force of 5Kg wt. acting on a body changes its velocity from 20 cm/s to 1000 cm/s in 20s. Calculate the mass of the body. Take $g = 9{\cdot}8\ m/s^2$. **(Ans.** 100 Kg)

13. If the resultant of the vectors $3\hat{i}+4\hat{j}+5k$ and $5\hat{i}+3\hat{j}+4\hat{k}$ makes an angle θ with *x*–axis, then find Cosθ. **(Ans.** 55°6′)

14. Find the value of $|\vec{A}\cdot\vec{B}|^2-|\vec{A}\times\vec{B}|^2$ **(Ans.** A^2B^2 Cos2θ)

15. Show that the vectors $9\hat{i}+\hat{j}+6\hat{k}$ and $4\hat{i}-6\hat{j}-5\hat{k}$ are perpendicular to each other.

16. Show that the vectors $2\hat{i}-10\hat{j}$ and $4\hat{i}-20\hat{j}$ are parallel to each other.

17. A car of mass 1000 Kg is moving with a velocity of 10 *m/s* and is acted upon by a forward force of 1000 *N* due to engine and a retarding force of 500 *N* due to friction. What will be its velocity after 10 seconds. **(Ans.** 15 *m/s*)

18. One of the rectangular components of a velocity of 80 Km/hr in 40 km/hr. Find other component. **(Ans.** 69·28 *km/hr*)

19. A body of mass 5 Kg is moving in a circle of radius 100 cm with an angular velocity of 2 rad/second. Find the centripetal force in Newton. **(Ans.** 20 *N*)

20. If coefficient of friction between the tyre and road is 0·5. What is the smallest radius at which car can turn on a horizontal road when its speed is 30 *km/hr*.

(Ans. 14·17*m*)

21. An aeroplane is flying due north with a velocity of 80 km/hr. If the wind is blowing from east at 20 *km/hr*, find the resultant velocity of the aeroplane with reference to the earth. **(Ans.** 82·46 *km/hr*)

22. A car moving along a straight highway with a speed of 126 *km/hr* is brought to stop within a distance of 200 *m*. What is the retardation of car and how long does it take for the car to stop. **(Ans.** $a = -3{\cdot}06\ m/s^2$, $t = 11{\cdot}4\ s$)

23. A shell of mass 0·02 kg is fired by a gun of mass 100 kg. If the muzzle speed of shell is 80 m/s. What is the recoil speed of gun ? **(Ans.** - 0·016 m/s)

Chapter

WPE

Work, Power and Energy

Work equals the force acting the distance through which it acts. It is note worthy that the value of force be that which acts in the **direction of motion.**

Power is the time rate of doing work.

Energy in Physics is the ability to do work. Physics is said to have one prevading theme. *i.e.,* concept of **energy.** The energy of a body or system of bodies is like the concept of financial worth of a person. A certain amount of energy involved may appear in many forms and may be converted from one form to another. Energy in the form of radiation falls upon the earth from the sun.

In plants the solar energy is converted into chemical energy, resulting in the formation of atoms and molecules in such a way as to form plant growth (wood). The wood may become coal and may be burned at a power station to release some energy as heat. This heat energy through pressure of steam may give mechanical energy. The turning of turbine rotors give us electrical energy, which is brought to our homes, where part of it may be converted to mechanical energy through motors and part to radiant energy through lamps, toasters, irons etc.

Energy can assume the following forms ; electrical, chemical, mechanical, radiation and heat. Another form of energy which should be included in the general discussion is *mass*. Mass itself is a form of energy. The mass energy equivalence is expressed as

$$E = mc^2$$

where c is the velocity of light.

If in the energy conversion chain mentioned above, we could measure the amount of energy present in **each form** including mass we would find that the total is always constant. This fact is commonly referred to as the *principle of conservation of energy*.

WPE—1. WORK

Work is said to be done by a force when it moves a body through a certain distance. It is measured by the product of the force (*in the direction of motion*) and the distance through which the body moves *i.e.*

Work done = Force (*in the direction in motion*) × Distance through which the force acts.

or $$\mathbf{W} = \mathbf{F} \times \mathbf{S}$$

where W = Work done

F = Force acting in the direction of motion

S = Distance through which the force acts

Examples, (*i*) When the body moves along a direction making an angle θ with the line of action of the force F, only the component $F \cos \theta$ does work. Let a force F acts along OA and moves the body (*horizontally*) through a distance S. Then,

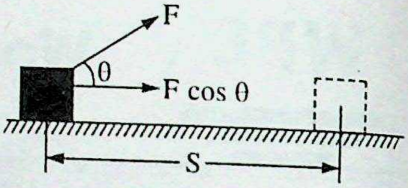


Fig. WPE—1

Work done = Force (*in the direction of motion*) × Distance

or $$\mathbf{W = F \cos \theta \times S}$$

(*ii*) When a body of mass m is lifted through a height h above the ground, the force required to lift is equal to the weight of the body *i.e.*,

$$\text{Force} = \text{Weight} = mg$$

$$\therefore \quad \text{Work done} = \text{Force } (\textit{in direction of motion}) \times \text{Distance}$$

or $$W = mg \times h$$

or $$\mathbf{W = mgh \text{ joules}}$$

(*iii*) The effect of gravity always acts vertically downwards and if a body acted upon by gravity moves in a horizontal direction, its motion is clearly at right angles to the line of action of the force *i.e.*, $\theta = 90°$.

$$\therefore \quad W = F \cos 90° \times S = 0 \times S = 0$$

Hence no work is done against gravity when a body moves in a horizontal direction.

It should be noted that a force does *no work* if it does not move the body on which it acts *i.e.*, if $S = 0$ then, $W = F \times 0 = 0$

Thus a man tending to push a wall is doing no work. In this case, though the force is being applied, yet the point of application of force (wall) does not move.

∴ *No displacement, no work.*

Units of Work :

The units of work depend upon the units of force and the units of distance. In *SI* system, the unit of force is *newton* and unit of distance is *metre*.

$$\therefore \quad \text{Work} = \text{newton} \times \text{metre} = N\,m$$

The unit of work, *newton metre* is called *joule* (J).

One joule *or* **newton metre** *is the work done by a constant force of one newton, if it moves the body, on which it acts, through a distance of one metre in the direction of force.*

Examples of different types of work done :

We know that the work done by a force $(\vec{F})$ which displaces a body through $\vec{S}$ is given by

$$W = \vec{F} \cdot \vec{S} = F\,S\cos\theta$$

Where θ is angle between $\vec{F}$ and $\vec{S}$.

(*a*) **Examples of positive work done :** (*i*) When a body falls freely under gravitational pull, the work done by gravity is positive.

(*ii*) When a horse pulls a cart on a level road, the work done is positive.

In both the above examples $\vec{F}$ and $\vec{S}$ are in same direction.

(*b*) **Examples of negative work done :** (*i*) When a body is made to slide over one surface, the work done by the frictional forces is negative.

(*ii*) When two similar poles of a magnet are brought close to each other, the work done by the magnetic force of repulsion is negative.

In both the above examples $\vec{F}$ and $\vec{S}$ are in opposite directions.

(*c*) **Examples of zero work done :** (*i*) when a person travels on a platform with a load on his head, work done by him is zero.

(*ii*) When a body moves along a circular path with the help of a string, the work done by tension in the string is zero.

In both the above examples $\vec{F}$ and $\vec{S}$ are perpendicular to each other.

WPE—2. POWER

The rate of doing work is called Power. In other words, the power of an agent is the work done per unit time.

$$\therefore \quad \text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

or

$$P = \frac{W}{t} = \frac{F \times S}{t} = F \times v$$

where P = Power

W = work done and t = time

F = force applied and v = velocity given to the body.

More power of an agent means, it is capable of doing more work in unit time.

Units of Power. The units of power depend upon the units of work and the units of time.

In *SI* units work is measured in *newton metres or joules* and time in *seconds.*

$$\therefore \quad \text{Power} = \frac{Nm \text{ or joules}}{\text{seconds}} = J/s = \text{watts}$$

∴ Units of power are **watt**

One Watt is the power of an agent doing one joule of work in one second.

WPE—3. ENERGY

The capacity of a body to do work is called the energy of the body.

The body can do work only if work is previously done on it. This work done is stored in the body and the body is said to possess energy.

The energy of the body is measured by the amount of work it can do.

Units of Energy. The units of energy are the same as the units of work *i.e.* **joules.**

In atomic and nuclear Physics, of energy is **electron volt** (eV) and $1\ eV = 1{\cdot}6 \times 10^{-19}$ joules.

Ex. WPE—1 : What is the work done by a man in carrying a suitcase weighing 30 kg over his head when he travels a distance of 10 m in :

(*i*) the horizontal direction ;

(*ii*) Vertically upward direction.

Ans. (*i*) Work done $= F \cos\theta \times S$

Here, $F =$ force exerted by the man $= mg = 30 \times 9{\cdot}8\ N$

$\theta = 90°$, Since the body moves in horizontal direction and force acts vertically upwards.

$\therefore \quad \cos\theta = \cos 90° = 0$

$S = 10\ m$

$\therefore \quad W.D. = 30 \times 9{\cdot}8 \times 0 \times 10 = 0$ (No work is done when the suitcase is moved in the horizontal direction)

(*ii*) Work done $= mgh$

Here, $m = 30$ kg

$g = 9{\cdot}8\ m/s^2$

and $h = 10m$

$\therefore \quad W.D. = 30 \times 9{\cdot}8 \times 10\ J = 2940\ J$ **(Ans).**

WPE—4. FORMS OF MECHANICAL ENERGY.

Mechanical energy has two forms, kinetic energy and potential energy.

Kinetic Energy : *The energy possessed by a body by virtue of its motion is called* **Kinetic Energy.**

It is measured by the amount of work the body can do due to its motion.

The units of $K.E.$ are *joules*

Example : (*i*) A moving ball possesses kinetic energy.

(*ii*) Water moving in a stream or river has kinetic energy, because it can wash away soil or can move other objects.

(*iii*) Blowing wind has $K.E.$

(*iv*) A moving bullet has $K.E.$ because it can pierce the target.

Potential Energy : *The energy possessed by a body by virtue of its position, state or configuration is called* **Potential Energy.**

It is measured by the amount of work the body can do due to its position or state.

The units of potential energy are *joules.*

Examples : (*i*) A stone raised to certain height has potential energy due to its raised position, because it can do work if allowed to fall.

(*ii*) Water at a higher level in a dam has potential energy due to its position.

(*iii*) A stretched rubber band has potential energy due to its state.

(*iv*) The wound up spring of a clock has potential energy due to its configuration.

WPE—5. EXPRESSIONS FOR POTENTIAL AND KINETIC ENERGIES

***(i)* Expression for Potential Energy :** Consider a body of mass '*m*' lying on the surface of earth. In this case it has zero potential energy. Let it be raised through a height '*h*'. The force *F* required to lift the body against gravity is equal to the weight of the body *i.e. mg*.

$\therefore \quad F = mg$

and $\quad S = h$

Thus, $\quad$ work done $= F \times S = mg \times h = mgh.$

This work done in lifting the body through a height *h* has been stored in it the form of potential energy. (or gravitational Potential energy)

$\therefore$ **Gravitational Potential Energy = mgh**

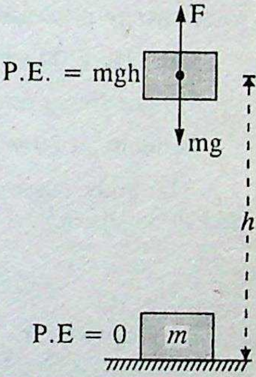


Fig. WPE—2

***(ii)* Expression for Kinetic Energy :** Consider a body of mass *m* at rest. It possesses zero kinetic energy in this case since $v = 0$. If a constant force *F* is applied to this body, the body begins to move with an acceleration *a*.

Therefore, according Newton's second law of motion $F = ma$. ...(*i*)

Let after travelling a distance, *S* the body attains a velocity, *v* then,

$$v^2 - 0^2 = 2a\,S \qquad (\because v^2 - u^2 = 2aS \text{ and } u = 0).$$

$$\therefore \quad a = \frac{v^2}{2S} \qquad ...(ii)$$

Now, $\quad F = ma$

Multiplying both sides by *S*, we get

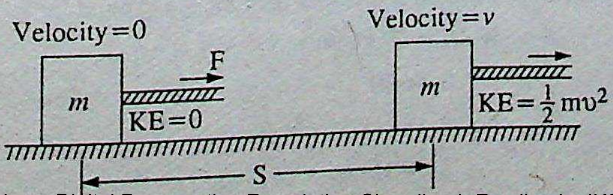


Fig. WPE—3

$$F \times S = m\,a \times S$$

or $$F \times S = m \times \frac{v^2}{2S} \times S = \frac{1}{2} mv^2$$

But $F \times S = W$, work done by a force F in giving a velocity v to the body.

$$\therefore \quad W = \frac{1}{2} mv^2$$

This work done is stored in the body in the form of kinetic energy.

$$\therefore \quad \textbf{Kinetic Energy} = \frac{1}{2} mv^2$$

Ex. WPE—2. A body of mass 5 kg is freely falling under gravity. It has got a velocity of 10 m/s at a height of 20 m above the reference level. Calculate the total energy possessed by the body at this point.

Sol. Here, Mass of the body, $m = 5$ kg

Height above reference level (earth), $h = 20\ m$

Velocity at 20 m height, $v = 10\ m/s$

The body possesses both kinetic energy due to its velocity and potential energy due to its position.

$$\therefore \quad K.E. = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times (10)^2 = 250\,J.$$

and $$P.E. = mgh = 5 \times 9{\cdot}8 \times 20 = 980\ J.$$

Total Energy of the body $= KE + PE = 250 + 980 = 1230\ J$ **(Ans.)**

Ex. WPE—3. A man weighing 50 kg caries a load of 10 kg on his head. Find the W.D. when he goes (i) 15 m vertically up (ii) 15 m on a levelled path on the ground.

Sol. Here, Mass of the man, $m_1 = 50\ kg$

Mass carried by the man, $m_2 = 10\ kg$

Total mass, $M = m_1 + m_2 = 50 + 10 = 60\ kg$.

(*i*) When the man goes vertically up,

Height through which rises, $h = 15\ m$.

$$\therefore \quad W.D. = Mgh$$

$$= 60 \times 9{\cdot}8 \times 15 = 2940\ J \text{ (Ans.)}$$

(*ii*) When the man goes on a levelled path on the ground.

$$W.D. = F \cos\theta \times S$$

Here, $$\theta = 90°$$

$$\therefore \quad \cos\theta = \cos 90° = 0$$

$$S = 15\ m.$$

$$\therefore \quad W.D. = F \times 0 \times 15 = 0$$ (No work done in moving horizontally)

Ex. WPE—4. **A body moves through a horizontal distance of 20 m under the action of a force of 15 N. Calculate the work done if the force makes an angle of.**

(i) **30° with the horizontal**

(ii) **60° with the horizontal.**

Sol. (*i*) Here,

Force, $F = 15\ N$

Distance, $S = 20\ m$

Angle with horizontal, $\theta = 30°$

Therefore, work done is given by ;

$$W = F\cos\theta \times S = 15N \times \cos 30° \times 20\ m$$

$$= 300 \times \frac{\sqrt{3}}{2} Nm = \textbf{259·8 joules (Ans).}$$

(*ii*) Here, $F = 15\ N$, $S = 20\ m$, $\theta = 60°$

Therefore, work done is given by,

$$W = F\cos\theta \times S = 15\ N \times \cos 60° \times 20\ m$$

$$= 15\,N \times \frac{1}{2} \times 20\,m = \textbf{150 joules (Ans).}$$

Ex. WPE-5 A force of 3N is exerted through a distance of 10m upon a body whose mass is 1·5 kg. The body is initially at rest. Find the kinetic energy of body.

Sol. The acceleration produced by force of 3N is

$$a = \frac{F}{m} = \frac{3}{1\cdot 5} = 2\,\text{m/s}^2$$

Velocity at the end of 10*m* is given by

$$\upsilon^2 - 0^2 = 2 \times 9 \times 10$$

$$\Rightarrow \qquad \upsilon^2 = 2 \times 2 \times 10 = 40$$

$$\text{K.E.} = \frac{1}{2} m\upsilon^2 = \frac{1}{2} \times 1\cdot 5 \times 40 = 30\,\text{J} \ \textbf{Ans.}$$

Ex. WPE-6 A neutron of mass $1{\cdot}7 \times 10^{-27}$ kg moving with a constant speed is found to pass two horizontal points 6m part in 2×10^{-4} s. Calculate its kinetic energy.

Sol. Given $m = 1{\cdot}7 \times 10^{-27}$ kg, distance (s) = 6m

Time taken (t) $= 2 \times 10^{-4}$ s

$$\text{Velocity } (\upsilon) = \frac{\text{distance}(s)}{\text{time}(t)} = \frac{6}{2 \times 10^{-4}} = 3 \times 10^4\ \text{m/s}$$

$$\text{K.E.} = \frac{1}{2}m\upsilon^2 = \frac{1}{2}\times 1\cdot 7\times 10^{-27}\times(3\times 10^4)^2$$

$$= \frac{1}{2}\times 1\cdot 7\times 10^{-27}\times 3\times 10^4\times 3\times 10^4$$

$$\text{K.E.} = 7\cdot 65\times 10^{-19}\text{ J. Ans.}$$

Ex. WPE-7 Calculate the velocity of bob of simple pendulum at its mean position, if it is able to rise to a vertical height of 10 cm given $g = 9{\cdot}8$ m/s^2.

Sol. Given $h = 10$ cm $= 0{\cdot}1$ m, $g = 9{\cdot}8$ m/s^2

At the mean position, kinetic energy of pendulum is equal to its potential energy *i.e.*

$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 9\cdot 8\times 0\cdot 1} = 1\cdot 4\text{ m/s}$$

Velocity of bob at mean position = 1·4 m/s.

Ex. WPE-8 How much mass is converted into energy per month in a nuclear plant operated at 100 MW ?

Sol.

$$\text{Energy} = 100\text{ MW} = 100\times 10^6\text{ W} = 10^8\text{ J/s}$$

$$\text{Energy in a day} = 10^8\times 24\times 60\times 60\text{ J}$$

$$\text{Energy in month} = 10^8\times 24\times 60\times 60\times 30\text{ J}$$

$$\text{Energy} = 2\cdot 592\times 10^{14}\text{ J/month}$$

Now $E = mc^2$, $c = 3\times 10^8$ m/s

$$m = \frac{E}{c^2} = \frac{2\cdot 592\times 10^{14}}{(3\times 10^8)^2} = 0\cdot 288\times 10^{-2}\text{ kg}$$

$$m = 2\cdot 88\times 10^{-3}\text{ kg} = 2\cdot 88\text{ g Ans.}$$

WPE—6. CONCEPT OF FRICTION :

Friction : *The opposing force which comes into play in between the surfaces of two bodies, when one is moved over the surface of another is called* **force of friction** *or* **friction.**

Whenever there is a relative motion between the two surfaces in contact, an opposing force comes into play to prevent the relative motion. This opposing force is called force of friction.

The surfaces of solid bodies are never perfectly smooth. Friction is produced by the interlocking of irregularities of one surface with that of the other surface in contact.

Static Friction : *The friction offered by the surfaces subjected to external forces until there is no motion between them is called* **static friction.**

Let us consider a block on a horizontal table at rest. The block is in equilibrium under the action of its weight W and an upward force R exerted by the table. (Here $W = R$)

When a small horizontal force F is applied towards right, the block does not move. The force R exerted by the table on the block becomes inclined towards the left. The component of force R parallel to the surface is called the force of static friction 'f_8' which balances the applied force F. Thus $F = f_8$. The component of R perpendicular to the surface is the normal reaction N exerted on the block which balances the weight W. Thus $N = W$.

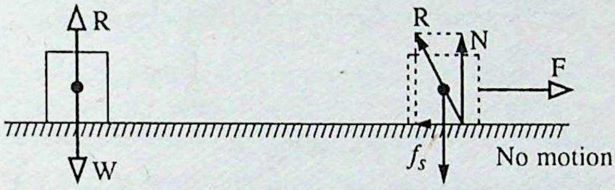


Fig. WPE—4

As the applied force F is increased, the force of static friction f_8 also increases and the equilibrium is maintained. Finally F reaches a maximum (limiting) value when f_8 can not increase and the block begins to move. Thus static friction f_8 can have a certain maximum value called the limiting friction.

Dynamic Friction : *The force acting between the surfaces when there is relative motion between the two surfaces in contact is called the force of* **dynamic** or **kinetic friction 'f_k'.** Dynamic friction is less than the static friction.

In daily experience it is observed that less force is required to keep a body in motion than it is required to start the body from rest.

WPE—7. CO–EFFICIENT OF FRICTION

The ratio of the limiting (maximum) force of friction to the normal reaction is called **co–efficient of friction.**

The force of limiting friction which comes into play when a body is just to move over the surface of another body is directly proportional to the normal reaction *i.e.*,

$$f \propto R$$

or
$$f = \mu R$$

where μ is constant of proportionality and is called the *co–efficient of friction.*

$$\therefore \qquad \mu = \frac{f}{R} = \frac{\text{Force of limiting friction}}{\text{Normal Reaction}}$$

WPE—8. LIMITING FRICTION

The maximum value of friction which comes into play when a body just starts moving over the surface of another body is called **limiting friction.**

Friction is a self adjusting force upto a certain limit *i.e.*, the force of friction goes on increasing as the pull on the body is increased till it just starts moving over the surface of another body.

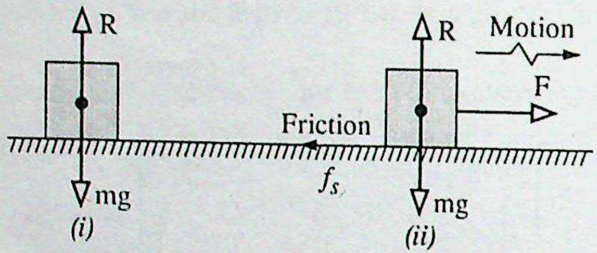


Fig. WPE—5

Consider a block of wood placed over the surface of a table as shown in fig. WPE—5. When no force is applied on the block, there is no force of friction between the surfaces in contact and the block is under the action of the following two forces :

(*i*) Weight of the block $W = mg$ acting vertically downwards.

(*ii*) Normal Reaction are acting vertically upward.

When some force is applied to pull the body, friction comes into play and the body does not move. If the pulling force is gradually increased, the force of friction also increases. When the body just starts moving, the opposing force attains a maximum value and is called the force of **limiting friction.** The various forces acting on the block when it just starts moving are :-

(*i*) Weight of the block $W = mg$ acting vertically downwards

(*ii*) Normal reaction R acting vertically upward

(*iii*) Pulling force F

(*iv*) Force of friction f acting opposite to F.

Since the body is in equilibrium under these forces as shown in fig. 8.5 (*ii*)., we have,

$$W = R, \text{ and } F = f$$

i.e., Weight of the body = Normal reaction

and, Pulling force, F = Force of friction, f

When the pulling force slightly increases the force of limiting friction, the body just starts moving.

WPE—9. LAWS OF FRICTION

Force of friction between two surfaces obey the following laws :

(*i*) *Friction is an opposing force and comes into play, only, if the body is in motion or tending to move over the surface of another body.*

(*ii*) *Force of friction acts along the tangent to the two surfaces at the point of contact.*

(*iii*) *The force of friction acts in a direction opposite to the direction of motion of the body.*

(*iv*) *The force of friction is independent of the areas of the surfaces in contact.*

(*v*) *The force of friction is directly proportional to the normal reaction between the two surfaces in contact i.e., $f \propto R$.*

WPE—10. FRICTION IS A NECESSARY EVIL

In most of the cases friction is found to be of disadvantage. Due to friction the energy given to a machine is not totally converted into useful work but a part of it is converted into heat. Friction causes wear and tear of the moving parts of the machines and therefore is an evil.

In many cases friction plays a very important role in our daily life. We are able to walk due to friction otherwise we will slip. We can not stop our cycle if there is no friction between the brakes and the rims. Thus friction is a necessity.

Therefore, sometimes friction is an evil and at some stages it is very necessary in daily life. Thus, friction is necessity as well as an evil.

Methods used to reduce Friction : The following methods are used for reducing friction :

1. Lubrication : When a thin film of oil or some other suitable lubricant is formed between two sliding or rolling surfaces, the surfaces are not allowed to come in contact. The surfaces slide on grease or oil film more easily than on dry surfaces. Thus friction is reduced by the use of lubricant.

2. Rolling : When one body rolls over another the friction is less. Thus sliding motion is changed into rolling motion. This is achieved by using ball bearings or wheels.

3. Polishing : Polished (smooth) surfaces have less friction than rough, irregular surfaces.

Ex. WPE—9. A body on the ground with a velocity of 1 metre/sec. comes to rest after travelling a distance of 5 metres. Find the co-efficient of friction.

Sol. Here, Initial velocity, $u = 1\ m\, s^{-1}$

Final velocity, $v = 0$

Distance over which it stopes, $S = 5\ m$

Using the relation, $v^2 - u^2 = 2aS$

$$\therefore \quad a = \frac{v^2 - u^2}{2S} = \frac{0-1}{2 \times 5} = -\frac{1}{10} = -0 \cdot 1 m s^{-2}$$

The force of friction, $f = -ma$

But, $f = \mu R = \mu\, mg$ where μ is co-efficient of friction

$$\therefore \quad \mu = \frac{f}{mg} = -\frac{ma}{mg} = -\frac{a}{g}$$

$$= -\frac{0 \cdot 1 m/s^2}{9 \cdot 8 m/s^2} = 0 \cdot 0102 \text{ **(Ans.)**}$$

WPE—11. WORK DONE AGAINST FRICTION

Whenever a body moves over the surface of another body, the force of friction opposes its motion. To keep the body in motion work will be done.

∴ Work done = Force required to overcome friction × Distance moved

The two cases are given below :-

(*i*) **Body moving over a rough horizontal surface :** Consider a body of mass m lying on a horizontal plane as shown in fig WPE—6. Let a force F be applied to the body so that it starts moving.

The various forces acting on the body are :

(*i*) Weight mg acting vertically downwards.

(*ii*) Normal reaction R acting vertically upwards.

(*iii*) Force of friction 'f' acting opposite to the direction of motion.

(*iv*) Pulling force F moving the body forward.

As the body does not move in upward or downward direction, therefore, normal reaction R balances the weight mg *i.e.*, $R = mg$

But $f = \mu R$ where μ is the co-efficient of friction

∴ $f = \mu\, mg$ $[\because R = mg]$

Here the body has to overcome only the force of friction 'f' *i.e.*,

$$F = f = \mu\, mg$$

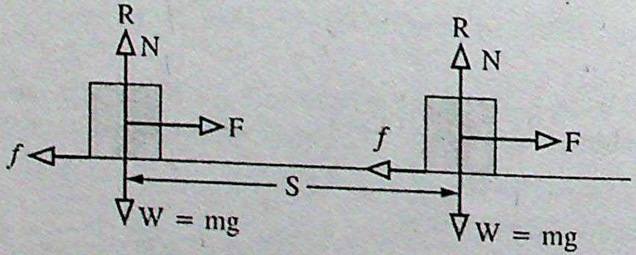


Fig. WPE—6

If the body moves through a distance S, then work done is given by, Work done = Force × Distance

$$= F \times S = \mu\, mg \times S$$

∴ Work done against friction on a horizontal plane = μ mg S

(*ii*) **Body moving up a rough inclined plane :** Consider a body of mass m being pulled up a rough inclined plane with a force F as shown in fig. WPE—7. Let the inclined plane make an angle θ with the horizontal and F be the force acting along the plane in the upward direction.

When the body is just to move, the various forces acting on it are :

(*i*) The weight mg acting vertically downwards.

(*ii*) Normal reaction R acting perpendicular to the plane (OB).

(*iii*) Force of friction f acting down the plane opposite to the direction of motion.

(*iv*) Pulling force F acting along the plane in the upward direction.

The weight mg can be resolved into two components :

(*a*) Component $mg \cos \theta$ perpendicular to the plane (OB).

(*b*) Component $mg \sin \theta$ acting downward along the plane.

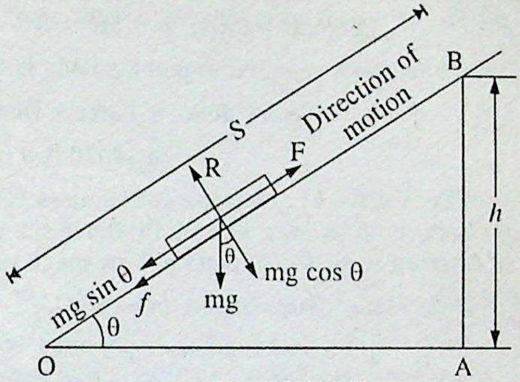


Fig. WPE—7

Now, normal reaction, R balances the component $mg \cos \theta$ as the body does not move in this direction *i.e.*

$$R = mg \cos \theta$$

Therefore, only two forces, $mg \sin \theta$ and f (force of friction) are acting downwards along the plane. The force F required to move the body up the inclined plane has to just overcome these forces *i.e.*

Force, $F = mg \sin \theta + f$

But $f = \mu R = \mu\, mg \cos \theta$

$\therefore$ $F = mg \sin \theta + \mu\, mg \cos \theta$

If the body moves through a distance S up the inclined plane, then

$$\text{Work done} = \text{Force} \times \text{Distance} = F \times S$$

$$= (mg \sin \theta + \mu\, mg \cos \theta)\, S$$

$\therefore$ $\textbf{Work done} = \mathbf{mg \sin \theta\, S + \mu\, mg \cos \theta\, S}$

or $\mathbf{W = mgh + \mu\, mg \cos \theta\, S}$ $(\because \sin \theta = h/S)$

Now mgh = Work done against gravity

and $\mu\, mg \cos \theta\, S$ = Work done against friction

$\therefore$ Work done = Work done against gravity + Work done against friction

Ex. WPE—10. How much work will be done in dragging a body of mass 100 kg through a distance of 100 m on a level road, if co–efficient of friction is 0·2 ?

Sol. Here,

Distance moved, $S = 100\, m$

Normal reaction, R = wt. of the body

$= mg = 100 \times 9{\cdot}81 = 981{\cdot}00\, N$

Co–efficient of friction, $\mu = 0{\cdot}2$

∴ Force of friction, $f = \mu R = 0{\cdot}2 \times 981\ N = 196{\cdot}2\ N$

Thus, force required to pull the body is $196{\cdot}2\ N$

$$\therefore \quad \text{Work done} = \text{Force} \times \text{Distance} = 196{\cdot}2\ N \times 100\ m$$

$$= \mathbf{19620\ Nm\ (\textit{or}\ joules)\ (Ans.)}$$

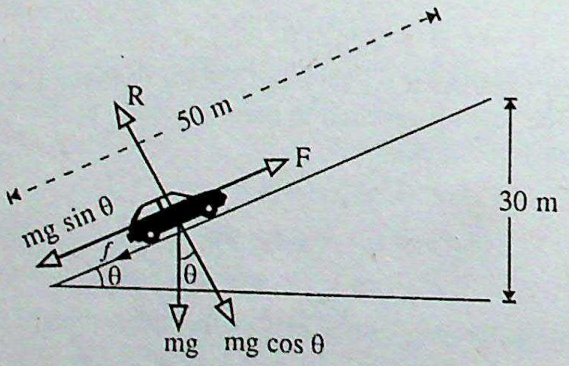
Ex. WPE—11. A small car of mass 500 kg is to be raised from the bottom of an incline 50 m long and 30 m above the ground at the top. Assuming the force of friction to be 0·3 N per kg, how much work will be done by the engine ?

So. Here, Mass of car, $m = 500$ kg

Force of friction $= 0{\cdot}3\ N/\text{kg}$

Distance moved along the incline, $S = 50\ m$

Distance moved vertically upwards, $h = 30\ m$



Total force of friction, f = Friction/kg × Mass of car

$= 0{\cdot}3\ N/\text{kg} \times 500\ \text{kg} = 150{\cdot}0\ N$

Let F be the force required to move the car up the inclined plane. Then,

$$F = mg \sin\theta + f$$

$$= 500\,\text{kg} \times 9\cdot 8 \frac{m}{s^2} \times \frac{30}{50} + 150\,N \qquad \left(\because \sin\theta = \frac{30}{50}\right)$$

$$= 2940\ N + 150\ N = 3090{\cdot}0\ N$$

$$\therefore \quad \text{Work done} = \text{Force} \times \text{Distance}$$

$$= 3090\ N \times 50\ m$$

$$= 154500\ \text{joules}\ \mathbf{(Ans.)}$$

WPE—12. POWER.

Power. *The rate of doing work is called* **power.**

$$\therefore \quad \text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If 'W' be amount of work done by an agent in time t, then the power 'p' of the against is given by

$$P = \frac{W}{t} \qquad \ldots(i)$$

or

$$P = \frac{F \times S}{t} \qquad [\because W = F \times S]$$

$$= F \times v$$

From equation (i), it is clear that smaller the time t in which the agent can do W amount of work, greater is the power P of the agent.

Thus more power of an agent means, it is capable of doing more work in a unit time.

Units of Power :-

In SI units, work is measured in *newton metres or joules* and time in *seconds.*

$$\therefore \quad \text{Power} = \frac{Nm \text{ or joules}}{\text{seconds}}$$

$$= Nm \text{ per second or joules per second}$$

$$= Nm/s \text{ or } j/s = \text{watt}$$

Therefore, units of power are **watts.**

One newton–metre per second or one joule per second is called the **watt (W).**

∴ 1 N m/s or 1 joule/second = 1 Watt

One watt : We know, $P = W/t$

If $W = 1$ joule, $t = 1$ second

Then, $$P = \frac{1 \text{ joule}}{1 \text{ second}} = 1 \text{ joule / second}$$

$$= 1 \text{ watt}$$

Hence **one watt** *is the power of an agent doing one joule of work in one second.*

A still bigger unit of power is **kilowatt (kW).**

1 kilowatt (kW) = 1000 watts

Thus **one kilowatt (kW)** *is the power of an agent doing* 1000 *joules of work in one second.*

The reader is familiar with the use of watts in electricity. The power of electric bulbs, electric heaters, fans etc., is expressed in **watts** or **kilowatts** *e.g.* 60 watt bulb, 1 **kW** heater etc.

The practical unit of power (*i.e.* the unit of power actually used in practice) in the *F.P.S.* system is the **Horse Power (H.P.).**

One Horse Power (H.P.) : In the early stages of science, it was found that an average horse does work at the rate of 550 ft. lbs, per second. This was called the horse power.

Thus **one horse power (H.P.)** *is the power of an agent doing 550 ft, lbs. of work in one second i.e.,*

$$1\ H.P. = 550 \text{ ft. lbs./second.}$$

The powers of scooters, motor cycles, electric motors, engines etc. is expressed in **horse power (H.P.)**

Relation between H.P. and Watt

$$1\ H.P. = \frac{550 \text{ ft.lbs.}}{1 \text{second}} = \frac{550 \times \text{foot} \times \text{lbs.}}{1 \text{second}}$$

$$= \frac{550 \times 0\cdot305 \text{m} \times 0\cdot454 \times 9\cdot81 \text{N}}{1 \text{ second}}$$

$$[\because 1 \text{ foot} = 0\cdot305\ m,\ 1 \text{ lb. (force)} = 0\cdot454 \text{ kg} \times 9\cdot81 \text{ m/s}^2 = 0\cdot454 \times 9\cdot8 \text{ N}]$$

$$= \frac{746 \text{ joules}}{1 \text{second}} = 746 \text{ watts.}$$

$$\therefore \quad \mathbf{1\ H.P. = 746\ watts}$$

Also, $$1 \text{ H.P.} = \frac{746}{1000} \text{KW} = 0\cdot746 \text{ KW}$$

WPE—13. FORMS OF ENERGY.

Energy exists in many forms such as Mechanical energy, Heat energy, Light energy. Electrical energy, Chemical energy and Nuclear energy. They are inter-related in the sense that the energy in one form may be transformed into energy of another form. For example when two bodies are rubbed against each other, heat is produced. Thus mechanical energy is converted into heat. Similarly chemical energy of coal is converted into heat energy on burning. This heat energy is responsibly for the running of a steam engine. Heat energy is thus transformed into mechanical energy.

When electric current passes through the filament of an electric bulb, the filament is heated to incandescence and light is emitted. The electrical energy is transformed into heat and light energies.

Thus it is found that different forms of energy are mutually convertible and many such examples of conversion are found in various natural phenomena. Different forms of energy are discussed below :

1. Mechanical Energy. Mechanical energy is of two types :-

***(i)* Potential Energy.** Refer WPE—4.

***(ii)* Kinetic Energy.** Refer WPE—4.

2. Light Energy : *Light may be defined as a form of radiant energy which produces in us the sensation of sight.*

The sources of light like the sun, electric lamp, candle etc. radiate light energy in the form of waves or photons. In the sun thermo-nuclear reactions are taking place and radiate light energy. In electric lamp light energy is emitted at the expense of electrical energy. Similarly in the burning of candle carbon atom combine with oxygen atoms to produce CO_2 gas and a large quantity of heat and light are produced (chemical energy converted into heat and light).

In a reverse process when light falls on matter it produces mechanical, chemical and heating effects. The example of mechanical effects is met within the emission of electrons from the surface of some metals when light falls on them (Photo electric effect). The phenomenon of photosynthesis in plants is a chemical effect, produced by light. Similarly if you focus sunlight on a piece of black paper with a lens, the paper gets burnt due to the heating effect of light.

The transfer of energy between light and matter can occur only in packets or quanta. This quantum of light energy is known as photon. A photon is an energy packet (E) which is proportional to the frequency (n) of the radiation.

$$\therefore \quad E \propto n$$

$$\text{or} \quad E = hn$$

where h is a universal constant, called Plank's constant and is equal to $6{\cdot}65 \times 10^{-27}$ erg. sec.

$$\therefore \quad \text{Light energy } E = hn \text{ ergess or joules}$$

3. Heat Energy : *Heat is a form of energy which produces in us the sensation of warmth.*

It has been established beyond any doubt that heat can perform useful work as any agent possessing energy can do. The most common example is met with in the steam engine which does mechanical work due to the heat present in steam.

Kinetic theory of matter explains that molecules of a substance are always in a rapid motion at various speeds. On this basis, if a molecule has a mass m and velocity v, its kinetic energy is $\frac{1}{2}mv^2$. Different molecules in a substance are in different conditions, therefore they move with different speeds and hence have different kinetic energies.

According to the modern concept, *heat of a body is the total kinetic energy of all its molecules.*

When heat is supplied to a body, the molecules start moving faster. This results in the increase of kinetic energy of the body. On the other hand if heat is taken from the body, the speeds of its molecules decrease and this results in the decrease in the kinetic energy of the body. Thus more heat in a body means that sum total of kinetic energy of all its molecules is more.

Flow of heat from a body at a higher temperature to a body at lower temperature is explained as a flow of energy. The molecules of a hotter body have greater energies than the molecules of the colder body. When the hot and cold bodies are placed in contact, their molecules collide and the energy passes from the faster molecules of the hot body to the slower molecules of the cold body.

We feel warm when we sit either near a furnace or in the sun. It is due to the heat energy that we receive from either source. Here, transfer of heat energy takes place by the process of radiation.

All the above examples show that heat is a form of **energy.**

4. Electrical Energy : Electricity plays a very important role in the lives of all of us. It provides us, light, heat, sound, transportation and entertainment. Thus useful work is done by electricity by giving light energy, heat energy, sound energy and mechanical energy required for transportation and running of fans etc.

Thus there are a number of evidences which suggest that electricity can do useful work and hence the existence of **electrical energy.**

Whenever a current is passed through a conductor, it becomes hot after some time. This means that electrical energy is being converted into heat energy.

Similarly an electric motor converts electrical energy into mechanical energy, which is used to revolve the fan, to operate the refrigerator, to run the factories and so on.

In our homes Electricity Department has installed a meter for measuring the electrical energy consumed by us during the month. Therefore, this meter is actually energy meter. It measures energy in kilowatt-hours (kWh). 1 kWh is the unit of electrical energy for which the electricity board charges money. For example you can calculate the number of kWh of energy consumed for operating a lamp during the month. Suppose you burn a lamp of power 60 watts for 5 hours a day, for 30 days then electrical energy consumed in kWh is given as

$$\text{No. of } k\text{W}h = \frac{\text{Power of lamp} \times \text{No. of hours per day}}{1000} \times 30$$

$$= \frac{60 \times 5}{1000} \times 30 = 9 \text{ units.}$$

Thus it is established beyond any doubt that electricity is a form of energy.

5. Chemical Energy. *The energy change that takes place in a chemical reaction is known as* **chemical energy.**

A chemical reaction involves only conversion of molecules of one form into molecules of another form. In other words, it involves breaking of bonds in reactant molecules and formation of new bonds in product molecules. Energy is evolved or absorbed when reactant molecules change into product molecules. This indicates that *each substance is associated with a definite amount of energy which is known as internal energy.*

The value of internal energy for a given substance depends upon its *state* (*i.e.*, temperature, pressure and volume) and upon its *chemical nature* (*i.e.*, number and nature of chemical bonds).

The value of internal energy or chemical energy of a substance is measured in the form of *heat, work, light* etc.

6. Nuclear Energy : The nuclei of heavy atoms like Uranium are unstable and are continuously disintegrating. Due to the disintegration of heavy nuclei, some energy is released. Einstein gave the relation between mass and energy as :

$$E = mC^2 \text{ joules}$$

where E = Energy released in joules

m = mass in kg

and C = velocity of light = 3×10^8 m/s.

Thus *during the conversion of mass into energy, a tremendous amount of energy is released. This is called* **nuclear energy.**

Ex. WPE—12. About 4×10^{10} kg of matter is converted into energy in the sun each second. What is the power output of the sun ?

Sol. Here, mass converted into energy, $m = 4 \times 10^{10}$ kg

Velocity of light, $C = 3 \times 10^8\ \text{ms}^{-1}$

Let, E be the energy produced.

Using the relation, $E = mC^2$ joules.

$$\therefore \quad E = 4 \times 10^{10}\ \text{kg} \times (3 \times 10^8\ \text{ms}^{-1})^2 = 36 \times 10^{26}\ \text{joules}$$

Now, Work done by the sun, $W = 36 \times 10^{26}\ J$

Time taken, $t = 1s$

$$\therefore \text{ Power output of sun, } P = \frac{W}{t} = \frac{36 \times 10^{26}\ J}{1s}$$

$$= 36 \times 10^{26}\ \text{watts} = 36 \times 10^{20}\ MW \textbf{ Ans.}$$

WPE—14. PRINCIPLE OF CONSERVATION OF ENERGY

The principle of conversion of energy states that :

"*Energy can neither be created nor it can be destroyed but may be converted from one form to another.*"

This is the fundamental law of Physics.

Examples :

(*i*) When a speeding car hits a brick wall, the car comes to rest. Here, the kinetic energy of car is not destroyed but is converted into heat energy, sound energy etc. If you somehow measure, it will be seen that kinetic energy of car before hitting is *equal* to the heat, sound energies etc. produced after hitting. This shows that total energy is the same though it has changed its form.

(*ii*) When a bullet hits a target, the kinetic energy of bullet is converted into sound, light, heat etc. *K.E.* of bullet is exactly *equal* to heat, sound, light etc, produced after hitting.

(*iii*) If a stone of mass *m kg* is raised through a height of *h* metres.

Potential energy of stone = *mgh*

When the stone is allowed to fall freely its potential energy starts converting into kinetic energy. The result is that potential energy goes on decreasing and kinetic energy goes on increasing. *But the total energy (P.E. + K.E.) of the stone is the same at any point i.e.,* energy is neither created nor destroyed.

Transformation of energy : In all the physical processes energy changes from one form to another. For example :

(*i*) In a heat engine, heat energy changes into mechanical energy.

(*ii*) In an electric bulb, the electric energy changes into light energy.

(*iii*) In an electric heater, the electric energy changes into heat energy.

(*iv*) In a fan, the electric energy changes into mechanical energy which rotates the fan.

(*v*) In the sun, mass changes into radiant energy.

(*vi*) In an electric motor, the electric energy is converted into mechanical energy.

(*vii*) In burning of coal, oil etc., chemical energy changes into heat & light energy.

(*viii*) When we rub two surfaces on each other, the mechanical energy changes into heat energy which heats the surfaces.

(*ix*) In a dam, potential energy of water changes into kinetic energy, then K.E. rotates the turbine which produces the electric energy.

(*x*) In an electric bell, electric energy changes into sound energy.

(*xi*) In a greater, mechanical energy is converted into the electric energy.

(*xii*) In a telephone sound energy is first converted into electric signal (energy) and then again at the reception point electric energy is changed into sound energy.

WPE 15. CONSERVATION OF ENERGY OF A FREELY FALLING BODY :

Consider a body of mass m to be at rest at a position A. Let h be the height of the body above the reference level which is of course close to the ground.

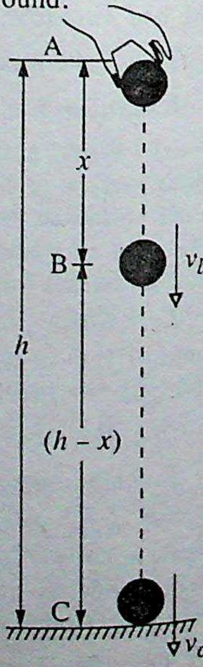


Fig. WPE—9

At the position A.

Kinetic energy of the body = 0

[The body is at rest.]

Potential energy of the body = mgh

Total energy of the body = $KE + PE$

$$= 0 + mgh = mgh \quad \ldots(1)$$

At the position B.

Let the body be at the position B at any instant after having fallen through a distance x.

∴ Potential energy of the body at B = $mg\,(h-x)$

If v_b be the velocity of the body at B, then

$$v_b^2 - 0^2 = 2gx \quad \text{or} \quad v_b^2 = 2gx$$

∴ Kinetic energy of the body $= \frac{1}{2} m v_b^2$

$$= \frac{1}{2} m \times 2gx = mgx$$

Total energy of the body $= mg\,(h - x) + mgx = mgh$ ...(2)

At the position C.

When the body is at position C, its height above the ground may be regarded as zero.

$\therefore$ Potential energy of the body $= 0$

If v_C be the velocity with which the body just touches the ground, then, $v^2{}_C - 0 = 2gh$

$$\text{or } v^2{}_C = 2gh$$

$\therefore$ Kinetic energy of the body $= \frac{1}{2} m v_C{}^2 = \frac{1}{2} m \times 2gh = mgh$

Total energy of the body $= K.E. + P.E. = mgh + 0 = mgh$ ...(3)

From equations (1), (2) and (3), it is clear that the sum of the kinetic and potential energies of freely falling body is constant at all states of motion. As the body falls, its potential energy decreases while its kinetic energy increases. However, the total mechanical energy is conserved.

The graph shown in fig. WPE—10 depicts the values of kinetic energy, and potential energy at different heights above the reference level. As is clear from the graph, the potential energy decreases linearly with decrease in height. On the other hand, the kinetic energy increase with decrease in height. The total mechanical energy is represented by a thick straight line parallel to the height-axis. This shows that the total mechanical energy is conserved.

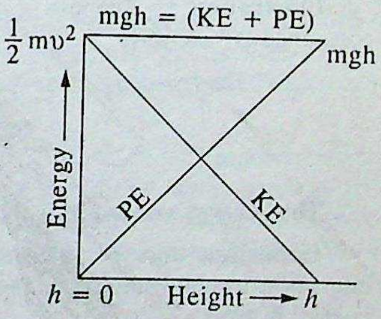


Fig. WPE—10

WPE—16. KILOWATT–HOUR (KWR) AS THE UNIT OF ENERGY.

A conveniently large unit of *energy in* common use in the *kilowatt-hour* (abbreviated as *kWh*).

One **kilowatt–hour** *is the work done in one hour by an agent working at a constant rate of one kilowatt.*

Since watt is the unit of power, therefore, people often think that kilowatt-hour is also the unit of power. But it is not so. Kilowatt jour is the unit of work (energy) as :–

$$1 \text{ watt} = 1 \text{ joule/second} \qquad ...(i)$$

Multiplying equation (*i*) by 1 second on both sides, we get,

1 watt × 1 second = 1 joule

i.e., **1 watt – second = 1 joule**

The watt-second is therefore, the unit of work or energy and is equal to 1 joule.

Again, considering the kilowatt,

$$1 \text{ kilowatt} = 1000 \text{ watts} = 1000 \text{ joules/second}$$

Multiplying both sides by 1 hour, we get,

$$1 \text{ kilowatt} \times 1 \text{ hour} = (1000 \text{ joules/second}) \times 1 \text{ hour}$$

$$= 1000 \frac{\text{joules}}{\text{second}} \times 3600 \text{ second}$$

$$= 3600000 \text{ joules} = 3{\cdot}6 \times 10^6 \text{ joules}$$

i.e., **1 kilowatt hour = $3{\cdot}6 \times 10^6$ joules**

The kilowatt–hour (kWh) therefore, is the unit of energy (work).

It should be noted that the State Electricity Board presents its bill for a certain number of kWh consumed *i.e.*, charges are actually made for *energy* and **not** for *power.*

$$1\ kWh = 1 \text{ unit (Board of Trade unit } i.e., \text{ B.O.T.)}$$

Thus, when we say that 20 units are consumed in a month, it means that 20 kWh of electrical energy is consumed.

Illustration : Suppose a 100 watt bulb operates for 5 hours. Then energy consumed is given by,

$$\text{Energy consumed} = \text{Power} \times \text{Time}$$

$$= 100 \text{ watt} \times 5 \text{ hours} = 500 \text{ watt hours}$$

$$= 0{\cdot}5\ kWh$$

Thus energy consumed is $0{\cdot}5\ kWh$ or $0{\cdot}5$ unit.

Important note : Since electricity is a form of energy, therefore, it can be expressed in conventional units *i.e.*, joules. But it is customary to express electrical energy in kWh.

WPE—17. MASS ENERGY EQUIVALENCE

By the term *energy* we mean energy in any of its forms, whether mechanical, thermal, electrical, or *mass*. Energy may be expressed in a number of ways; E, mc^2, $h\upsilon$, eV, $\frac{1}{2}mv^2$, joules, calories, British thermal unit (Btu). In nuclear reactions energy may appear in any of these forms and we must be able to convert them all to their values in one common form if we are to add them up. Therefore, in nuclear reactions it is customary to express *all energies* in terms of *mass units* or electron volts (eV).

Thus by the *mass–energy* relationship we know that the mass of one electron is equal to 5,11,000 electron volts of energy. This is called its *annihilation energy*. The annihilation energy of 1 mass unit (*amu*) is 931 MeV (million electron volt).

Mass is one form of energy and according to the conservation of energy, the table below shows mass energy equivalence in different forms.

$$1 \text{ kg} = 8{\cdot}99 \times 10^{16} \text{ joules}$$

$$1\ eV = 1{\cdot}6 \times 10^{-19} \text{ joules}$$

$$1 \text{ electron mass} = 5{\cdot}11 \times 10^5\ eV$$

$$1\ amu = 931\ MeV$$

At $\lambda = 1$ Å, $1\ hn = 12,\ 336\ eV$.

WPE—18. MASS ENERGY CONSERVATION LAW

Before Einstein's time it had been accepted that mass and energy are two different entities and it was the accepted view that matter could not be destroyed. Another law called the **law of conservation of energy** states that the total quantity of energy in the universe is also constant and can neither be created nor destroyed.

According to Einstein's theory of relativity **the mass of a body is a measure of the quantity of energy contained in it.** For instance in fission reactions sum of the masses of atoms produced as a result of fission is slightly less than the mass of the original uranium nuclei. This difference in mass is converted into energy which we call nuclear energy. If this difference in mass be Δm, then equivalent energy ΔE is given by

$$\Delta E = \Delta mc^2 \qquad \ldots(i)$$

Here c is the velocity of light. Equation (i) is called **Einstein's mass–energy equation.** This equation shows that mass and energy are independent. If mass in the universe decreases, its equivalent energy increases. We now accept the view that **sum of mass and energy in the universe remains constant or conserved.** This is called **conservation law of mass and energy.**

Mass Defect. In general the mass of nucleus is less than the sum of masses of protons and neutrons which constitute the nucleus. The difference is mass is called **mass defect.** If Δm is the decrease in mass when a number of protons and neutrons combine to form a nucleus, then according to Einstein's mass–energy relation, this mass will be converted into energy which is released when nucleus is formed. If ΔE be this energy, then

$$\Delta E = \Delta m \times c^2 \qquad \ldots(ii)$$

Mass defect Δm = (Sum of the masses of nucleons) – (mass of the nucleus).

Thus *mass defect is the amount of mass which would be converted into energy if a particular nucleus were to be assembled from protons and neutrons.* The same amount of energy would be needed to break the atom into its constituent particles and the energy equivalent of mass defect is therefore a measure of binding energy of the nucleus.

WPE—19. NUCLEAR FISSION

German scientists Hahn and Strassman discovered that when uranium (Z = 92) was bombarded with neutrons, it split up into two fragments–Barium (Z = 56), and Krypton (Z = 36). These two fragments travel in opposite directions with tremendous speeds. It was also found that enormous energy is released when uranium is split up in this way. This process of splitting uranium with the release of enormous amount of energy is called **Nuclear Fission.** Above reaction may be represented by the following equation.

$$_{92}U^{235} + {}_0n^1 \rightarrow 92U^{236} = {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3{}_0n^1 + \text{energy}$$

(Uranium) (Neutron) (Barium) (Krypton)

Here $92U^{236}$ is unstable and hence atonce splits up into two fragments (Barium and Krypton) and neutrons with large amount of energy.

In fission process a heavy atom is split up into two light atoms. The nuclides which can be fissioned by low energy neutrons called thermal neutrons are (*i*) U^{235}, (*ii*) U^{233}, (*iii*) Pu^{239} (Plutonium). U^{233} can be fissioned by fast neutrons. The total amount of energy of released called the nuclear energy is about 200 MeV in the fission of one U^{235} nuclide. Out of this energy about 5 MeV is carried by 3 neutrons produced in each fission.

WPE—20. NUCLEAR ENERGY

When a heavy nucleus undergoes fission process, a large amount of energy is released together with the emission of a number of neutrons which produce further fission. This energy is known as **Nuclear energy**. This is also called **Atomic energy.**

Uses of Nuclear Energy :

(*i*) **Nuclear energy is utilised in producing large amount of electricity.**

(*ii*) Large quantities of nuclear explosives are used in mining, in the excavation of ditches and in the recovery of petroleum.

(*iii*) The energy is utilised in producing **isotopes** which are very useful in curing diseases like cancer, in increasing production in agriculture and in research.

(*iv*) Big factories can be dependent upon this energy. Thus in future it will replace coal and petrol.

WPE—21. NUCLEAR REACTION

Nuclear reaction is defined as a process of strong interaction of an atomic nucleus with an elementary particle or with another nucleus resulting in the transformation of a nucleus. The most wide-spread kind of nuclear reaction is the interaction of a light particle a with a nucleus X, the result being the formation of a light particle b and a nucleus Y.

$$X + a \rightarrow Y + b.$$

Light particles may be neutron, proton, deuteron, alpha particle, gamma particles. Nuclear reactions may result, either in the liberation or the absorption of energy. The amount of liberated energy is called reaction energy. It is determined by the difference between masses of the initial and final nuclei.

WPE—22. NUCLEAR FUSION

It is a process in which two very light nuclei with $A < 8$ combine to form a heavy nucleus with simultaneous release of a large amount of energy. Such nuclear fusion reactions are taking place in the sun and other stars. The temperature in the interior of stars is about 10^7 K which leads to complete ionisation of atom. Such a system contains bare nuclei and electrons and is called **plasma.** These nuclei are moving with such high speeds that when they colide, can overcome coulomb repulsion and combine together by attractive nuclear forces with a release of energy.

For example if two deuterons are brought together to form an α–particle the following reactions take place with the liberation of about 21·6 MeV energy.

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 40\ MeV \text{ (energy)}$$

(Deuteron)(Deuteron) (Tritium) (Proton)

Nucleus of tritium (${}_1H^3$) again fuses with another nucleus of deuteron as follows :

$${}_1H^3 + {}_1H^2 \rightarrow {}_2H^4 + {}_0n^1 + 17{\cdot}6\ MeV.$$

An almost inexhaustible supply of deuteron is available in the form of heavy water. This can be utilized for fusion purposes.

WPE—23. SUN'S ENERGY

The problem of energy sources of sun and stars was a puzzle for a long time that engaged the minds of scientists.

Sun is known to radiate enormous quantity of energy. What is the source of this energy ? With the knowledge of nuclear reactions it became possible to give a satisfactory explanation of the origin of energy radiated by sun and other stars.

Two sets of thermo–nuclear reactions have been proposed as sources of energy in the sun and other stars. One is called *proton–proton chain reaction* and the other is *carbon cycle*. In both reactions enormous energy is liberated.

Ex. WPE—13. A man whose mass is 75 kg walks up 10 steps each 0·2 m high in 5 seconds, find the power he develops.

Sol. Here, mass of the man, $m = 75$ kg

Height moved, $h = 10 \times 0{\cdot}2 = 2\ m$

Time taken, $t = 5$ seconds.

$\therefore$ Work done against gravity, $W = mgh$

$$= 75 \text{ kg} \times 9{\cdot}81\ m\ s^{-2} \times 2\ m = 1471{\cdot}50\ J$$

$\therefore$ Power developed, $P = \dfrac{W}{t} = \dfrac{1471 \cdot 5 J}{5 s} = 294 \cdot 3\ W$ **(Ans.)**

Ex. WPE—14. A man weighing 65 kg lifts a mass of 45 kg to the top of a building 10 metres high in 12 seconds. Find :–

(i) Total work done by him.

(ii) The Power developed by him.

Sol. Mass of the man, $m_1 = 65$ kg

Mass lifted, $m_2 = 45$ kg

Total mass raised, $m = m_1 + m_2 = 65 + 45 = 110$ kg

Height through which raised, $h = 10\ m$

time taken, $t = 12$ seconds.

(*i*) Total work done by the man, $W = mgh$

$$= 110 \times 9{\cdot}81 \times 10 = 10791{\cdot}0\ J$$

(*ii*) Power developed, $P = \dfrac{W}{t} = \dfrac{10791 J}{12 s} = 899 \cdot 25\ W$ **(Ans.)**

Ex. WPE—115 **Calculate the power of an engine which can maintain a speed of 50 m s^{-1} for a train of mass 3 × 10^7 kg on a rough level line. Given co-efficient of friction = 0·05 and g = 10 m s^{-2}.**

Sol. Here, mass of train, $m = 3 \times 10^7$ kg

Co-efficient of friction, $\mu = 0 \cdot 05$

Acc. due to gravity, $g = 10\ m\ s^{-2}$

Velocity of train, $v = 50\ m\ s^{-1}$

∴ Force applied to pull the train, F = force of friction

or $$F = f = \mu R = \mu\ mg$$

$$= 0 \cdot 05 \times 3 \times 10^7 \times 10 = 15 \times 10^6\ N$$

Now Power of engine, $P = F \times v = 15 \times 10^6\ N \times 50\ m\ s^{-1}$

$= 75 \times 10^7$ watts. $= 75 \times 10^4\ kW$ **(Ans.)**

Ex. WPE—16. A bullet of mass 20 g travelling with a velocity of 16 m s^{-1} penetrates a sand bag and is brought to rest in 0·05 s. Find,

(a) The depth of penetration in metres.

(b) The average retarding force of the sand in newtons.

Sol. Here, mass of bullet, $m = 20\ g = 0 \cdot 002$ kg

Initial velocity of bullet, $u = 16\ ms^{-1}$

Final velocity of bullet, $v = 0\ ms^{-1}$

Time taken to stop, $t = 0 \cdot 05\ s^{-1}$

∴ Loss in $K.E.$ $$= \frac{1}{2} mu^2 - \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times 0 \cdot 002 (16^2 - 0^2) = 0 \cdot 256\ J$$

Let retardation produced, a

∴ $$v = u + at$$

or $$0 = 16 + a \times 0 \cdot 05$$

or $$a = -\frac{16}{\cdot 05} = -320\ m\ s^{-2}$$

(a) Depth of penetrations, $s = \dfrac{v^2 - u^2}{2a}$

∴ $$s = \frac{0 - (16)^2}{2 \times -320} = \frac{-256}{-640}$$

$= 0 \cdot 4\ m$ **(Ans.)**

(*b*) Let F be the average retarding force of sand bag, then

$$\text{Work done } = F \times S$$

Now work done is equal to the loss in kinetic energy

$$\therefore \quad F \times 0{\cdot}4 = 0{\cdot}256 \quad \text{Or} \quad F = 0{\cdot}64\ N \text{ (Ans.)}$$

Ex. WPE—17. A cricket ball of mass 0·2 kg reaches a field–man's hand with a velocity *v*. The field–man stops the ball by moving his hands backwards through a distance of 0·1 m and applies a force of 100 N. Calculate the velocity v of the ball.

Sol. Here,

Force applied, $F = 100\ N$

Distance in which ball is stopped, $S = 0{\cdot}1\ m$

Mass of ball, $m = 0{\cdot}2$ kg

Velocity of the ball, $v = ?$

Kinetic energy of the ball $= \frac{1}{2}mv^2$

As the field–man moves his hands backwards by $0{\cdot}1\ m$, therefore, the ball does work against a force of $100\ N$ over $0{\cdot}1\ m$ before coming to rest.

$$\therefore \quad \text{Work done on ball } = F \times S = 100\ N \times 0{\cdot}1 = 10 \text{ joules}$$

But the kinetic energy of the ball is equal to the work done on it.

$$\therefore \quad \frac{1}{2}mv^2 = 10 \text{ joules}$$

$$\text{or} \quad \frac{1}{2} \times 0 \cdot 2 \times v^2 = 10$$

$$\text{or} \quad v^2 = 100$$

$$\text{or} \quad v = \mathbf{10\ ms^{-1}} \text{ (Ans.)}$$

Ex. WPE—18. A car of mass 500 kg on a level road takes one minute to attain a speed of 120 km/hr. starting from rest. If it accelerates at a uniform rate and the co–efficient of friction between tyres and road is 0·05, find the H.P. of the engine applied.

Sol. Here,

Mass of car, $m = 500$ kg

Velocity of car, $v = 120\ km/hr = \frac{120 \times 1000 m}{60 \times 60 \text{second}} = \frac{100}{3} ms^{-1}$

The engine of car has to overcome :

(*i*) Force of friction.

(*ii*) Maintain uniform acceleration

∴ Total force required by car engine,

= Force of friction + Force to maintain uniform acceleration or $$F = f + F_1$$

(*i*) Force of friction, $f = \mu R$ [R = Normal reaction]

$$= \mu\, mg = 0{\cdot}05 \times 500 \text{ kg} \times 9{\cdot}8\ ms^{-2}$$

$$= 245\ N$$

(*ii*) *Force F_1 required to maintain uniform acceleration*

Acceleration of car, $a = \dfrac{v-u}{t}$

Here, $u = 0,\ v = \dfrac{100}{3}\ m/s,\ t = 1$ minute $= 60$ seconds

$$\therefore \quad a = \frac{v-u}{t} = \frac{100/3-0}{60} = \frac{100}{180}\, ms^{-2}$$

Force required to maintain uniform acceleration,

$$F_1 = ma = 500 \text{ kg}\ \ 100/180\ ms^{-2} = 277{\cdot}78\ N$$

∴ Total force required by car engine,

$$F = f + F_1$$

$$= 245 + 277{\cdot}78 = 522{\cdot}78\ N$$

Distance S travelled in one second,

$$S = ut + \frac{1}{2}at^2 = 0 \times t + \frac{1}{2} \times 100/180 \times (1)^2$$

$$= 0 + \frac{1}{2} \times 100/180 \times 1 = 5/18\, m$$

∴ Work done/second = Total force applied × Distance travelled per sec.

$$= 522{\cdot}78\ N \times 5/18\ m/s$$

or Power = 145·55 joules/second.

= 145·55 watts = **0·194 H.P. (Ans.)**

Ex. WPE—19. Calculate the power of an engine required to carry a mass of 1300 kg up an inclined surface of 50 in 130 at a speed of 90 km/hr. The force of friction is 0·08 N/kg.

Sol. Here,

Mass, $m = 1300$ kg

Normal reaction, $R = mg\cos\theta$

$$= 1300 \times 9 \cdot 8 \times \frac{120}{130} = 11760\ N$$

Speed $= 90\ km/hr$

$$= \frac{90 \times 1000\, m}{60 \times 60\,\text{sec.}} = 25\, ms^{-1}$$

Total force of friction, $f = 0{\cdot}08 \times 1300 = 104\ N$

Now the engine has to overcome force of friction f and rise against $mg \sin \theta$ component of gravity.

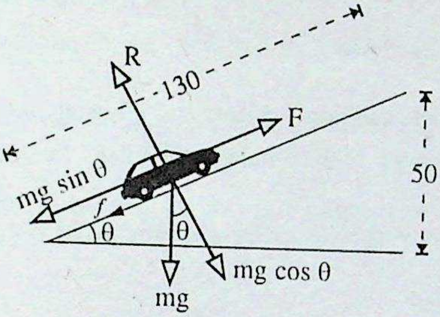


Fig. WPE—11

∴ Total force required by the engine

$$= mg \sin \theta + f$$

$$= 1300\,\text{kg} \times 9 \cdot 8 \frac{m}{s^2} \times \frac{50}{130} + 104\ N$$

$$= 4900\ N + 104{\cdot}00\ N = 5004\ N$$

$$\therefore \quad \text{Work done/second} = \text{Total force} \times \text{Distance/sec.}$$

$$= 5004\ N \times 25\ m/s = 125100\ N\,m\,s^{-1}$$

i.e. $\quad \text{Power} = 125100 \text{ joules}/s = 125100 \text{ watts}$

$$= \mathbf{125{\cdot}1\ kW} \ \textbf{(Ans.)}$$

QUESTIONS FOR SELF TEST

1. Explain the term work with suitable examples. Mention its units.
2. 'A person walking on a horizontal road with a load on his head does no work'. Explain.
3. Define the term joule. Find an expression for the work done when a body of mass m is raised through a height h.
4. Explain the terms :–

   (*i*) Friction

   (*ii*) Co–efficient of friction

   (*iii*) Limiting friction

   (*iv*) Laws of friction
5. Justify the statement that friction is a necessary evil.
6. Derive an expression for the work done against friction along :–
7. Explain the term power with suitable examples. Mention its units.

8. *Define* :-

   (*i*) Joule

   (*ii*) watt

   (*iii*) horse-power

   (*iv*) kilowatt.

   Establish a relation between horse-power and kilowatt.

9. Explain the term energy with suitable examples. Mention different forms of energy.

10. A body of mass *m* moving with speed *v* is brought to rest. Derive an expression for its kinetic energy. *(Haryana, May, 1986)*

11. Define :-

    (*i*) Kinetic energy

    (*ii*) Potential energy.

    Derive an expression for the kinetic energy of a body of mass *m* moving with a uniform velocity *v*.

12. Explain the terms work, power and energy and give the units for measuring these quantities.

13. What is meant by 'Conservation of Energy' ? Show that the sum of kinetic and potential energies of a freely falling body is constant at all points of its motion.

14. What is the principle of conservation of energy ? Give any five examples of transformation of energy from one form into another. A ball rolling on the ground comes to rest. Where does its kinetic energy go ?

PROBLEMS

1. A body requires a force of 25 *N* to move it through a horizontal distance of 30 *m*. Calculate the work done if the force makes an angle of 30° with the horizontal.

   (Ans. 649·5 joules)

2. A man carrying a suitcase of mass 30 kg walks on horizontal road through a distance of 10 *m* and then rises 10 *m* in the vertical direction through a staircase. Calculate the total work done. **(Ans.** 2940·0 joules)

3. A truck of mass 1800 *kg* is raised from the bottom up an incline of 1 in 100 through a distance of 10 *m*. Calculate the total work done if the coefficient of friction is 0·25. **(Ans.** 3·6912 joules)

4. Find the power of an engine required to move a train of mass 50000 kg with a velocity of 40 *km/hour* on a level track. The force of friction is 8 *N*/100 kg.

   (Ans. 44·4 *kW*)

5. A solid mass of 0·5 kg is allowed to drop from a height of 100 metres. Calculate the amount of kinetic energy gained by the body, $g = 9{\cdot}8\ m/s^2$. **(Ans.** 490 J)

6. A hammer pile of 10 kg mass falls through 1*m* and drives a nail through 0·1 *m*. Find the resisting force. **(Ans.** 980 *N*).

7. A bullets of mass 20g is found to pass two points 30 m apart in 4s. Assuming its speed to be constent, find its K.E. **(Ans.** 0·5625 J)

8. A bag of wheat weighing 90 kg, is raised to a height of 5m. Find the potential energy. **(Ans.** 4410 J)

9. How much mass is converted into energy in a nuclear plant operated at 200 MW.

   (Ans. 5·76 gram)

10. A bullet moving with a velocity of 200 m/s pieces a wooden target upto 6 cm. What velocity should it have to be able to piece it up to 4 cm.

    (Ans. $100\sqrt{6}$ m/s)

11. Find the power of the engine which is required to lift 90 metricton of coal per hour from a mine 200 m deep. **(Ans.** 49 KW)

Chapter

RM

Rotational Motion

RM—1. RIGID BODY

A rigid body may be defined as a body which can not be deformed. In a rigid body the separations between all pairs of particles remain constant

Concept of a Body. When an extended body, such as a wheel, rotates about its axis, the motion cannot be analyzed by considering the body as a particle. It is because, at any given time different parts of the wheel will have different velocities and accelerations. Due to this reason, it is convenient to consider a body to be made up of a *large number of particles,* each having its own velocity and acceleration.

All real bodies in nature are deformable to some extent. Thus approach to a rigid body model is a situation where deformation is negligible.

RM—2. ROTATIONAL MOTION

The rotation of a body about a fixed axis is called **rotational motion.**

In this case the body turns around a fixed axis. The different particles of the body describe concentric circles. The speeds of different particles are different but all the particles have same angular velocities. A body may possess both translational as well as rotational motion. A spinning cricket ball when thrown by a player is an example of a body having both translational and rotational motions.

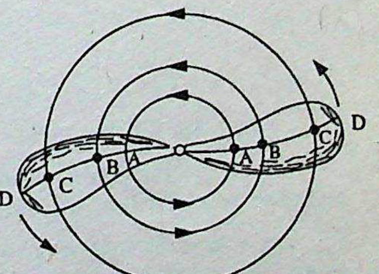


Fig. RM—1

Examples of Rotational Motion :

(i) Motion of a wheel about its axle.

(ii) Rotation of earth about its axis.

(iii) Rotation of the disc of a gramophone record player.

Translational Motion or Linear Motion. *The motion of a body as a whole along a straight line is called* **translational or linear motion.**

In this case the body moves as a whole from one place to another. All the particles of the body move along parallel lines with the same speed.

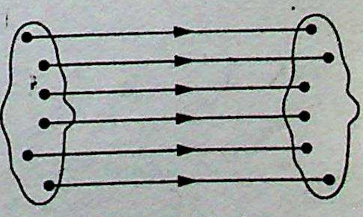


Fig. RM—2

Examples of Translational or Linear Motion :

(i) Motion of a car on the road.

(ii) Motion of an aeroplane.

(iii) Motion of your body while running a race.

RM—3. IMPORTANT TERMS USED IN ROTATIONAL MOTION :

(i) **Angular Displacement (θ) :** *The angle described by a rotating particle with respect to a fixed line in a given time is called* **angular displacement**. It is denoted by θ.

Let the particle moves through a distance l along the circumference of the circle of radius r in a time t and θ be the angle described at the centre of the circle, then,

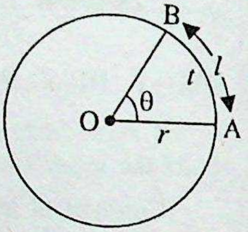


Fig. RM—3

$$\text{Angular displacement, } \theta = \frac{l}{r}$$

or

$$l = r\theta$$

∴ Linear displacement = radius × angular displacement

(ii) **Angular Velocity (ω).** *The angle described by a rotating body per unit time is called* **angular velocity.** It is denoted by ω.

Let the rotating body describe an angle θ at the centre of the circle in time t, then,

$$\text{Angular velocity} = \frac{\text{Angle described (in radians)}}{\text{Time}}$$

or

$$\omega = \frac{\theta}{t} \text{ rad/s}$$

or

$$\theta = \omega t \text{ radians}$$

∴ Angular displacement = angular velocity × time.

(iii) **Angular Acceleration (α).** *The rate of change of angular velocity is called* **angular acceleration.** It is denoted by α.

Let a force acting on a rotating body changes its angular velocity from ω_1 to ω_2 in a time t, then

$$\text{Angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Time}}$$

or

$$\alpha = \frac{\omega_1 - \omega_2}{t} \text{ rad/s}^2$$

or

$$\omega_2 - \omega_1 = \alpha t$$

or

$$\omega_2 = \omega_1 + \alpha t$$

∴ **Final angular velocity** = Initial angular velocity + Angular acceleration × Time *(iv)* **Average Angular Velocity.** If the angular velocity of a body moving with uniform angular acceleration changes from ω_1 to ω_2, then average angular velocity ω is given as,

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

(v) **Angular velocity in terms of time period.** Let a body executing uniform circular motion has time period T and angular velocity ω. The angle described by the body at the centre of the circle in one revolution is 2π radians and time taken is T seconds.

$$\text{Thus, angular velocity} = \frac{\text{Angle described}}{\text{Time taken}}$$

$$\therefore \qquad \omega = \frac{2\pi}{T}$$

$$\text{Hence Angular velocity} = \frac{2\pi}{\text{Time period}}$$

(vi) **Angular velocity in terms of frequency :**

Let, n = frequency and

ω = angular velocity of a body executing uniform circular motion.

Angle described by the body in 1 revolution = 2π rad.

Angle described by the body in n revolutions = $2\pi \times n$ rad.

But n revolutions are made in 1s (frequency) and the angle described in 1s is th angular velocity ω.

$$\therefore \qquad \omega = 2\pi n$$

Hence Angular velocity = 2π × frequency.

***(vii)* Angular velocity in terms of R.P.M. (N) :**

Let a body executing circular motion has,

ω = angular velocity

N = revolutions per minute (R.P.M.)

Angle described by the body in 1 revolution = 2π *rad.*

$\therefore$ Angle described by the body in N revolutions = $2\pi N$ *rad.*

or Angle described in 1 minute = $2\pi N$ *rad.*

$$\text{Hence, Angle described in 1 second} = \frac{2\pi N}{60}\ rad.$$

But angle described in 1 second is angular velocity (ω).

$$\therefore \qquad \omega = \frac{2\pi N}{60}\,\text{rad / s} = \frac{2\pi \times R.P.M.}{60}\,rad\,/\,s$$

Imp.

RM—4. MOMENT OF INERTIA (M.I.)

Moment of Inertia *of a rotating body about an axis is defined as the sum of the products of the masses and squares of the distances of different particles of the body from the axis of rotation.*

When a body is freely rotating about an axis, it has a tendency to keep itself in the same state of motion. This inertness of the rotating body to oppose changes produced by external agencies in its state of rotatory motion is called *rotational inertia* or *moment of inertia* about the given axis of rotation. The kinetic energy of rotation of the body depends not only on its mass and angular velocity but also depends upon the distribution of mass about the axis of rotation.

Expression for the Moment of Inertia :

Suppose a body of mass M revolves about any axis XY. Let, the body be supposed to be made of large number of particles of masses m_1, m_2, m_3 etc. at distances r_1, r_2, r_3 etc. from the axis XY. The moment of inertia of the particle m_1 about XY is $m_1r_1^2$ and that of particle m_2 is $m_2r_2^2$ and so on. Therefore, moment of inertia I, of the whole body, about the axis XY is equal to the sum of $m_1r_1^2, m_2r_2^2, m_3r_3^2$........etc.

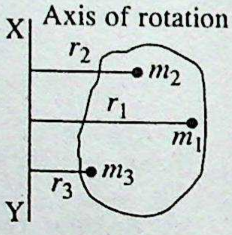


Fig. RM—4

$$\therefore \quad \text{Moment of inertia,} \quad I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots..$$

$$= \Sigma mr^2$$

or

$$I = MK^2$$

Where K is the **radius of gyration** of the body about the axis XY. The *S. I.* units of moment of inertia are *kg* m^2.

Physical Significance of Moment of Inertia. Moment of inertia has the same physical significance in rotatory motion as mass in linear motion *i.e.*, moment of inertia is analogue of mass in linear motion.

RM—5. RADIUS OF GYRATION (K).

Radius of gyration of a body, may be defined as the distance of a point from the axis of rotation at which whole mass of the body is supposed to be concentrated, so that moment of inertia about the axis remains the same. It is denoted by K.

If the mass of the body is M, the moment of intertia *(I)* of the body in terms of radius of gyration is given as,

$$I = MK^2$$

Also

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots = \Sigma mr^2$$

If mass of each particle is m and there are n molecules in all, then

$$M = mn$$

Also

$$MK^2 = \Sigma mr^2 = mn\left(\frac{r_1^2 + r_2^2 + r_3^3 + \ldots..}{n}\right)$$

$$\therefore \qquad K^2 = \left(\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots}{n}\right)$$

or

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \ldots\ldots}{n}}$$

RM—6. ANGULAR MOMENTUM

Imp.

Angular momentum of a rotating body about its axis of rotation is the algebraic sum of the moments of the linear momenta of its particles about that axis.

In linear motion, the *momentum* of a body is the product of its *mass and linear velocity*. On the same analogy in the case of **rotational motion,** the product of the moment of *inertia and angular velocity* is **angular momentum** of a rotating body.

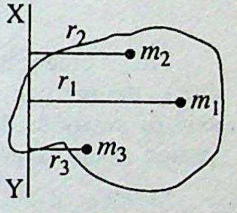


Fig. RM—5

To find an expression for angular momentum, consider a body rotating about an axis with angular velocity ω. Then all the particles of the body have the *same* angular velocity ω, but their linear velocities depend upon their respective distances from the axis of rotation.

Consider the particles with masses m_1, m_2, m_3...etc. of a body at distances r_1, r_2, r_2...etc. from the axis of rotation and having linear velocities v_1, v_2, v_3...etc. respectively.

Therefore, linear momentum of the particles are, m_1v_1, m_2v_2, m_3v_3...etc.

Now moment of momentum = linear momentum × distance

$$\therefore \quad \text{Angular Momentum} = m_1v_1r_1 + m_2v_2r_2 + m_3v_3r_3 + \ldots\ldots$$

If ω is the angular velocity of all the particles of the body,

then $v_1 = r_1\omega$, $v_2 = r_2\omega$ and $v_3 = r_3\omega$.

$$\begin{aligned}\therefore \quad \text{Angular Momentum} &= m_1r_1\omega.r_1 + m_2r_2\omega.r_2 + m_3r_3\omega.r_3 + \ldots\ldots \\ &= m_1r_1^2\omega + m_2r_2^2\omega + m_3r_3^2\omega + \ldots\ldots \\ &= (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots)\,\omega\end{aligned}$$

$$\therefore \qquad L = I\omega \qquad \text{(where } I = \text{Moment of Inertia)}$$

Thus angular momentum = Moment of Inertia × Angular velocity.

Analogy. We have $L = I\omega$ (angular momentum)

Also $M = mv$ (linear momentum)

The analogue of **linear momentum** (mv) in rotatory motion is called **angular momentum** ($I\omega$).

The analogue of *mass* in rotatory motion is *moment of inertia* and that of linear velocity *(v)* is angular velocity (ω).

RM—7. TORQUE

It is defined as the algebraic sum of the moments of the forces acting on the rotating particles constituting the body. It is denoted by

To accelerate a moving body we have to apply force. Similarly to accelerate the rotation of a body, free to rotate about an axis, we have to apply a couple on it. *The moment of the couple, so applied, is called torque.*

Expression for Torque :

Consider a rigid body consisting of particles of masses $m_1, m_2, m_3.....$ lying at distances $r_1, r_2, r_3.....$ respectively from the axis of rotation XY. When an external couple is applied to the body, it acquires an angular acceleration α. The linear acceleration of the various particles will be $r_1\alpha$, $r_2\alpha$, $r_3\alpha.....$ respectively.

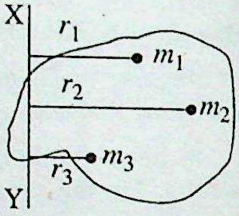


Fig. RM—6

Force acting on the particle of mass $m_1 = m_1 r_1 \alpha$

Moment of the force about the axis = Force × distance

$$= m_1 r_1 \alpha \times r_1 = m_1 r_1^2 \alpha$$

This moment is equal to the moment of the couple acting on the particle because due to the inertial reaction there is an equal; opposite and parallel force acting at the axis of rotation. These two forces form a required couple which accelerates the motion of the rotating body.

Moment of the couple for second particle $= m_2 r_2^2 \alpha$

$\therefore$ Total moment of the couple for the rotating body

$$= m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + ...$$

This quantity is called the torque or the moment of the couple acting on the body about the axis of rotation XY.

$$\therefore \quad \text{Torque} = \Sigma m r^2 \alpha$$

$$\text{or} \quad \tau = I\alpha$$

$$\text{Torque} = \text{Moment of Inertia} \times \text{Angular acceleration.}$$

Hence, *torque is the product of the moment of inertia and the angular acceleration.*

In the relation,

$$\text{Torque} = I\alpha$$

$$\text{or} \quad \text{Torque} = I\frac{d\omega}{dt} \qquad \left[\because \alpha = \frac{d\omega}{dt}\right]$$

This relation corresponds to the relation

$$\textit{Force} = \textit{mass} \times \textit{acceleration}$$

Thus in rotatory motion, *torque, moment of Inertia* and *angular acceleration* are analogues of *force, mass* and *linear acceleration* respectively in the case of linear motion,

In case a force F acts on a body and moves it in a circle of radius r, then

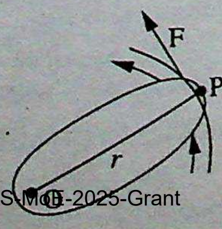


Fig. RM—7

$$\text{Torque} = \text{Force} \times \text{Radius}$$

$$\text{or} \quad \tau = F \times r$$

Ex. RM—1. What torque will produce an acceleration of 2 rad/s^2 in a body of moment of inertia 500 $kg\ m^2$?

Sol. Here, $I = 500\ kg - m^2$

and $\alpha = 2\ rad/s^2$

Now, torque, $\tau = I\alpha$

$\therefore \quad \tau = 500\ kg - m^2 \times rad/s^2 = 1000\ kg\ m^2\ s^{-2}$

$= 1000\ Nm$ or joules. **Ans.**

Ex. RM—2. An engine is rotating at the rate of 1500 rev. per minute. Find its angular velocity.

Sol. Here, *R.P.M.* of engine, $N = 1500$

$\therefore$ Angular velocity, $\omega = 2\pi \frac{N}{60} = 2 \times \frac{22}{7} \times \frac{1500}{60}$

$= 157{\cdot}1$ rad/s **Ans.**

Ex. RM—3. A fly wheel of mass 500 kg. and diameter 1 metre revolves about its axle. Its frequency is increased by 18 in 5 sec. Calculate torque applied.

Sol. Here, Mass of wheel, $M = 500\ kg$

Diameter of wheel, $D = 1\ m$

$\therefore$ Radius, $R = 0{\cdot}5\ m$

Moment of inertia of wheel, $I = \frac{1}{2} MR^2 = \frac{1}{2} 500 \times (\cdot 5)^2$

$= 62{\cdot}50\ kg\ m^2$

Again, let initial frequency, be n_1 and final frequency, n_2

$\therefore$ Increase in frequency, $(n_2 - n_1) = 18\ Hz$

Time, $t = 5\ s$

Angular acceleration, $\alpha = \frac{\text{Change in angular velocity}}{\text{Time}}$

$$\therefore \quad \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi n_2 - 2\pi n_1}{t}$$

$$= \frac{2\pi}{t}(n_2 - n_1) = \frac{2 \times 22}{5 \times 7}(18) = 22{\cdot}63\ rad/s^2$$

Torque applied, $\tau = I\alpha$

$\therefore \quad \tau = 62{\cdot}50\ kg - m^2 \times 22{\cdot}63\ rad/s^2$

$= 1414{\cdot}375\ kg\ m^2\ s^{-2}$ **Ans.**

Ex. RM—4. How large a torque is needed to accelerate a wheel, for which $I = 2\ kg - m^2$, from rest to 30 r.p.s. in 20 seconds.

Sol. Here, Moment of inertia, $I = 2\ kg - m^2$

R.P.S. after 20 sec., $n = 30$

Initial velocity, $\omega_1 = 0$

Final velocity, $\omega_2 = 2\pi \times 30 \; rad/s$

$$\therefore \text{ Angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi \times 30 - 0}{20} = 9{\cdot}43 \; rad/s^2$$

$$\text{Hence, torque required, } \tau = I \times \alpha = 2 \; kg - m^2 \times 9{\cdot}43 \; rad/s^2$$

$$= 18{\cdot}86 \; kg \; m^2 \; s^{-2} \text{ Ans.}$$

Ex. RM—5. A grinding stone has moment of inertia of 200 kg m^2, and a radius of 25 cm. On applying a torque, it is found to acquire a speed of 300 rev. per minute starting from rest position in a time of 10 sec. Find the torque acting on the stone.

Ans. Here, Moment of Inertia, $I = 200 \; kg - m^2$

Initial angular velocity, $\omega_1 = 0$

Speed after 10 *s*, $N = 300 \; R.P.M.$

$$\therefore \text{ Angular velocity after 10 } s, \; \omega_2 = 2\pi \frac{N}{60} = 2 \times \frac{22}{7} \times \frac{300}{60} = 31{\cdot}4 \; rad/s$$

$$\text{Angular acceleration, } \alpha = \frac{\omega_2 - \omega_1}{t} = \frac{31 \cdot 4 - 0}{10} = 3{\cdot}14 \; rad/s^2$$

Now, Torque $= I \times \alpha$

$$\therefore \quad \tau = 200 \; kg \; m^2 \times 3{\cdot}14 \; rad/s^2$$

$$= 628{\cdot}00 \; kg \; m^2 \; s^{-2} \text{ or } Nm \text{ Ans.}$$

RM—8. LAW OF CONSERVATION OF ANGULAR MOMENTUM

Law of conservation of angular momentum states that angular momentum of a rotating body about an axis remains constant, if no external torque be applied to it.

Proof. Let I be the moment of inertia and α the angular acceleration, then torque is given by,

$$\tau = I\alpha$$

$$\text{But} \quad \alpha = \frac{d\omega}{dt}, \text{ therefore } \tau = I \frac{d\omega}{dt} \qquad \ldots(i)$$

The angular momentum is given relation $L = I\,\omega$

$$\text{Differentiating, we get} \quad = \frac{dL}{dt} = I \frac{d\omega}{dt} \qquad \ldots(ii)$$

$$\text{Comparing eqns. } (i) \text{ and } (ii), \text{ we get } \frac{dL}{dt} = \tau$$

If external torque on the body is zero *i.e.*, $\tau = 0$, then $\frac{dL}{dt} = 0$

which is possible only if $L = I\omega$ is constant.

Thus "In the absence of external torque angular momentum of a body remains constant."

This is called law of conservation of angular momentum.

Applications. The use of this principle is employed by acrobats divers, ballet-dancers and ice-skaters. Since moment of inertia depends upon square of the distance of the parts of the body from the axis of rotation, by extending or pulling in limbs of the body he can vary the value of moment of inertia according to his need. Thereby angular velocity can be increased or decreased accordingly. A cat manages to land on its feet after a fall by using the same principle, its tail serving as a useful limb.

Ex. RM—5. A boy is standing on a 'rotating plateform with his arms extended and rotating with a frequency 0·40. When he draws his hands in, the frequency is 0·64. Compare the M.I. in the two cases.

Ans. In this case let I_1 be the moment of inertia of the system and ω_1 the angular velocity, then

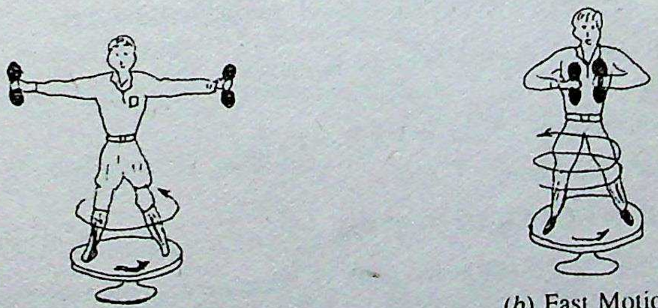
$$\text{Angular Momentum} = I_1\,\omega_1 \qquad \ldots(i)$$

When he draws his arms in (fig RM—8 (*b*) let the moment of inertia decreases to I_2 and angular velocity increases to ω_2

$$\therefore \quad \text{Angular momentum} = I_2\,\omega_2 \qquad \ldots(ii)$$

According to the law of conservation of angular momentum,

$$I_1\,\omega_1 = I_2\,\omega_2$$



(*a*) Slow motion

(*b*) Fast Motion

Fig. RM—8.

Here, $n_1 = 0{\cdot}40$ and $n_2 = 0{\cdot}64$

$\therefore \quad \omega_1 = 2\pi \times n_1 = 2\pi \times 0{\cdot}40$

and $\quad \omega_2 = 2\pi \times n_2 = 2\pi \times 0{\cdot}64$

$\therefore \quad I_1 \times 2\pi \times 0{\cdot}40 = I_2 \times 2\pi \times 0{\cdot}64$

Hence, $\quad \dfrac{I_1}{I_2} = \dfrac{2\pi \times 0\cdot64}{2\pi \times 0\cdot40} = \dfrac{64}{40} = \dfrac{8}{5}$

$\therefore \quad I_1 : I_2 = 8 : 5$ **Ans.**

RM—9. KINETIC ENERGY OF ROTATION

Kinetic Energy of rotation of a rigid body is the sum of kinetic energies of all the constituent particles of the body rotating about a given axis.

Expression for K.E. of rotation :

Consider that a body rotates about an axis xy, m_1, m_2, m_3,..... are the masses of its particles ; r_1, r_2, r_3,...... are their respective distances from the axis of rotation see fig. **RM—9.** Let v_1, v_2, v_3,...... be the linear velocities of the particles.

∵ According to definition, kinetic energy of rotation of the body,

$$\text{K.E.} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + ...$$

Let ω be the angular velocity of the body, then all the particles have the same angular velocities but as their distances from the axis of rotation differ, their linear velocities are different thus,

$$v_1 = r_1\omega,\ v_2 = r_2\omega,\ v_3 = r_3\omega.....$$

Substituting these values, we get

$$\text{K.E.} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + ...$$

$$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ...)\omega^2 = \frac{1}{2}I\omega^2$$

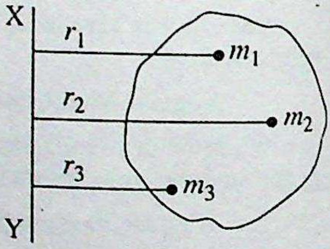


Fig. RM—9

Where, $I = \Sigma mr^2$ = the moment of inertia of the body.

This relation is similar to $\text{K.E.} = \frac{1}{2}mv^2$ for translational motion.

Torque : Since the angular velocity is constant, therefore, the body has no acceleration *i.e.*, $\alpha = 0$

$$\text{Hence torque,} = I\alpha = I \times 0 = 0$$

No torque acts on the body.

RM—10. WORK IN ROTATING SYSTEM

Consider a rigid body fixed at the point O. Suppose a single external force F is applied at the point P. The work done by force F as the body rotates through a small distance, $ds = r\,d\theta$ in a time dt is given as,

Work done, $dw = F \times ds = F \sin\phi\, rd\theta$. Where, $F \sin\phi$ is the tangential component of F or the component of force along the displacement.

$$\therefore \quad \text{Work done} = \text{force} \times \text{displacement}$$

$$= F \sin\phi\, r \times d\theta$$

$$\text{But } F \sin\phi \times r = \text{torque} = \tau$$

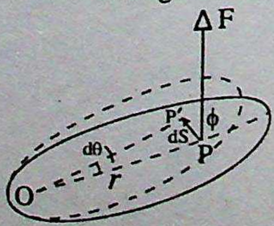


Fig. RM—10

$$\text{Work done, } dw = \tau\, d\theta \quad ...(i)$$

or Work done = torque × angular displacement.

RM—11. POWER IN ROTATING SYSTEM

Power is the rate of doing work. Therefore, the rate at which work is being done by the force F for rotation about the fixed axis is obtained by dividing *(i)* both sides by dt.

$$\therefore \quad \frac{dw}{dt} = \frac{d\theta}{dt} = \omega \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$$

$$\therefore \quad \text{Power} = \tau \times \omega$$

$$\text{or} \quad \text{Power} = \text{torque} \times \text{angular velocity}$$

Thus work in linear system is given by the product of *force* and *linear displacement* $[W = F \times S]$, where as in rotational motion work is given by the product of torque and *angular displacement*.

∴ Torque in rotational motion has the same significance as force in linear motion.

Similary, power $= F \times v$ (linear system)

and Power $= \tau \times \omega$ (rotating system)

RM—12. ROLLING, COMBINATION OF ROTATIONAL AND TRANSLATIONAL MOTION

When bodies having high degree of symmetry like a *disc, a cylinder* or a sphere roll down on a smooth horizontal surface without slipping, have two types of kinetic energies.

(i) K.E. of rotation due to its rotation about an axis through the centre of mass.

(ii) K.E. of translation due to its linear motion.

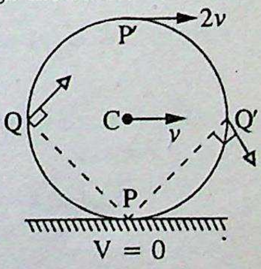


Fig. RM—11

Let a body of mass M and moment of inertia I (about its axis of rotation) rolls down on a horizontal surface with a uniform linear velocity υ.

If angular velocity of rotation is ω, then

$$K.E. \text{ of rotation of the body} = \frac{1}{2} I \omega^2$$

$$\text{and} \quad K.E. \text{ of translation} = \frac{1}{2} M \upsilon^2$$

∴ Total $K.E.$ of rolling body $= K.E.$ of rotation $+ K.E.$ of translation.

$$= \frac{1}{2} I \omega^2 + \frac{1}{2} M \upsilon^2 = \frac{1}{2} MK^2 \omega^2 + \frac{1}{2} M \upsilon^2$$

$$= \frac{1}{2} M(K^2 \omega^2 + \upsilon^2)$$

Now, $\omega^2 = \dfrac{\upsilon^2}{r^2}$ where r is the radius of the body

$$\therefore \quad \text{Total } K.E. \text{ of the body} = \frac{1}{2} MK^2 \left(\frac{\upsilon^2}{r^2}\right) + \frac{1}{2} M \upsilon^2 = \frac{1}{2} M \upsilon^2 \left[\frac{K^2}{r^2} + 1\right]$$

For a solid sphere $K^2 = \frac{2}{5}r^2$

$$\therefore \quad \text{Total } K.E. = \frac{1}{2}M\,\upsilon^2\left(\frac{2}{5}r^2 \times \frac{1}{r^2}+1\right)$$

$$= \frac{1}{2}M\,\upsilon^2\,\frac{7}{5} = \frac{7}{5} \times (K.E. \text{ of translation})$$

Thus rolling is a combination of rotational and translational motion having *K.E.* of rotation as well as *K.E.* of translation.

RM—13. ROLLING OF A CYLINDER DOWN A ROUGH INCLINED PLANE.

Let a cylinder of mass *M* rolls freely down an inclined plane, of inclination θ to the horizontal. The plane is rough, therefore, there is no slipping and no work is done against friction.

Let υ be the velocity acquired by the cylinder after falling through a vertical height *h*.

∴ Loss of *P.E.* of the body = *Mgh*

The loss of *P.E.* must be equal to the *K.E.* gained by the cylinder.

The rolling cylinder has *K.E.* of rotation and *K.E.* of translation.

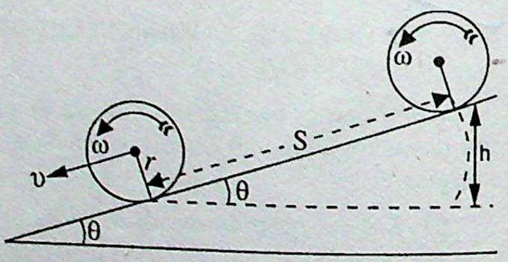


Fig. RM—12

$$\text{Now,} \quad K.E. \text{ of rotation} = \frac{1}{2}I\,\omega^2$$

$$\text{and} \quad K.E. \text{ of translation} = \frac{1}{2}M\,\upsilon^2$$

$$\therefore \quad \text{Total } K.E. \text{ gained} = \frac{1}{2}I\,\omega^2 + \frac{1}{2}M\,\upsilon^2 = \frac{1}{2}MK^2\,\omega^2 + \frac{1}{2}M\,\upsilon^2$$

$$= \frac{1}{2}MK^2\frac{v^2}{r^2} + \frac{1}{2}M\,\upsilon^2 = \frac{1}{2}M\,\upsilon^2\left[\frac{K^2}{r^2}+1\right]$$

According to the law of conservation of energy, *K.E.* gained = *P.E.* lost.

$$\therefore \quad \frac{1}{2}M\,\upsilon^2\left[\frac{K^2}{r^2}+1\right] = Mgh$$

$$\text{or} \quad \upsilon^2\,(k^2+r^2)/r^2 = 2gh \quad \text{or} \quad \upsilon^2 = \frac{2gh\,r^2}{(k^2+r^2)}$$

Comparing this with the equation $\upsilon^2 - 0^2 = 2\,aS$ for a body starting from rest, we have,

$$\frac{2gh\,r^2}{k^2+r^2} = 2\,aS = 2a\frac{h}{\sin\theta} \qquad \left[\sin\theta = \frac{h}{S}\right]$$

$\therefore$ Acceleration of the cylinder down the plane, $a = \dfrac{gr^2 \sin\theta}{k^2 + r^2}$

The acceleration is proportional to $\dfrac{r^2}{k^2 + r^2}$ for a given angle of inclination θ.

This shows that,

(i) The greater the value of K, as compared to r, the smaller will be the acceleration of the body rolling down the plane. Thus the body takes more time to roll down.

(ii) The acceleration and, therefore, the time of descent are independent of the mass of the body.

Hence, a solid sphere, for which $K^2 = \dfrac{2\,r^2}{5}$ will roll down faster than a disc, for which $K^2 = r^2/_2$, and similarly a disc will roll down faster than a loop, for which $K^2 = r^2$. K^2 for a hollow sphere is greater than for a solid sphere of the *same mass and radius,* they can be distinguished from each other by allowing them to roll down the inclined plane.

Ex. RM—6. A ball with a diameter of 25 cm and weight of 5 kg rolls along a floor at 5 m/s. Calculate its total kinetic energy.

Sol. A rolling ball has both, energy of rotation as well as energy of translation.

Here, Mass of ball, $M = 5\ kg$

Radius of ball, $r = \dfrac{25}{2} = 12 \cdot 5\,cm = 0 \cdot 125\,m$

Linear velocity, $\upsilon = 5\ m/s$

$\therefore$ Angular velocity, $\omega = \upsilon / r = \dfrac{5}{0 \cdot 125} = 40 \text{ rad / s}$

Moment of inertia of the ball (sphere) about its diameter,

$$I = \frac{2}{5} \times \text{mass} \times (\text{radius})^2$$

$$= \frac{2}{5} \times 5 \times (0 \cdot 125)^2 = 0{\cdot}031250\ kg\ m^2$$

Now $K.E.$ of translation $\dfrac{1}{2} = \dfrac{2}{5} M\,\upsilon^2 = \dfrac{1}{2} \times 5 \times 25 = 62 \cdot 5$ joules

and $K.E.$ of rotation $= \dfrac{1}{2} I\,\omega^2 = \dfrac{1}{2} \times 0 \cdot 03125 \times (40)^2 = 25 \cdot 0$ joules

$\therefore$ Total energy $= K.E.$ of translation $+$ $K.E.$ of rotation

$= 62{\cdot}5 + 25{\cdot}0$

Total energy $=$ 87·5 joules (Ans.)

Ex. R.M.-8 **A 40 Kg flywheel in the form of a uniform circular disc 100 cm in radius is making 120 rev/min. Calculate the angular momentum of wheel. Given moment of inertia of disc = $\frac{1}{2}$ MR^2.**

Sol. Given, $M = 40$ Kg, $R = 1$m,

$$I = \frac{1}{2} \times MR^2 = \frac{1}{2} \times 40 \times 1^2 = 20 \text{ Kg } m^2.$$

Also $n = \frac{120}{60} = 2$ rev/s.

$$\omega = 2\pi n = 2 \times 3{\cdot}142 \times 2 = 12{\cdot}57 \text{ rad/s.}$$

Thus angular momentum, $L = I\omega = 20 \times 12{\cdot}57 = 251{\cdot}4$

$$L = 251{\cdot}4 \text{ Kg}m^2 s^{-1}.$$

Ex. R.M.-9 **A wheel of grindstone has applied at its axle 2cm in radius a constant tangential force of 600 N. Find the angular momentum and torque on it after 8 seconds.**

Sol. Given $R = 2 \text{ cm} = \frac{2}{100} = 0{\cdot}02\ m.$

Force, $F = 600\ N.$, time $t = 8$ seconds.

$$\tau = F \times R = 600 \times 0{\cdot}02 = 12\ N.$$

Change in angular momentum = $\tau t = 12 \times 8 = 96\ Nms.$

Initial angular momentum = 0

∴ Angular momentum acquired = 96 $Nms.$

Ex. R.M.-10 **Calculate the kinetic energy of rotation of a circular disc of mass 1 Kg and rota 0·2 m relating about an axis passing through the centre and perpendicular to its plane. The disc is making $\frac{30}{\pi}$ rotations per minute.**

Sol. Given, $M = 1$ Kg, $R = 0{\cdot}2\ m$

∴ Moment of inertia of disc about an axis passing through its centre and perpendicular to its plane is

$$I = \frac{1}{2} MR^2 = \frac{1}{2} \times 1 \times (0{\cdot}2)^2 = 0{\cdot}02 \text{ Kg } m^2$$

$$n = \frac{30}{\pi} \text{ rot./min} = \frac{30}{\pi \times 60} = \frac{1}{2\pi} \text{ rot/s}$$

$$\omega = 2\pi n = 2\pi \times \frac{1}{2\pi}\ 1 \text{ rad/s}$$

The $K.E.$ of rotation $= \frac{1}{2} I\omega^2 = \frac{1}{2} \times 0{\cdot}02 \times (1)^2$

$K.E.$ of rotation $= 0{\cdot}01\ J$ **Ans.**

RM—14. PARAMETERS IN TRANSLATIONAL AND ROTATIONAL MOTION

The following table gives the analogy between the various parameters in translational and rotational motion.

Translational motion of a body	*Rotational motion of a rigid body*
Linear displacement s	Angular displacement θ
Linear velocity $v = \frac{ds}{dt}$	Angular velocity $\omega = \frac{d\theta}{dt}$
Linear acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
Translatory inertia or mass $= m$	Rotational inertia or $M.I. = I$
Force $F = ma$	Torque (moment of couple), $\tau = I\alpha$
Work $W = F.S.$	Work $W = \tau(\theta_2 - \theta_1)$
$K.E.$ of translation $= \frac{1}{2}mv^2$	$K.E.$ of rotation $= \frac{1}{2}I\omega^2$
Linear momentum $= mv$	Angular momentum $= I\omega$
Power in linear motion $= F \times v$	Power in rotatory motion $= \tau\omega$
Equations of linear motion :	Equations of rotatory motion :
(i) $v = u + at$	*(i)* $\omega = \omega_0 + \alpha t$
(ii) $s = ut + \frac{1}{2}at^2$	*(ii)* $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
(iii) $v^2 - u^2 = 2as$	*(iii)* $\omega^2 - \omega_0^2 = 2\alpha\theta$
Law of conservation of *linear* momentum, $MV = mv$	Law of conservation of *angular* momentum $I_1\omega_1 = I_2\omega_2$

QUESTIONS FOR SELF TEST

1. Explain the terms moment of inertia and radius of gyration of a rigid body. What is their physical significance ?
2. Define the explain angular momentum. Show that the rate of change of angular momentum is equal to the torque acting on the rotating body.
3. Explain the term kinetic energy of rotation. Derive an expression for the kinetic energy of rotation of a rigid body about an axis.
4. What is torque ? Find the value of torque required to produce a given angular acceleration in a body rotating about an axis. Hence deduce the definition of moment of inertia and radius of gyration.
5. Derive an expression for the work done by a torque in rotating a body through an angle θ.
6. State and prove the law of conservation of angular momentum while a body is rotating about its axis of rotation.
7. Discuss the motion of a rolling body and derive an expression for kinetic energy of a rolling body on a horizontal surface.

PROBLEMS

1. A grinding stone has a moment of inertia of 1600 $kg\ m^2$. A constant couple is applied and the grinding stone is found to have a speed of 150 *rpm* in 10 seconds after starting from rest. Calculate the couple ? **[Ans. 800π Nm]**

2. A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t seconds, show that the angular acceleration is given by, $\alpha = \frac{4\pi n}{t^2}$ rad s^2.

3. A child stands at the centre of a turntable with his two arms outsretched. The turntable is set rotating with frequency of 40 *rev/min*. How much is the frequency of revolution of the child if he folds his hands back and thereby reduces his moment of inertia to $\frac{2}{5}$ times the initial value ? Assume that the turntable rotates without friction. **[Ans.** 100 *rpm*]

4. To maintain a rotor at a uniform angular speed of 200 *rad/s*, an engine needs to transmit a torque of 180 *N m*. What is the power of the engine required ?

[Ans. 36 *kW*]

5. A solid spherical ball on a table. Find the ratio of its translational and rotational kinetic energies. Moment of intertia of a sphere about the diameter = 2/5 mass × radius^2. **[Ans.** 2·5]

6. Energy of 484 joules is spent in increasing speed of a flywheel from 60 *rpm* to 360 *rpm*. Calculate the moment of inertia of the wheel. **[Ans.** 0·7 *kg m*^{2}]

7. A circular disc of mass M and radius r is set rolling on a table. If ω is the angular velocity, show that its total energy E is given by $\frac{3}{4} Mr^2\omega^2$. [Given MI of disc $= \frac{1}{2} Mr^2$]

8. How large a torque is needed to accelerate a wheel for which $I = 2$ Kg m^2 from rest to 3 rev/s in 20*s*. **[Ans.** 0·6π *Nm*)

9. An engine flywheel of 6 cm radius whose moment of inertia is 144000 g cm^2 has tangential force of 1500 g weight applied on its rim. What is the angular acceleration produced ? **(Ans.** 61·25 rad/s^2)

10. The distance of the moon from earth is $3{\cdot}84 \times 10^5$ Km. and it rotates around the earth in 27·3 days. What is the angular momentum of moon around earth ? Mass of moon $= 7{\cdot}3 \times 10^{22}$ Kg. **(Ans.** $2{\cdot}867 \times 10^{34}$ Kg m^2/s)

11. What will be duration of the day, if earth suddenly shrinks to $\frac{1}{64}$ of its original Volume, mass remaining constant ? **(Ans.** 1·5 hours)

12. Calculate the kinetic energy of earth due to its rotation about its iron axis rotating at the rate one revolution per day. Given $R = 6400$ Km.

$M = 6 \times 10^{24}$ Kg and $I = \frac{2}{5} MR^2$. **(Ans.** $2{\cdot}599 \times 10^{29}$ *J*.)

13. A uniform solid sphere has a diameter of 50 cm and a mass of 100 Kg. It rotates at 720 revolution per minute about its axis through its centre of mass. Find its angular momentum and rotational kinetic energy.

(Ans. 188·4 Kgm^2/s, 7106·11 *J*)

14. A solid sphere of mass 10 Kg rolls without slipping with a velocity of 50 cm/s. Calculate its total energy. Moment of inertia $= \frac{2}{5} MR^2$ **(Ans.** 1·75 *J*)

Chapter

SHM

Simple Harmonic Motion and Vibrations

The motion of a body can be predicted if the external forces acting on it are known. If a force varies with time, the velocity and acceleration of the body will also change with time. A very important and special type of motion occurs when the force acting on a body is proportional to the displacement of the body from the mean (equilibrium) position. If this force always acts towards the mean position, a back-and-forth motion will result about this position. Such a motion is called **periodic** or **oscillatory motion.**

We are familiar with several examples of periodic motions, such as the motion of a pendulum, oscillations of a mass attached to a spring, and the vibrations of the strings of a musical instrument. The number of systems that exhibit oscillatory motion is very large. For example oscillations of molecules in solids, electromagnetic waves such as light waves, radar and radio waves originate due to the oscillating electric and magnetic field vectors. In alternating current circuits, voltage, current and electrical charges vary periodically with time.

Simple Harmonic Motion is the simplest form of periodic motion. In this type of motion, an object oscillates between two spacial positions for indefinite time, with no loss in mechanical energy. In actual practice retarding (frictional) forces are always present. Such forces reduce the mechanical energy of the system and the oscillations die out. Such oscillations are said to be *damped.* If an external driving force is applied to overcome this loss in energy, we call this motion as a *forced oscillation.*

SHM—1. PERIODIC MOTION

Any back-and-forth motion, which repeats itself after equal intervals of time is called a periodic motion *or oscillatory motion.*

Most familiar examples of periodic or oscillatory motions are :

(i) Pulse beat of a person.

(ii) Oscillations of a mass on a spring.

(iii) The motion of a pendulum.

(iv) Vibrations of the strings of a musical instrument.

(v) Molecules of a solid oscillating about their mean positions.

SHM—2. SIMPLE HARMONIC MOTION (S.H.M.)

A body is said to execute Simple Harmonic Motion, if its acceleration is proportional to its displacement from a fixed point (mean position) and is always directed towards that point.

A simple harmonic motion may be *(i) linear* or *(ii)* angular, according as the body moves along a linear path under the action of a force which always acts towards the mean position or rotates about an axis under the action of a constant torque or couple.

In S.H.M. the object oscillates between two fixed positions for an indefinite period of time.

Examples of S.H.M.

(i) To and fro motion of the piston of an engine.
(ii) Oscillations of mercury column in a U-tube.
(iii) Vibrations of the prongs of a tuning fork.

S.H.M., a projection of uniform circular motion :

A linear S.H.M. may be considered as the projection of a uniform circular motion on the diameter of the circle. This circle is called the circle of reference.

To illustrate, consider a point P moving along the circumference of a circle of radius a and centre O, with a uniform speed v, in the anti-clockwise direction as shown in fig. SHM—1. Let XX' and YY' be the two perpendicular diameters. Suppose the particle is in position P after a time, t. If ω is the angular velocity, then the displacement θ in time t is given by,

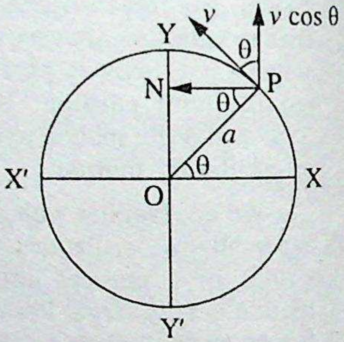


Fig. SHM—1

$$\theta = \omega t$$

From P draw PN perpendicular to YY'. Now as P moves from X to Y, the foot of the perpendicular N moves from O to Y. As P moves further from to Y to X', from X' to Y' and back again to X, the point N moves from Y to O, from O to Y' and back again to O. During the time the point P completes one rotation along the circumference, the point N, moves to and fro along YOY' and completes one vibration about O. The motion of the projection N along the diameter YY' is **simple harmonic.**

Hence simple harmonic motion is the projection of a uniform circular motion on a diameter of the circle of reference.

SHM—3. CHARACTERISTICS OF A LINEAR SIMPLE HARMONIC MOTION.

The following are the characteristics of an S.H.M.

1. Displacement (y) : The distance travelled by the vibrating particle N from its mean position is known as its displacement. If the displacement of the particle along Y-axis by y when the particle is at P, then

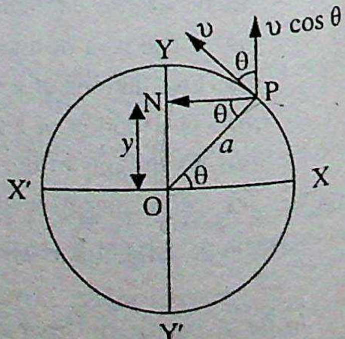


Fig. SHM—2

$$ON = OP \sin\theta$$
$$= OP \sin \omega t$$

or
$$y = a \sin \omega t$$

2. Amplitude. *(a)* : *The* **amplitude** *of a particle executing S.H.M. is its maximum displacement from the mean position to one extreme position.*

The maximum displacement of the vibrating particle is given by $OY = OY' = a$.

3. Velocity (*v*). The linear velocity of the particle P is v and acts **along** the tangent at P. This can be resolved into two components.

(i) $v \cos \theta$ in a direction parallel to OY and

(ii) $v \sin \theta$ in a direction perpendicular to OY.

The component $v \sin \theta$ produces no effect on the motion of the point N. **Hence,**

$$\text{Velocity of } N = v \cos \theta$$

$$= a\omega \cos \omega t = a\omega \cdot \frac{PN}{OP}$$

$$= a\omega \frac{\sqrt{a^2 - y^2}}{a} = \omega \sqrt{a^2 - y^2}$$

When the particle is at the mean position, $y = 0$

$\therefore$ Velocity of $N = a\omega = v$

which is the same as that of P and is maximum.

When the particle is at the extreme position, $y = a$

$\therefore$ Velocity of $N = 0$

which is obviously the minimum.

4. Acceleration. The angular acceleration of the particle P acts along PO and is equal to $\frac{v^2}{a}$. This can be resolved into two components ;

(i) $\frac{v^2}{a} \cos \theta$ along PN, perpendicular to OY and

(ii) $\frac{v^2}{a} \sin \theta$ along NO.

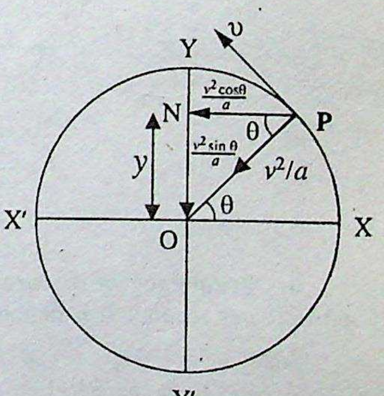


Fig. SHM—3

The perpendicular component $\frac{v^2}{a} \cos \theta$ produces no effect on the motion of N

Hence, $\quad$ Acceleration of $N = -\frac{v^2}{a} \sin \theta$ along NO

$$= -a\omega^2 \sin \omega t \qquad [\because v = a\omega$$

$$= -\omega^2 y \qquad [\because y = a \sin \omega t$$

or $\quad$ Acceleration of $N = -\omega^2 \times$ **displacement**

Now the particle is moving with uniform speed, therefore, the angular velocity ω is constant. Hence acceleration is proportional to displacement. The negative sign indicates that the acceleration is always directed towards the mean position.

Thus when a body moves such that its acceleration is always directed towards its mean position and varies directly as the distance from this mean position, it is said to move with simple harmonic motion.

It is clear that when the particle passes through the mean position $y = 0$. Hence the acceleration of N is zero. At the extreme position $y = a$ and acceleration of N is $a\omega^2$ which is maximum and is the same as that of P.

$$\therefore \quad a_{max} = a\omega^2$$

$$a_{min} = 0$$

5. Time period (T). *The time taken by a vibrating particle to move from its mean position to one extreme, then to the other extreme and back again to the mean position is called the* **time period.** It is denoted by. T.

During the time the point N completes one vibration, the particle P completes one rotation in the circle of reference with an angular velocity ω. Hence, if T is the time period, then,

$$\omega = \frac{2\pi}{T}$$

Now acceleration of $N = \omega^2 \times$ displacement of N

$$\therefore \quad \omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$\text{or} \quad \frac{2\pi}{T} = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

6. Frequency *of a vibrating particle is the number of vibrations made by the particle per second.* It is denoted by n.

Also $\quad n = 1/T.$

$$\therefore \quad n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

SHM—4. EQUATION OF SIMPLE HARMONIC MOTION. ($y = a \sin wt$).

Let there be a particle P, moving in the anticlockwise direction along the circumference of a circle (see fig. SHM—4).

Let, v = Uniform speed of the particle

ω = Angular velocity of the particle.

a = Radius of the circle.

= amplitude of the particle executing S.H.M.

The projection N of the moving particle on the diameter YOY' executes S.H.M. Let θ be angular displacement at any instant t.

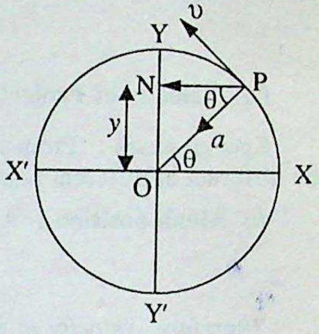


Fig. SHM—4

$$\therefore \text{ In } \Delta\, ONP, \quad \frac{ON}{OP} = \sin\theta$$

$$= \sin\omega t \qquad \left(\because \omega = \frac{\theta}{t}\right)$$

or $\qquad ON = OP \sin\omega t$

or $\qquad y = a\sin\omega t$

Thus Displacement = Amplitude × Sin ωt.

The above equation gives the value of displacement of a particle executing S.H.M. in terms of its amplitude a and angular velocity ω at any instant t. Therefore, this equation is called the equation of motion of a particle executing S.H.M.

SHM—5. EXPRESSION FOR VELOCITY OF A PARTICLE EXECUTING S.H.M.

To find an expression for the velocity of a particle moving with S.H.M. consider a particle P moving in the anticlockwise direction in a circle with an angular velocity ω radians/sec. (see fig. SHM—5).

Let after time, t this particle be at P and N be the projection of P on YOY'. The projection N executes S.H.M. along diameter YOY' and it is desired to find its velocity V.

Let the linear velocity of particle is v and acts along the tangent to the circle at P. This linear velocity of the particle can be resolved into two components.;

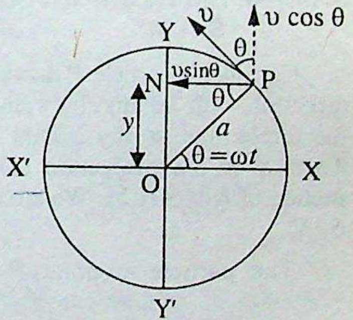


Fig. SHM—5

(i) Component $v\cos\theta$ along YOY' directed towards OY.

(ii) Component $v\sin\theta$ perpendicular to YOY' along PN.

The component $v\sin\theta$ is perpendicular to YOY', therefore, it produces no effect on the motion of N.

But the component $v\cos\theta$ acts along YOY' and gives the velocity of N.

$\therefore$ Velocity 'V' of N is given by ;

$$V = v\cos\theta = v\cos\omega t \qquad [\because \theta = \omega t]$$

$$= a\,\omega\cos\omega t \qquad [\because v = a \times \omega]$$

$$= a\,\omega\frac{NP}{OP} \qquad \left[\because \text{In rt.} \angle d\ \Delta ONP, \frac{NP}{OP} = \cos\theta = \cos\omega t\right]$$

$$= a\,\omega\frac{\sqrt{a^2 - y^2}}{a} = \omega\sqrt{a^2 - y^2}$$

$$\therefore \qquad V = \omega\sqrt{a^2 - y^2}$$

i.e., **Velocity of Projection N = angular velocity** $\sqrt{(amplitude)^2 - (displacement)^2}$

Special cases : From the above expression, it is clear that the velocity of *N* will be different at different points along its path *YOY'*

(i) **Mean position :** At the mean position, $y = 0$

$$\therefore \qquad V = \omega\sqrt{a^2 - 0} = a\omega$$

Therefore, velocity at mean position is zero (point O) is maximum and is given by ;

$$V_{max} = a\omega = v, \text{ the same as that of P}$$

(ii) **Extreme position.** At extreme position, $y = a$

$$\therefore \qquad V = \omega\sqrt{a^2 - a^2} = 0$$

Therefore, velocity is zero at the extreme position.

SHM–6. EXPRESSION, FOR ACCELERATION OF A PARTICLE EXECUTING S.H.M.

Consider a particle *P* moving in the anticlockwise direction with an angular velocity ω rad/sec. along the circle. Let at any instant this particle be at *P*. Then *N* is the projection of *P* on *YOY'* and the motion of *N* is S.H.M. We are to find the acceleration of *N*.

The particle at point *P* is acted upon by a centripetal acceleration $\frac{v^2}{a}$ along the radius *PO*. This acceleration can be resolved into two components :-

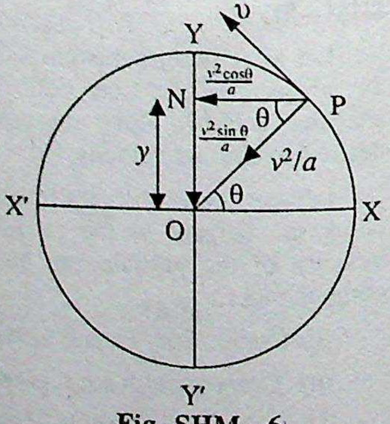


Fig. SHM—6

(i) $\frac{v^2}{a}\cos\theta$ along *PN* perpendicular to *YOY'*

(ii) $\frac{v^2}{a}\sin\theta$ along *YOY'* directed toward *NO*.

The component $\frac{v^2}{a}\cos\theta$ being perpendicular to *YOY'* produces no effect on the motion of N. But the component $\frac{v^2}{a}\sin\theta$ acts along *YOY'* and hence gives the acceleration of N.

$$\therefore \quad \text{Acceleration of } N = \frac{v^2}{a} \sin\theta \text{ along } NO$$

$$= \frac{(a\omega)^2}{a} \times \sin\theta \qquad [\because v = a\omega]$$

$$= \omega^2 \times a \sin \omega t = \omega^2 y \qquad [\because \theta = \omega t] \quad [y = a \sin \omega t]$$

$$\therefore \quad \text{Acceleration} = \omega^2 y$$

***i.e.*, Acceleration = (angular velocity)^2 × displacement**

Since ω is constant therefore. **Acceleration $\propto$ displacement (y).**

Special cases : From the expression, it is clear that the acceleration on N will be different at different points along its path YOY'.

(i) **Mean position.** At mean position, $y = 0$

$$\therefore \quad \text{Acceleration} = \omega^2 \times 0 = 0$$

Thus, acceleration at mean position (point O) is zero.

(ii) **Extreme position.** At extreme position, $y = a$

$$\therefore \quad \text{Acceleration} = \omega^2 \times a = a\,\omega^2$$

i.e., Acceleration is maximum

Thus, acceleration is maximum at the extreme position.

SHM—7. EXPRESSION FOR TIME PERIOD.

Let a Particle be Executing S.H.M.

Where, T = Time period in seconds.

When the particle completes one revolution, it describes an angle of 2π radians at the centre. During this time, the projection completes one vibration and, therefore, takes time T seconds.

$$\text{Now,} \quad \text{Angular velocity} = \frac{\text{Angle described}}{\text{Time taken}}$$

$$\therefore \quad \omega = \frac{2\pi}{T}$$

$$\text{Acceleration of } N = \omega^2 y = \omega^2 \times \text{displacement}$$

$$\text{or} \quad \omega = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

$$\text{or} \quad \frac{2\pi}{T} = \sqrt{\frac{\text{Acceleration}}{\text{Displacement}}} \qquad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\text{Hence,} \quad T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

Expression for Frequency :

We know that, frequency, $n = \frac{1}{T}$

$$\therefore \qquad n = \frac{1}{2\pi}\sqrt{\frac{\text{Acceleration}}{\text{Displacement}}}$$

Units of Frequency. The unit of frequency is **hertz** (*Hz*). It is 1 vib./sec.

$$\therefore \qquad 1 \text{ Hz} = 1 \text{ vibration/sec.} = 1 \text{ cycle/sec.}$$

SHM—8. GRAPHICAL REPRESENTATION OF SHM.

Displacement Time Graph. *Displacement time curve is a graph which shows the relation between the time that elapes since the particle was at its mean position O, and its displacement from O during this time.*

The displacement of a particle executing S.H.M. is given by the relation ;

$$y = a \sin \omega t$$

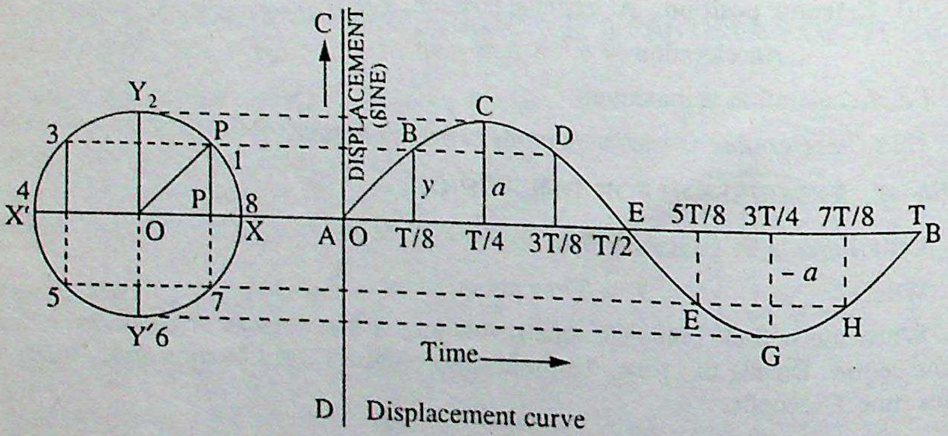


Fig. SHM—7

Since amplitude a is constant ; therefore

$$y \propto \sin \omega t.$$

This shows that the displacement of the particle changes with time. If a curve is drawn between displacement (y) and time (t) it will be a sine curve as shown in fig. SHM—7.

To draw time–displacement graph, consider a circle of reference and divide it into 8 equal parts, each part representing equal intervals of time, $\frac{T}{8}$. Mark these intervals along the axis AT taking A as origin or starting point.

At time $t = 0$, the particle is at X, and the displacement is zero. This is represented by the point A on the time–displacement graph.

At time $t = \frac{T}{8}$, the particle reaches the position number 1 and the displacement is *ON*. This is represented by the point *B* on the time displacement graph.

At time $t = \frac{T}{4}$, the particle reaches the position number 2 (extreme position), the displacement is maximum and is equal to *amplitude*. This is represented by point C on the graph.

At time $t = \frac{3T}{8}$, the particle reaches the position number 3 and the displacement is *ON* again. This is represented by point *D* on the graph.

At time $t = \frac{T}{2}$, the particle reaches the position number 4 and the displacement is zero. This is represented by the point *E* on the graph.

Similarly at times $\frac{5T}{8}, \frac{6T}{8}, \frac{7T}{8}$, the particle reaches at position 5, 6, 7 and the respective displacements are represented by points *F, G, H* on the graph.

Again at time $t = T$ *i.e.*, after one revolution, the particle reaches the position number 8 (starting point) and the displacement is again zero. This is represented by the point *T* on the graph.

If all the points *A, B,.....T* are joined through a smooth curve **the resultant graph will be a *sine curve* (see fig. SHM—7).**

Velocity–time graph for S.H.M.

Velocity of a particle (projection of uniform circular motion on a diameter) executing S.H.M. is given by the relation.

Velocity, $$V = \omega\sqrt{a^2 - y^2}$$

Thus velocity of the particle is different at different points along its path, depending upon its displacement.

Therefore, *(i)* when $y = 0$ (at mean position *O*)

Velocity $= \omega\sqrt{a^2 - 0} = \omega a = v$, same as that of particle *P* moving in a circle and its projection on a diameter executing *S.H.M.*

Also. *(ii)* when $y = a$ (at Y and Y′)

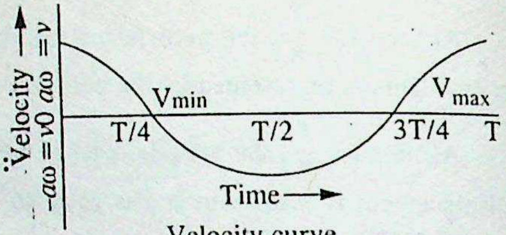
Velocity $= \omega\sqrt{a^2 - a^2} = \omega \times 0 = 0$

Thus, *velocity of a particle executing S.H.M. varies inversely as its displacement.*

The velocity of *N* varies from maximum (v) at *O*, to minimum (zero) at Y then increases from zero to v at O. Then velocities of *N* at different times are shown in table below :

Time	0	*T*/4	*T*/2	3*T*/4	*T*
Velocity	v *max.*	0 *min.*	$-v$ *max.*	0 *min.*	v *max.*

The graph between time and velocity is as shown in fig. SHM—8. The time is plotted along *X*–axis and velocity along *Y*–axis. The curve obtained is a *cosine curve.*



Velocity curve

Fig. SHM—8

Acceleration–time graph for S.H.M. :

Acceleration of a particle executing S.H.M. is given by the reltation,

$$\text{Acceleration} = -\,\omega^2 y$$

Since ω is constant, therefore, Acceleration ∝ Displacement (y)

Thus acceleration has its maximum value $a\omega^2$ = acceleration of P (centripetal acc.). at the extreme position Y and Y' *i.e.*, when $y = a$, and has its minimum value, *i.e.*, zero, at O, where $y = 0$.

The acceleration of the particle at different times is given below in the table :

Time	0	$T/4$	$T/2$	$3T/4$	T
Acceleration	0 (*min.*)	$-\,a\omega^2$ (*max.*)	0 (*min.*)	$a\omega^2$ (*max.*)	0 (*min.*)

When time is plotted along *X*–axis and acceleration along *Y*–axis, we get the acceleration curve of the particle and is shown in fig. SHM—9. The graph is similar to displacement curve, but is reciprocal in form. This shows that acceleration is directed in the opposite direction of the dislacement.

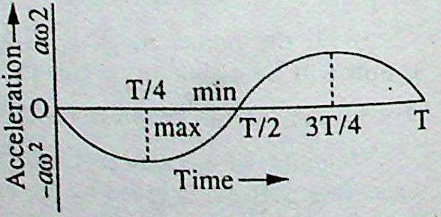


Fig. SHM—9

Ex. SHM—1. A particle executes S.H.M. of amplitude 10 cm. When the particle is 6 cm. from its mean position its acceleration is found to be 96 cm/s^2. Calculate :

(*i*) Velocity at the same instant.

(*ii*) Time period.

(*iii*) Maximum velocity.

Sol. Here, amplitude, $a = 10\ cm$

displacement, $y = 6\ cm$

$$\text{Acceleration} = 96\ cm/s^2$$

Acceleration of a particle executing S.H.M. is given by the relation,

$$\text{Acceleration} = \omega^2 y$$

$$\therefore \quad 96 = \omega^2 \times 6$$

$$\text{or} \quad \omega = \sqrt{96/6} = \sqrt{16}$$

$$= 4\ rad/s$$

(i) Now, velocity, $V = \omega\sqrt{a^2 - y^2} = 4\sqrt{10^2 - 6^2}$

$= 4\sqrt{64} = 4 \times 8 = 32\ cm/s$ **Ans.**

(ii) Time period, $T = \frac{2\pi}{\omega} = 2 \times \frac{22}{7} \times \frac{1}{4} = \frac{11}{7} = 1{\cdot}57\ sec.$ **Ans.**

(iii) Maximum velocity, $V_{max} = a\omega$

$= 10 \times 4 = 40\ cm/s$ **Ans.**

Ex. SHM—2. A body is executing S.H.M. of amplitude 0·01 m. Its velocity while passing through neutral point is 0·1 m/s. Find its frequency.

Sol. Here, amplitude, $a = 0{\cdot}01\ m$

Velocity at neutral point or $V_{max} = 0{\cdot}1\ m/s$

We know that, $V_{max} = a\omega$

$\therefore \quad \omega = \frac{V_{max}}{a} = \frac{0\cdot 1}{0\cdot 01} = 10\ rad/s$

Now, $\omega = 2\pi \times n$ where n is the frequency

$\therefore$ Frequency, $n = \frac{\omega}{2\pi} = \frac{10 \times 7}{2 \times 22} = 1{\cdot}59\ Hz$ **Ans.**

Ex. SHM—3. A body of mass 0·50 kg is executing S.H.M. If time period is 0·1s and amplitude is 10 cm. Find the acceleration and force acting on it while it is 5 cm apart from its mean position.

Sol. Here, Time period, $T = 0{\cdot}1\ s$

$\therefore$ Angular velocity, $\omega = \frac{2\pi}{T} = 2 \times \frac{22}{7} \times \frac{1}{0\cdot 1} = \frac{440}{7} = 62{\cdot}86\ rad/s$

Displacement, $y = 5\ cm$

$\therefore$ Acceleration $= \omega^2 y = \left(\frac{440}{7}\right)^2 \times 5 = 19755\ cm/s^2$

$= 197{\cdot}55\ m/s^2$ **Ans.**

Now, mass of body, $m = 0{\cdot}5\ kg$

$\therefore$ Force = mass × Acceleration

$= 0{\cdot}5 \times 197{\cdot}55 = 987{\cdot}75\ N$ **Ans.**

Ex. SHM—4. The maximum velocity in S.H.M. is 1·00 m s^{-1}, while maximum acceleration is 1·57 m s^{-2}. Calculate the periodic time.

Ans. Here. Max. velocity, $V_{max.} = 1{\cdot}00\ m\ s^{-1}$

and Max. acceleration $= 1{\cdot}57\ m\ s^{-2}$

Let, a = amplitude

ω = angular velocity

and $$V_{max} = a\omega = 1 \cdot 00 \qquad \ldots(i)$$

and $$\text{Max. Acc.} = a\omega^2 = 1 \cdot 57 \qquad \ldots(ii)$$

Dividing *(ii)* by *(i)*, we get,

$$\frac{a\omega^2}{a\omega} = \frac{1 \cdot 57}{1 \cdot 00}$$

or $$\omega = 1 \cdot 57 \text{ rad/s}$$

Now $$\omega = \frac{2\pi}{T} \text{ where } T \text{ is the periodic time}$$

$$\therefore \qquad T = \frac{2\pi}{\omega} = 2 \times \frac{22}{7} \times \frac{1}{1 \cdot 57} = 4 \cdot 0 \; s \; \textbf{Ans.}$$

Ex. SHM—5. A body executes simple harmonic motion of period 20 seconds. Its velocity is found to be 5 m/s seconds after it has passed through its mean position. Find amplitude of the body.

Sol. Here, time period, $T = 20$ s

$$\therefore \qquad \text{Angular velocity, } \omega = \frac{2\pi}{T} = \frac{2 \times \pi}{20} = \frac{\pi}{10} \text{ rad/s}$$

Velocity of the body when $t = 2$ s is given to be 5 *m/s*

Now, displacement, $y = a \sin \omega t$

or $$y = a \sin\left(\frac{\pi}{10} \times 2\right) = a \sin \frac{\pi}{5}$$

Velocity at any time is given as,

$$V = \omega\sqrt{a^2 - y^2} \text{ where } a \text{ is the amplitude}$$

$$\therefore \qquad 5 = \frac{22}{70}\sqrt{a^2 - a^2 \sin^2 \frac{\pi}{5}} = \frac{22}{70}\sqrt{1 - \sin^2 \frac{\pi}{5}} \times a$$

or $$5 = \frac{22}{70} \times \sqrt{1 - \cdot 36} \times a = \frac{22}{70} \times 0 \cdot 8 \times a$$

$$\therefore \qquad a = \frac{5 \times 70}{22 \times 0 \cdot 8} = 20 \; m \; \textbf{(Ans.)}$$

Ex. SHM—6. If the displacement of a moving particle at an instant is given by the relation, $y = a \cos \omega t + b \sin \omega t$

Where a and b are constant. Prove analytically that the motion of the particle is simple Harmonic.

Sol. Here, the displacement of a vibrating particle is given by the relation,

$$y = a \cos \omega t + b \sin \omega t$$

Putting $a = c \sin \theta$ and $b = c \cos \theta$

Squaring and adding,

$$a^2 + b^2 = c^2 \sin^2 + c^2 \cos^2 \theta = c^2 [\cos^2 \theta + \sin^2 \theta] = c^2$$

$$\therefore \qquad c = \sqrt{a^2 + b^2}$$

and dividing we get, $\dfrac{c \sin \theta}{c \cos \theta} = \dfrac{a}{b}$ or $\tan \theta = \dfrac{a}{b}$

$$\therefore \qquad \theta = \tan^{-1}\left(\frac{a}{b}\right) \quad [\because \sin A \cos B + \cos A \sin B = \sin(a + B)]$$

Now,

$$y = c \sin \theta \cos \omega t + c \cos \theta \sin \omega t$$

$$= c [\sin \omega t \cos \theta + \cos \omega t \sin \theta]$$

or

$$y = c \sin (\omega t + \theta)$$

This is the equation which gives the displacement of a body executing *S.H.M.*

where, $c = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\dfrac{a}{b}\right)$

Thus the motion of the particle represented by $y = a \cos \omega t + b \sin \omega t$ is a simple Harmonic Motion.

Ex. SHM—7. A body is executing SHM of amplitude 10 cm and frequency 10 vibrations per sec. Calculate the following after 0·16 sec :

***(i)* Displacement ; *(ii)* Velocity and *(iii)* Acceleration.**

Sol. Here, $a = 10\ cm,\ n = 10\ vib/s$

$$\therefore \qquad \omega = 2\pi n = 2\pi \times 10 = 20\pi \text{ rad}/s$$

and $t = 0{\cdot}16\ s$

(i) Displacement,

$$y = a \sin \omega t = 10 \sin 20\pi \times 0{\cdot}16$$

$$= 10 \sin 3{\cdot}2\pi = 10 \sin 576^\circ$$

$$= 10 \sin (360 + 216) = 10 \sin 216$$

$$= 10 \sin (180 + 36)$$

$$= -10 \sin 36 = -10 \times 0{\cdot}5878$$

$$= -5{\cdot}878\ cm \textbf{ (Ans.)}$$

(ii) Velocity,

$$V = \omega \sqrt{a^2 - y^2} = 20\pi\sqrt{100 - (-5 \cdot 878)^2}$$

$$= 20\pi\sqrt{100 - 34 \cdot 8} = 20 \times \frac{22}{7}\sqrt{65 \cdot 2}$$

$$= 20 \times \frac{22}{7} \times 8 \cdot 07 = 506 \cdot 00\ cm/s \textbf{ Ans.}$$

(iii) Acceleration

$$= -\omega^2 y = -(20\pi)^2 \times (-5{\cdot}878)$$

$$= 400 \times \frac{22}{7} \times \frac{22}{7} \times 5 \cdot 878$$

$$\text{Acceleration} = \mathbf{1137980{\cdot}8\ cm/s^2\ Ans.}$$

Ex. SHM-8 Find the period of vibrating particle which has acceration of 48 m/s^2, when displacement from mean position in 12m. Assume motion to be S.H.M.

Ans. Given, Acceleration, $a = 48\ m/s^2$

Displacement, $y = 12m.$

Now acceleration in a S.H.M. is given by, $a = \omega^2 y$

$$\Rightarrow \qquad \omega^2 = \frac{a}{y} = \frac{48}{12} = 4$$

i.e. $\omega = \sqrt{4} = 2$ rad/s

Thus $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi = 3\cdot142\ s.$ **Ans.**

Ex. SHM-9 A bob executes simple harmonic motion of period 20 s. If its velocity is 5 *m/s* after 2 seconds it has passed through its mean position. Find amplitude of the bob.

Sol. $T = 20s,\ \ \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{20} = \dfrac{\pi}{10}$ rad/s

Also $t = 2s, \quad v = 5m/s, \quad r = ?$

Applying, $v = \omega r \cos \omega t$, we get,

$$5 = \frac{\pi}{10} \times r \times \cos\left(\frac{\pi}{10} \times 2\right) = \frac{r\pi}{10} \text{Cos}\left(\frac{\pi}{5}\right)$$

$$5 = \frac{r\pi}{10} \cos (36°)$$

$$5 = r \times \frac{\pi}{10} \times 0\cdot8090$$

$$r = \frac{\cdot50}{\pi \times 0 \cdot 8090} = 19\cdot67\ m$$

$$r = 19\cdot67\ m \ \textbf{Ans.}$$

SHM—9. SIMPLE PENDULUM.

A simple pendulum consists of a heavy point mass suspended by a weightless, inextensible and perfectly flexible string from a rigid support about which it oscillates without friction.

In practice a small heavy metallic bob is suspended by a long flexible cotton thread.

Length of a simple pendulum is the distance from the point of suspension to the centre of gravity *(C.G.)* of the bob. It is also called the effective length of the pendulum.

If the bob of pendulum is drawn to one side and then let free, it comes back to the mean position *A*, and overshoots the mark due to momentum gained by it and moves nearly an equal distance on the other side up to *B*. It will again come back to the mean position, and then move to the point C and back again to A. In other words it begins to vibrate about its mean position.

SHM—10. TERMS USED

The following are the terms used in SHM.

Vibration : *The motion of the bob from mean position to one extreme position then to the other extreme position and back again to the mean position is called a* **vibration.**

Amplitude : *The maximum distance to which the bob moves from its mean position to the extreme position on either side is called the* **amplitude.** It is also equal to the maximum displacement.

In fig. SHM—10 the amplitude is represented by the distance AB. On account of air resistance, the amplitude of a simple pendulum goes on gradully decreasing as the bob continues to oscillate.

Time period *of a simple pendulum is the time taken by it to complete one vibration. It is denoted by T*

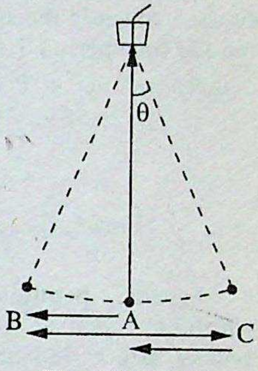


Fig. SHM—10

It has been observed that if the amplitude is small then, for a given length the time period remains unchanged although as stated above, the amplitude gradually decreases due to the air resistance. The vibrations of a simple pendulum are, therefore, called *isochronous*.

Frequency *is the number of vibrations made by a pendulum in one second.* It is the *reciprocal* of the time period.

Expression for Time period of a Simple Pendulum.

To find the time period of a simple pendulum consider the bob to be displaced from the mean position A to the position B through a small angle θ as shown in fig. SHM—11. The various forces acting on the bob in the position B are :

(i) Its weight mg acting vertically downwards.

(ii) The tension T in the thread acting along BO towards the point of suspension O.

The weight mg can be resolved into two rectangular components *(i)* $mg \cos \theta$ along OB produced and *(ii)* $mg \sin \theta$ perpendicular to OB along the tangent to the arc, AB, as shown in fig, SHM—11.

The component $mg \cos \theta$ is equal and opposite to the tension T and completely balances it.

The bob moves along the arc AB under the action of the force $mg \sin \theta$. If a is the linear acceleration produced in the bob towards A, then

$$ma = mg \sin \theta$$

or

$$a = g \sin \theta$$

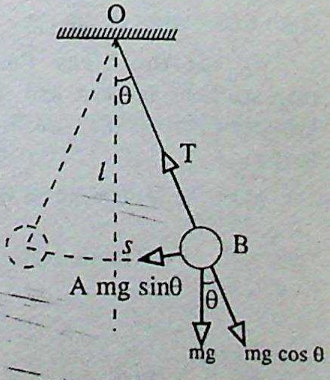


Fig. SHM—11

If the amplitude is small θ is also small and $\sin\theta = \theta$

$$\therefore \qquad a = g\theta = g\frac{s}{l}$$

where, $l = OA$ is the length of the pendulum and $s = AB$ is the displacement.

As $\frac{g}{l}$ is constant, the acceleration a is proportional to the displacement s and is directed towards the mean position A. Hence *motion of the bob is simple harmonic* and the time period T is given by,

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi\sqrt{\frac{s}{gs/l}} = 2\pi\sqrt{\frac{l}{g}}$$

$$\text{Hence, time period} = 2\pi\sqrt{\frac{\text{length of pendulum}}{\text{Acc. due to gravity}}}$$

Second's pendulum. *A second's pendulum is a pendulum which completes one vibration in two seconds.* Such a pendulum completes one *beat* or one *swing* in *one second.* If $g = 980$ cms/sec^2. then the length of the second's pendulum is given by,

$$2 = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{980}}$$

$$\text{or} \qquad l = \frac{4 \times 980}{4\pi^2} = 99{\cdot}29 \text{ cms.}$$

Thus if the length of the pendulum is 99·29 *cm.* then it completes one vibration in 2 seconds.

SHM—11. ENERGY IN S.H.M.

The acceleration of a particle in S.H.M. is always directed towards the mean position in direction opposite to the direction in which displacement (y) of the particle increases. Thus work is done during displacement. Therefore the particle possesses potential energy. The particle also possesses velocity and thus has kinetic energy as well. The sum of the two energies remains constant at any instant if there is no dissipation of energy.

(i) **At the Mean Position.** When the particle is in the mean position, its displacement is zero and the velocity is maximum so the whole of the energy is kinetic. If 'm' is the mass of the particle, 'a' its amplitude and ω its angular velocity, then,

Maximum velocity of the particle $= a\omega$

$$\therefore \qquad \text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2}m(a\omega)^2$$

$$= \frac{1}{2}ma^2\omega^2$$

The potential energy in this position is zero as the displacement is zero.

$$\therefore \qquad \text{Total energy} = K.E. + P.E.$$

$$= \frac{1}{2}ma^2\omega^2 + 0 = \frac{1}{2}ma^2\omega^2 \qquad \ldots(i)$$

Thus, $K.E. \propto a^2$ *i.e.*, (amplitude)^2.

(ii) At extreme position whole of energy is potential,

$$= \frac{1}{2}m\,\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2$$

(iii) For any position of the particle when its displacement is 'y' the linear velocity is given as,

$$\text{Velocity '}v\text{'} = \omega(\sqrt{a^2 - y^2})$$

$$\therefore \text{ Kinetic energy of the particle} = \frac{1}{2}m\,\omega^2(a^2 - y^2)$$

The potential energy is equal to the amount of work done in moving the particle from the mean position to a displacement 'y'.

$$\text{Now acceleration} = \omega^2 y \qquad [\because F = ma]$$

$$\therefore \qquad \text{Restoring force} = m\,\omega^2 y$$

$$\text{Hence, work done for a displacement, } y = \frac{1}{2}m\,\omega^2 y^2$$

$$\therefore\ P.E. \text{ of the particle for a displacement, } y = \frac{1}{2}m\,\omega^2 y^2$$

Hence, $P.E. \propto y^2$ *i.e.*, (displacement)^2

$$\therefore \qquad \text{Total energy} = K.E. + P.E.$$

$$= \frac{1}{2}m\omega^2(a^2 - y^2) + \frac{1}{2}m\,\omega^2 y^2 = \frac{1}{2}m\,\omega^2 a^2$$

If n is the frequency, then $\omega = 2\pi n$

$$\text{Thus total energy,} \qquad E = \frac{1}{2}m\,a^2(2\pi n)^2$$

$$\therefore \qquad E = 2\pi^2 a^2 n^2 m$$

According to the law of conservation of energy, the energy remains constant at any point, wheather it is kinetic, potential or both at any point

From the above equation we observe that the energy of a body in S.H.M. is

(*a*) directly proportional to the mass of the body.

(*b*) directly proportional to the square of its frequency.

(*c*) directly proportional to the square of its amplitude.

SHM—12. TRANSFORMATION OF ENERGY IN S.H.M.

Vibrations in bodies are produced due to their elastic properties. The body is deformed by applying force, and on releasing the deforming force the body is set in a state of vibrations. For example when a tuning fork vibrates it is due to the property of elasticity.

When a body is deformed, work is done by the external deforming force in producing initial displacement against the internal elastic forces of the body which resist deformation. This work done is stored up as potential energy in the deformed part of the body. When the deforming force is removed, the *P.E.* stored is used up to bring it back to its original position. In this process the velocity of the body increases and becomes maximum at the mean position. The *P.E.* of the body is transformed into *K.E.* and there is no deformation. Due to inertia the body overshoots the mark and swings to the other side. The *K.E.* is converted into *P.E.* at the extreme position and the body is again deformed. The body again tries to regain its original position and the *P.E.* is converted into *K.E.*

This process goes on repeating and the body continues to vibrate.

If there is no dissipation (loss) of energy possessed by the body, the body would continue to vibrate for ever with the same amplitude. But in actual practice the *K.E* is dissipated to air partly in the form of waves of compressions and rarefactions and partly in the form of heat due to internal friction. Due to these losses the energy of the system goes on decreasing and the vibrations of the body gradually die out. The body finally comes to rest when the whole of the energy is lost.

To understand the generation of waves due to transformation of energy, let us take the example of vibrating tuning fork. Let one prong of the vibrating tuning fork is considered.

When the prong moves from position *A* (mean position) to position A_1, it compresses the layer of air in front of it. The pressure of air increases in this region. This compressed layer of air further compresses the air in front of it. Thus a wave of compression moves in the forward direction.

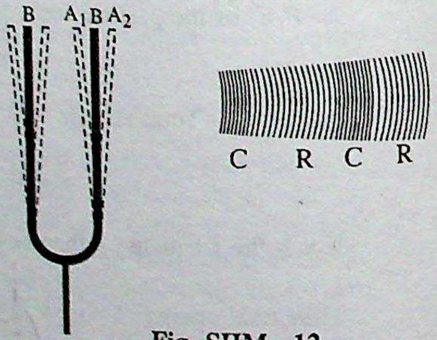


Fig. SHM—12

Similarly, when the prong moves from position A_1 to position A_2, the layer of air in front of it expands and the pressure decreases. The molecules of air start moving in the backward direction and a wave of rarefaction moves in the forward direction. As the tuning fork vibrates continuously, compressions and rarefactions are produced which constantly move in the forward direction. Thus waves are generated due to the vibrations of bodies or transformation of energy.

Ex. SHM—10. What for the following relations stand for ?

(i) $y = a \sin \omega t$ ***(ii)*** $V = \omega\sqrt{a^2 - y^2}$

(iii) $Acc. = \omega^2 y$ ***(iv)*** $E = 2\pi^2 m a^2 n^2$

(v) $\omega = \frac{2\pi}{T}$ *(vi)* $\omega = 2\pi n$.

Ans. *(i)* **$y = a \sin \omega t$:** It is the equation of *S.H.M.* It gives the displacement of a body executing *S.H.M.* at any instant t, in terms of amplitude "a" and angular velocity ω.

(ii) $v = \omega\sqrt{a^2 - y^2}$: This is the expression for velocity of a body moving with *S.H.M.* where,

ω = angular velocity

a = amplitude and

y = displacement at any instant.

(iii) **Acc. = $\omega^2 y$:** The relation gives the acceleration in terms of angular velocity and displacement. Since ω is constant

$\therefore$ Acc. $\propto$ displacement (y).

(iv) **$E = 2\pi^2 ma^2 n^2$:** This is the relation for the energy of a particle moving with *S.H.M.* where,

E = energy possessed

m = mass of the body

a = amplitude and

n = frequency of vibration.

(v) $\omega = \frac{2\pi}{T}$. This relation gives angular velocity (ω) in terms of time period (T).

(vi) $\omega = 2\pi n$. This relation gives angular velocity (ω) in terms of frequency (n).

Ex. SHM—11. Calculate the velocity of the bob of a simple pendulum at its mean position, if it is able to rise to a vertical height of 10 cm. Given, $g = 9{\cdot}8\ m/s^2$.

Sol. Let, m = mass of the bob.

and v = velocity of the bob at mean position,

$\therefore$ *K.E.* of bob at mean position $= \frac{1}{2}mv^2$

At the highest point whole of the energy is in the from of *P.E.*

Here, $h = 10\ cm$

$g = 9{\cdot}8\ m/s^2 = 980\ cm/s^2$

$\therefore$ $P.E. = mgh$

Now we know, *K.E.* at the mean position = *P.E.* at the highest point

$\therefore$ $= \frac{1}{2}mv^2 = mgh$

or $v^2 = 2gh = 2 \times 980 \times 10$

$\therefore$ $v = \sqrt{19600} = 140\ cm/s = 1{\cdot}40\ m/s$ **Ans.**

Ex. SHM—12. **A particle of mass 80 gm. executes S.H.M. of a period of 2π sec. If its velocity, $\pi/4$ sec. after passing the mean position be $8\sqrt{2}$ cm. per sec., Calculate :**

(i) Amplitude ; (ii) P.E. ; (iii) K.E. ; and

(iv) Total energy of the harmonic oscillator.

Sol. Here, mass, $m = 80\ gm$

Time period, $T = 2\pi\ sec.$

Velocity after time $\frac{\pi}{4}\ sec = 8\sqrt{2}\ cm/s$

Hence, $t = \frac{\pi}{4} sec$ and $V = 8\sqrt{2}\ cm/s$

Now, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1\ rad/s$

$\therefore$ Displacement, $y = a \sin \omega t$

$$= a \sin\left(1 \times \frac{\pi}{4}\right) = a \times \frac{1}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

Now, $V = \omega\sqrt{a^2 - y^2}$

$$\therefore \quad 8\sqrt{2} = 1\sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2} = a\sqrt{1 - \frac{1}{2}} = \frac{a}{\sqrt{2}}$$

(i) Amplitude, $a = 8\sqrt{2} \times \sqrt{2} = 16\ cm$ Ans.

(ii) $P.E. = \frac{1}{2} m\,\omega^2 y^2 = \frac{1}{2} \times 80 \times 1^2 \times (8\sqrt{2})^2$

$$= \frac{1}{2} \times 80 \times 64 \times 2 = 5120\ ergs \text{ Ans.}$$

(iii) $K.E. = \frac{1}{2} m\omega^2 (a^2 - y^2)$

$$= \frac{1}{2} \times 80 \times (1)^2 \times [(16)^2 - (8\sqrt{2})^2]$$

$$= \frac{1}{2} \times 80 \times 1 \times [256 - 128]$$

$$= \frac{1}{2} \times 80 \times 128 = 5120\ ergs$$

(iv) Total energy of the harmonic oscillator

$$= K.E. + P.E. = 5120 + 5120 = 10240\ ergs \text{ Ans.}$$

Ex. SHM-13 **Total energy of a particle executing SHM of period $2\pi s$ is 10240 erg. The displacement of particle $\frac{\pi}{4} s$ is $8\sqrt{2}$ cm. Calculate the amplitude of the motion and mass of the particle.**

Sol. $T = 2\pi s$ and Total Energy, $E = 10240$ ergs.

$$t = \frac{\pi}{4} s, \quad y = 8\sqrt{2} \text{ cm}, \quad r = ?, \quad m = ?$$

Displacement of particle excuting SHM is given by,

$$y = r \sin \omega t = r \sin\left(\frac{2\pi t}{T}\right) \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

or $$8\sqrt{2} = r.\sin\frac{2\pi}{2\pi}.\frac{\pi}{4} = r \sin 45° = \frac{r}{\sqrt{2}}$$

or $$r = 8\sqrt{2} \times \sqrt{2} = 8 \times 2 = 16 \text{ cm } \textbf{Ans.}$$

Thus amplitude of vibrations is 16 cm

$\therefore$ $$\text{Total energy, } E = \frac{2m\pi^2 r^2}{T^2}$$

$\Rightarrow$ $$m = \frac{ET^2}{2\pi^2 r^2} = \frac{10240 \times 4\pi^2}{2\pi^2 \times 16 \times 16}$$

or $$m = \frac{10240 \times 2}{16 \times 16} = 80 \text{ g}$$

Hence mass of the particle is 80 gram. **Ans.**

SHM—13. VIBRATIONS OF A VERTICAL SPRING MASS SYSTEM.

Consider a light and elastic spring suspended vertically from a rigid support (fig, SHM—13*a*). Let a mass *m* be suspended from the lower end of the spring such that the spring suffers an extension *P* in its length (fig. SHM—13*b*) due to the applied downward force F_a.

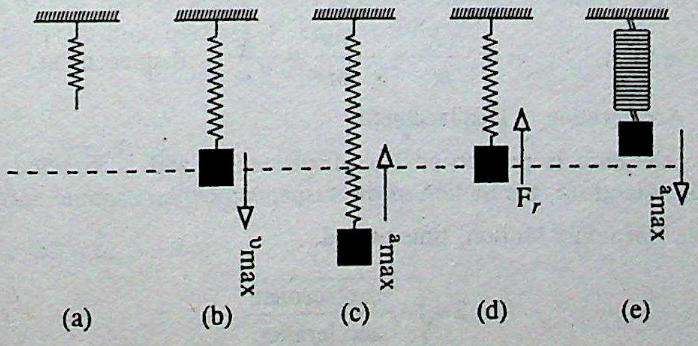


Fig. SHM—13

A restoring force F_r is developed in the spring.

$$\therefore \quad F_r = -F_a$$

According to Hooke's Law

$$\text{Extension} \propto \text{Tension}$$

or $\quad P \propto F_a \quad$ or $\quad F_a \propto P$

or $\quad F_a = Kp \quad \therefore \quad F_r = -Kp \quad \ldots(i)$

Where K is constant and is called *spring constant* or *stiffness factor* of the spring. Negative sign indicates that extension p is directed downward whereas restoring force F_r is directed upwards,

If extension $p = 1\ m$, then, From *(i)*,

$$F_r = -K \quad \text{or} \quad K = \frac{-F_r}{p} = \frac{F_a}{p} = F_a$$

Thus, **stiffness factor** *of a spring may be defined as the force required to stretch the spring through 1 m.* It is measured in *N/m*

Expression for time period :

Let the mass be pulled down through a small distance x ($<p$) by applying an additional force (ΔF_a) as shown in fig. SHM—13 (*c*).

Now, the total extension is $(P + x)$ and $(F_a + \Delta F_a)$ is the total applied force.

$\therefore$ Restoring force acting in the upward direction. $= -(F_a + \Delta F_a)$

Hence, $\quad F_a + \Delta F_a = -K(p + x) \quad \ldots(ii)$

Subtracting *(i)* from *(ii)*, we get,

$$\Delta F_a = -Kx$$

When the mass m is released it begins to move up and down (vertical vibrations) with Simple Harmonic Motion. The additional restoring force produces an acceleration in the suspended mass. Let a be the acceleration, then,

$$\Delta F_a = ma$$

or $\quad a = \dfrac{\Delta F_a}{m} \quad$ But $\quad \Delta F_a = -Kx$

$\therefore$ Acceleration, $\quad a = \dfrac{\Delta F_a}{m} = -\dfrac{Kx}{m} = -\dfrac{K}{m} \times \text{displacement.}$

Thus, $\quad$ Acceleration $\propto$ Displacement.

Since acceleration is proportional to displacement and is directed towards the mean position, therefore, the motion of the suspended mass is simple harmonic.

In Simple Harmonic Motion, time period,

$$T = 2\pi\sqrt{-\frac{\text{displacement}}{\text{acceleration}}}$$

Now, $$\frac{\text{Displacement}}{\text{Acceleration}} = -\frac{x}{a}$$

But $$\frac{x}{a} = \frac{m}{K}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{m}{K}}$$

Frequency. If n is the frequency of vibration, then

$$n = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{m}{K}}} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Ex. SHM—14. A spring elongates by 10 cm when a mass of 1 kg is hung from it. Find the restoring force and stiffness factor.

Sol. Here, elongation, $x = 10\ cm = 0{\cdot}10\ m$

Restoring force, $F_r = -mg = -1\ kg \times 9{\cdot}8\ m\ s^{-2} = -9{\cdot}8\ N$ **Ans.**

Now $F_r = -Kx$

$$\therefore \quad K = \frac{F_r}{-x} = \frac{-9\cdot 8 N}{-0\cdot 10 m} = 98\ N\ m^{-1}$$ **Ans.**

Ex. SHM—15 A body of mass 0·5 Kg is made to oscillate on a spring of force constant 16 N/m. Find the frequency of oscillation.

Sol. $m = 0{\cdot}5\ \text{Kg},\ \text{K} = 16\ Nm^{-1}$

$$v = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

$$v = \frac{1}{2\pi}\sqrt{\frac{16}{0\cdot 5}} = \frac{1}{2\times 3\cdot 142}\sqrt{32} = 0{\cdot}9 Hz$$

$\therefore$ Frequency $= 0{\cdot}9\ H_z$ **Ans.**

Ex. SHM—16 A mass of 0·5 Kg stretches a spring through 20 cm. Find the force constant of the spring. If the mass is pulled down an additional 5·0 cm and then relieved find the time period. Given $g = 9{\cdot}8\ m/s^2$.

Sol. $F = 0{\cdot}5\ \text{Kg}\ wt = 0{\cdot}5 \times 9{\cdot}8 = 4{\cdot}9\ N,\ m = 0{\cdot}5\ \text{Kg}.$

Stretching $= x = 20\ \text{cm} = \frac{20}{100} = 0{\cdot}2\ m.$

Now $F = \text{K}x$

Where K is force constant so

$$\text{K} = \frac{F}{x} = \frac{4\cdot 9}{0\cdot 2} = 24{\cdot}5\ N/m$$

The time period is given by,

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{0\cdot 5}{24\cdot 5}}$$

$$T = 2\pi\sqrt{\frac{1}{49}} = \frac{2\pi}{7} = \frac{2\times 3\cdot 142}{7} = \frac{44}{49}s \text{ Ans.}$$

SHM—14. VIBRATIONS OF BODIES SUPPORTED ON SPRINGS

When a body of mass M is supported by springs, there is compression produced in the springs. If the body is pushed downwards and then released, it begins to move up and down executing *S.H.M.*

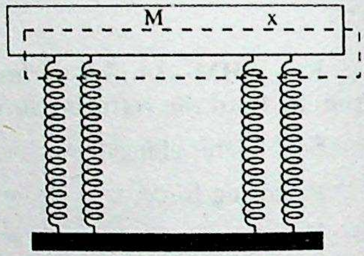


Fig. SHM—14

The most familiar examples of springs in such a case are shockers of scooters, motor cycles, cars, buses etc.

Let a body of mass M be supported by identical springs as shown in fig. SHM–14.

Let, K = Stiffness factor of each spring

and x = Compression produced.

Total force on the springs = wt. of mass, $M = Mg$.

Due to this force, each spring gets compressed by x.

But $F_a = Kx$ or $Mg = Kx$

$$\therefore \quad K = \frac{Mg}{x}$$

Time Period T. The time period T is given by,

$$T = 2\pi\sqrt{\frac{Mass}{K}} = 2\pi\sqrt{\frac{M}{\frac{Mg}{x}}} = 2\pi\sqrt{\frac{x}{g}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{\text{Compression}}{\text{Acc.due to gravity}}}$$

Frequency. If n is the frequency of vibrations, then,

$$n = \frac{1}{T} = \frac{1}{2\pi\sqrt{x/g}}$$

$$\therefore \quad n = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$$

Effect of Additional Mass

Let, an additional mass m is placed on the previous spring mass system as shown in fig. SHM—15.

The addition of mass simply increases the compression.

Therefore, let x_1 be the additional compression.

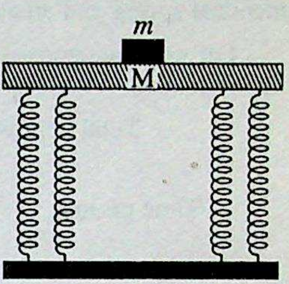


Fig. SHM—15

$\therefore$ Total compression $= x + x_1$

Total force on the springs = Wt. of mass $(M + m)$

$$= (M + m)g$$

But, $F_a = K(x + x_1)$

or $(M + m)g = K(x + x_1)$

$$\therefore \quad K = \frac{(M+m)g}{x+x_1}$$

Time Period. The addition of mass increases the compression and hence the time period increases.

The time period T is given by,

$$T = 2\pi\sqrt{\frac{mass}{K}} = 2\pi\sqrt{\frac{M+m}{K}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{M+m}{\frac{(M+m)g}{x+x_1}}} = 2\pi\sqrt{\frac{x+x_1}{g}}$$

Frequency. The frequency n is given as,

$$n = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{x+x_1}{g}}} = \frac{1}{2\pi}\sqrt{\frac{g}{x+x_1}}$$

Thus by increasing the mass, time period increases and the frequency decreases.

By increasing the number of springs, the compression decreases and also the time period decreases which results in the increase in frequency.

SHM—15. VIBRATIONS OF BUSES

(i) **Vibrations of an Empty Bus.** Consider an empty bus of mass M supported by identical springs as shown in fig. SHM—16.

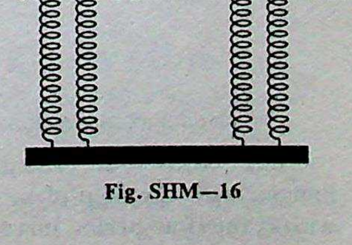


Fig. SHM—16

Let x be depression produced.

$\therefore$ Time period, $T = 2\pi\sqrt{\frac{x}{g}}$

Frequency, $n = \frac{1}{2\pi}\sqrt{\frac{g}{x}}$

(ii) **Vibrations of a Loaded bus.** Consider a bus of mass M supported by identical spring and an additional load of mass m as shown in fig. SHM—17. Fig.

Let x be the depression produced due to empty bus and x_1 an additional depression due to load m.

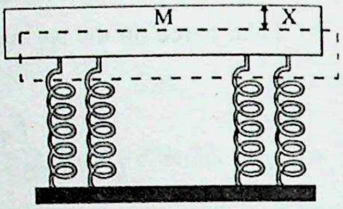


Fig. SHM—17

Total depression $= x + x_1$

$\therefore$ Time period, $$T = 2\pi\sqrt{\frac{x+x_1}{g}}$$

Frequency, $$n = \frac{1}{2\pi}\sqrt{\frac{g}{x+x_1}}$$

(iii) **Loaded bus is more comfortable than an empty bus.** A bus is supported on a number of springs to absorb sudden shocks. These springs set up vibrations during sudden shocks due to their elastic properties. The frequency of these vibrations depends upon the load supported by the springs and is given by,

Frequency, $$n = \frac{1}{2\pi}\sqrt{\frac{K}{\text{Mass of the bus}}}$$

where K is the stiffness factor of each spring.

Thus, in the case of an empty bus, the mass is less and hence frequency of vibrations is more. On the other hand, a loaded bus has greater total mass (*mass of empty bus + additional mass of passengers*) and therefore, the frequency of vibrations is comparatively less.

More frequency means that passenger seats are subjected to quicker up and down movements, which is uncomfortable, whereas in case to loaded bus frequency is less and passenger seats are not subjected to quicker up and down movements. Hence a loaded bus is more comfortable than an empty bus.

Ex. SHM—17. A mass of 10 kg is supported on a spring of spring constant 980 N/m. Find its compression.

Sol. Here, mass $M = 10\ kg$

and spring constant, $K = 980\ N/m$

Let the compression be x

Now, $F_a = Mg = 10 \times 9{\cdot}8 = 98\ N$

We know that, $F_a = -Kx$

$$\therefore \quad 98 = -980 \times x$$

or $$x = \frac{98}{-980} = -\frac{1}{10} = -0\cdot 10\,m$$

Hence compression $= 0{\cdot}10\ m = 10\ cm$ **(Ans.)**

Ex. SHM—18. The time period of a mass m is 2 sec. while it is being hanged with the help of an ideal spring is S.H.M. If 2 kg mass is also attached with it the time period increases by 1 sec. Find the value of m.

Sol. Let K be the spring constant of the spring, and m mass attached.

Here, time period, $T_1 = 2\ s.$

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{m}{K}}$$

$$\text{Mass added} = 2\ kg \qquad \ldots(i)$$

$$\therefore \quad \text{Total mass} = (m + 2)\ kg$$

$$\text{Increase in time period} = 1\ s.$$

$$\therefore \text{ Time period,} \quad T_2 = 2 + 1 = 3\ s$$

$$\therefore \quad T_2 = 2\pi\sqrt{\frac{m+2}{K}} \qquad \ldots(ii)$$

Dividing *(ii)* by *(i)*,

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{\dfrac{m+2}{k}}}{2\pi\sqrt{\dfrac{m}{k}}} = \frac{\sqrt{\dfrac{m+2}{k}}}{\sqrt{\dfrac{m}{k}}} = \frac{\sqrt{m+2}}{\sqrt{m}}$$

$$\text{or} \quad \frac{3}{2} = \frac{\sqrt{m+2}}{\sqrt{m}}$$

$$\text{Squaring,} \quad \frac{9}{4} = \frac{m+2}{m}$$

$$\text{or} \quad 9m = 4m + 8$$

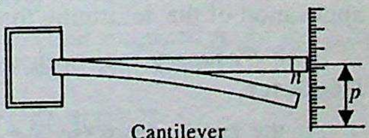
$$\therefore \quad m = \frac{8}{5} = 1{\cdot}6\ kg \ \textbf{(Ans.)}$$

SHM—16. CANTILEVER

A beam fixed horizontally at one end loaded at the other end is called a **cantilever.**

A weightless rod fixed at one end to a rigid support S and kept in the horizontal position is a cantilever. It can be loaded by placing a mass m at the free end. The load mg bends the rod producing a depression p.

When the mass is pushed down and then released, the rod beings to vibrate executing Simple Harmonic Motion as shown is fig. SHM—18.



Cantilever

Fig. SHM—18

Expression for time period : When the rod is depressed through a distance p, a restoring force comes into play which tries to bring the cantitlever to its original position.

The restoring force is proportional to the displacement p and is in a direction opposite to the direction of displacement.

$$\therefore \quad F = -kp \qquad \ldots(i)$$

Where k is proportionality constant or stiffness factor.

When the rod vibrates, it is under acceleration.

Now, Force = mass × acceleration

$$\therefore \quad \text{Acceleration} = \frac{\text{Force}}{\text{mass}} = \frac{F}{m} = \frac{-kp}{m} = -\frac{k}{m} \times p \qquad \ldots(ii)$$

$$\text{or} \quad \text{Acceleration} = -\frac{k}{m} \times \text{displacement}$$

$$\therefore \quad \frac{\text{Acceleration}}{\text{Displacement}} = -\frac{k}{m}$$

Now time period of a body executing *SHM* is given as

$$T = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$\therefore \quad T = 2\pi\sqrt{-\frac{m}{k}} \qquad \ldots(iii)$$

The restoring force, $F = mg = -kp$

$$\therefore \quad -\frac{m}{k} = \frac{p}{g} \qquad \ldots(iv)$$

Substituting the value of $-\frac{m}{k}$ in *(iii)*, we get,

$$T = 2\pi\sqrt{\frac{p}{g}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{\text{depression}}{\text{Acc. due to gravity}}}$$

SHM—17. FREE, FORCED, DAMPED AND RESONANT VIBRATIONS

(i) **Free Vibrations.** *Vibrations in which no external force acts after the initial displacement and motion is maintained by the internal elastic forces are called* **free vibrations.**

Free vibrations should continue for ever as no exeternal force acts after the application of the deforming force.

The frequency with which a body vibrates freely at its own is called **natural frequency.**

If the energy possessed by a vibrating system does not get dissipated, the body continues to vibrate with its natural frequency.

In practice the amplitude of the vibrating body gradually decreases to zero due to frictional forces.

Example *(i)* A stretched string when plucked, executes free vibrations.

(ii) A tuning fork when struck with a rubber pad executes free vibrations.

(iii) **Forced Vibrations.** *When a body is maintained in a state of vibrations by an external periodic force of frequency other than the natural frequency of the body the vibrations are called* **forced vibrations.**

The natural frequency of body decreases as a result of dissipative forces. To compensate for the energy loss and to maintain the vibrations, we have to apply external force. The energy can be put into the system by an applied force that acts in the direction of motion of the vibrating body. For example, a child on a swing can be kept in motion by appropriately timed pushes.

The frequency of the forced vibrations is different from the natural frequency of the body. It is equal to the frequency of the applied force. The amplitude of the forced vibrations is determined by the difference between the frequency of the applied force and the natural frequency of the body.

Examples : *(i)* Press the stem of a vibrating tuning fork against the diaphragm of a *tabla*. The diaphragm of *tabla* will be forced to vibrate. These vibrations of *tabla* are forced vibrations.

(ii) In case of a simple pendulum. the natural frequency can be changed by giving a strong blow of compressed air exactly when the bob of the pendulum is at one of its extreme positions every time. The pendulum can be made to vibrate with the frequency other than its natural frequency.

(iii) **Damped Vibrations :** *The vibrations in which amplitude of the vibrating body goes on decreasing and finally the body comes to rest are called* **damped vibrations.**

In vibrating systems, the initial energy imparted to the body gets dissipated in overcoming air and other frictional resistances. Consequently the amplitude of vibrations goes on decreasing with time and finally the vibrations die out.

Examples : Swinging of a simple pendulum is a familiar example of damped vibrations. During each vibration a part of the energy is lost in overcoming air friction. The amplitude goes on decreasing and finally the pendulum stops vibrating and comes to rest.

(iv) **Resonant Vibrations.** *When a body is maintained in a state of vibrations by periodic force having the same natural frequency as that of the body, the vibrations are called* **resonant or sympathetic vibrations.**

If we bring a vibrating tuning fork near another stationary tuning fork of the same natural frequency as that of vibrating tuning fork, we find that stationary tuning fork also starts vibrating. Such vibrations are known as the resonant **frequency** vibrations and this phenomenon is known as **resonance.**

During an earthquake, certain buildings, whose natural frequencies are the same as the frequency of the earthquake collapse due to **resonant vibrations.**

QUESTIONS FOR SELF TEST

1. Define Simple Harmonic Motion and derive a relation for velocity and acceleration of a particle executing S.H.M.
2. Discuss the theory of a simple spring mass system and derive an expression for its time period and frequency.

3. Define and explain :

   (i) Free vibrations *(ii)* Forced vibrations *(iii)* Damped vibrations and *(iv)* Resonance

4. Discuss the characteristics of simple harmonic motion.

5. A loaded bus is more comfortable than an empty bus. Explain.

6. What is cantilever ? Derive an expression for the time period of a cantilever.

PROBLEMS

1. Calculate the time period of a body executing SHM if it has an acceleration of 2 $m\,s^{-2}$ when its displacement is 10 *cm*. **(Ans.** 1·4 *s*)

2. A body executing S.H.M. has amplitude of 1 *cm*. Its velocity at the mean position is 10 *cm* per second. Find its frequency. **(Ans.** 1·59 Hz)

3. A body is executing SHM with an amplitude of 15 *cm* and frequency of 4 vib/s. Calculate the time required to move from mean position to a point 12 *cm* away from it. **(Ans.** 0·0369 *s*)

4. A mass of 100 *kg* is supported on a spring of stiffness factor 980 *N/m*. Find its compression and time period of vibration. **(Ans.** 1 *cm*., 0·2 *s*)

5. A 0·5 *kg* mass is hung from the lower end of a coiled spring and causes the spring to elongate by 0·05 *m*. If the mass is set to vibrate, how many complete vibrations will it make in one minute ? **(Ans.** 133)

6. A spring of stiffness factor *K* is making vibrations in S.H.M. When the displacement is one–half the amplitude, what fraction of the total energy is kinetic and what fraction is potential ?

   At what displacement is the energy half kinetic and half potential ?

   $$\left(\text{Ans.}\frac{3}{4},\frac{1}{4},A/\sqrt{2}\right)$$

7. A small body of mass 0·1 *kg* is undergoing simple harmonic motion of amplitude 1·0 metre and period 0·20 second.

   (*a*) What is the maximum force acting on the body ?

   (*b*) If the oscillations are produced by spring, what is the spring constant ?

   (Ans. (*a*) 25 *N* (*b*) 99 *N/m*)

8. A body is executing SHM of amplitude 10 and frequency 10 vibration per second. Calculate its acceleration after 0·015*s*. **(Ans.** 31940 ms^{-2})

9. The shortest distance travelled by a particle executing SHM from mean position in 2*s* is equal to $\frac{\sqrt{3}}{2}$ of its amplitude. Find its time period. **(Ans.** 12*s*)

10. A bob of simple pendulum of mass 1 kg is oscillating with a frequency of 5 revolutions per second and its amplitude is 3*m*. Find the kinetic energy in the lowest position **(Ans.** 4441·5 *J*)

Chapter

ST

Surface Tension

The attractive force of one molecule for another is responsible for many interesting phenomena. These forces between like molecules are called cohesive forces and are effective in solids as well as liquids. In liquids the common observation is the tendency to form droplets.

If the attracting force between a liquid molecule and a solid (surface) molecule (adhesion) is greater than between liquid molecules (cohesion), we say that the liquid wets the surface. If the liquid molecules are more attractive by each other than by surface molecules, we find the liquid pulling up into droplets and do not wet the surface.

These molecular forces predict the behaviour of liquid due to which their free surfaces at rest tend to contract.

The most important example of cohesion and adhesion is met with water in a glass tube of small diameter (capillary tube) where the column of water is raised up in the tube until its weight just balances the upward adhesive forces of the glass molecules. In very find capillary bores the height of water becomes much more.

This capillary action in porous structures explains the lifting of liquids by blotters, sponges, trees etc.

ST—1. MOLECULAR FORCES AND SPHERE OF INFLUENCE.

The forces of attraction or repulsion which exist between the molecules are as given below :

(i) **Molecular Forces :** *The forces of attraction or repulsion existing between the molecules of various substances are called* **molecular forces.**

Every substance is composed of tiny particles called **molecules.** These molecules attract or repel each other depending upon the distances between them. Such forces existing between the molecules are called *molecules forces.* The physical properties of a substance *i.e.* surface tension, viscosity, elasticity etc. depend upon the nature and magnitudes of these molecular forces. The forces of attraction exist not only between molecules of the same substances but also between the molecules of different substances.

(ii) **Force of Cohesion :** *It is the force of attraction between the molecules of the same substance.* Force of cohesion is also called **cohesive force** and the corresponding phenomenon is called *cohesion.*

The force of cohesion keeps the molecules of solids packed very closely. Therefore, solids have a definite shape.

Example. Force of cohesion is quite strong in steel molecules. Therefore, steel girders or axles can withstand very heavy loads.

(iii) **Force of Adhesion :** *It is the force of attraction between the molecules of two dissimilar substances.* It is also known as **adhesive force** and the corresponding phenomenon is called *adhesion*.

Example. When we dip glass rod in water, it sticks to the glass rod due to the force of adhesion between glass and water molecules. Again if we dip a glass rod in mercury, it does not stick to the glass rod which shows that the force of adhesion between mercury and glass is less than the force of cohesion between mercury molecules.

This explains why water wets the surfaces and mercury does not wet the surfaces.

(iv) **Molecular Range :** *The maximum distance upto which a molecule can exert force of attraction on another molecule is called* **molecular range.** In case of solids and liquids the range is of the order of 10^{-9} m and is different for different pairs of substances.

(v) **Sphere of Influence :** *It is a sphere drawn around a particular molecule as the centre and molecular range as the radius.*

The molecules lying within the sphere of influence are attracted by the central molecule but no molecule beyond the sphere of influence is attracted by the molecule at the centre.

Surface film. *The top most layer of a liquid having thickness equal to the molecular range is called surface film.*

ST—2. SURFACE TENSION

Surface tension is the property of a liquid by virtue of which the free surface of the liquid at rest behaves like a stretched elastic membrane tending to contract to have minimum surface area.

Concept of surface tension :

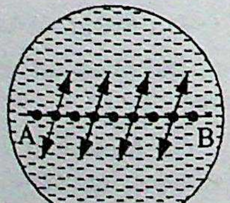


Fig. ST—1

To understand the concept of surface tension, consider an imaginary line AB drawn on the free surface of a liquid at rest. The molecules lying just on one side of AB try to pull the molecules lying exactly on the line. Similarly the molecules lying on the other side of AB pull the molecules lying on the line towards the other side. Thus the molecules on the line are pulled on both sides by forces which lie along the tangent to the surface of liquid and perpendicular to the line AB (See fig. ST—1). Thus every molecule on the line is pulled equally in all directions. Therefore, the molecules on the free surface of the liquid at rest are in a state of tension and the surface tends to contractor to minimum area.

If F is the total force on either side of the line AB of length l, then

$$\text{Surface tension, } T, = \frac{\text{Force}}{\text{length}} = \frac{F}{l}$$

Thus surface tension is measured as the force per unit length acting on either side of a line in the liquid surface at rest. The direction of the force being tangential to the surface and perpendicular to the line.

The *SI* units of surface tension are newtons per metre Nm^{-1}

ST—3. MOLECULAR THEORY OF SURFACE TENSION :

Each molecule exerts an attractive force on the molecules surrounding it upto a certain distance. Beyond this distance the force of attraction is considered to be zero. The assumption of the molecular range provides an explanation for the phenomenon of surface tension.

The explain this consider four molecules *A*, *B*, *C* and *D* of the liquid with their *spheres of influence*. Molecule *A* is well within the liquid. Molecule *B* is in the plane *RS* drawn parallel to the free surface *PQ* of the liquid at a distance equal to the molecular range. The layer between the plane *RS* and free surface *PQ* is called **surface film.**

Molecules *A* and *B* are attracted equally in all directions by other molecules in their spheres of influence. Hence no resultant force acts on them and they move freely in the liquid. Same is the case with other molecules well within the liquid.

Let molecule *C* lies in the surface film above the plane *RS*, therefore, has lesser number of molecules in the upper half of its sphere of influence than in the lower half. Thus if experiences a **resultant downward** or **inward force.** Similarly, all the molecules within the planes *RS* and *PQ* experience resultant downward forces. The downward force is minimum for molecules just above the plane *RS* but goes on increasing in going from plane *RS* to plane *PQ*. It being maximum for molecules in the plane *PQ*.

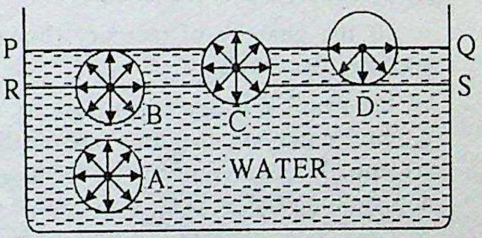


Fig. ST—2

The molecule *D* is on the free surface of the liquid and there is no molecule in the upper half of its sphere of influence. Thus there is a maximum downward force acting on molecule *D* and the same is true for all the molecules lying on the free surface.

All the molecules lying the surface film experience a net downward force which is minimum for molecules at the bottom of the surface film and maximum for molecules at the top of the film.

Now to move a molecule from within the liquid to the surface film, work has to be done against the inward pull.

Thus work done on the molecule is stored in it in the form of *potential energy*. Thus the potential energy of the molecules in the surface film is greater than those inside the liquid. The potential energy per unit area of the surface film is called **surface energy.**

As a system always tends to have minimum potential energy, surface contracts to have the minimum area. Since the surface film has a fixed thickness. Therefore, there are least number of molecules in the surface film for the potential energy to be minimum. This explains why the free surface of a liquid tends to contract to have minimum surface area.

For a molecule to occupy a position in the surface film space has to be provided by pulling molecules apart against the force of cohesion. In other words, work has to be done to increase the surface area of a liquid just as work as to be done to stretch a rubber sheet. This explains why the surface of a liquid at rest behaves like a stretched membrane or an elastic skin.

ST—4. SMALL DROPS OF MERCURY ARE ROUND (SPHERICAL) :

We observe that liquids when in small quantities always assume spherical shapes. For example rain drops, dew drops, mercury drops are all spherical (see fig. ST—3).

If a mercury drops is disturbed by applying small pressure from finger, it returns to its original shape (spherical) when pressure is removed. This shows that the mercury drop behaves as if the skin stretched over its surface was under tension. This phenomenon is **surface tension.**

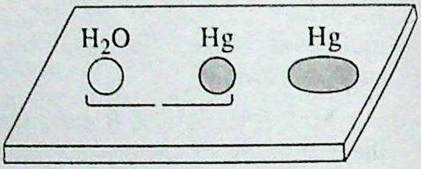


Fig. ST—3

If the quantity of mercury (liquid) is taken in large quantity, the bigger drop becomes flat (oval).

The reason is that when mercury is in small quantity, the external forces such as gravity acting on the drop are negligible in comparison to the **force of surface tension.** When mercury is taken in large quantity the force of gravity is appreciable and the big drop acquires a flat shape due to the weight of mercury.

Thus when liquids are in small quantities and free from external forces, they assume spherical shapes. And as for a given volume the surface area of the sphere is minimum, therefore, liquids assume spherical shape to have minimum area.

ST—5. EXPERIMENTS TO DEMONSTRATE SURFACE TENSION :

1. Take a camel hair paint brush and dip into water. Its hair get separated from each other. Now take the brush out of water. Its hair will cling together as the free surfaces of water films round the hair try to contract due to surface tension (see. fig. ST—4).

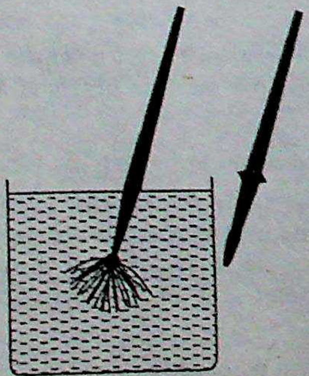


Fig. ST—4

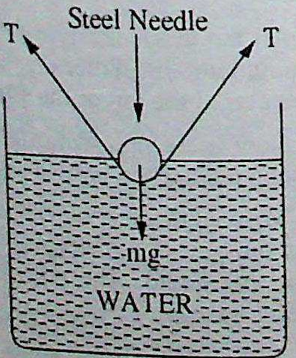


Fig. ST—5

2. When a small needle is gently placed on the surface of water, it floats. The water surface below the needle gets slightly depressed. The force of surface tension does not act horizontally but along an inclined direction. The vertical component of the force of surface tension balances the weight *mg* of the needle (see fig. ST—5).

3. Take a circular wire frame with a handle. Dip the frame in soap solution so as to form a thin soap film. Take a cotton thread loop and place it in an irregular shape on the film very gently. It will lie in an irregular way (see fig. ST—6(*a*)). Now prick the film within the loop by a sharp needle. The film in the loop disappears and the loop of the thread takes a circular shape. For a given perimeter, a circle has maximum area. Thus soap film possesses minimum surface area. (see fig. ST—6 (*b*)).

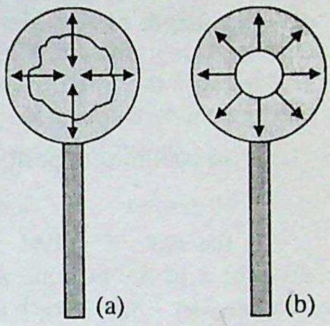

(a) (b)

Fig. ST—6

4. A rain drop or a globule of mercury assumes a spherical shape due the force of surface tension. External forces such as gravity etc. are not appreciable in such cases. The external forces will play an important role in the case of large drops.

5. An interesting experiment can be performed as. Take a small wooden boat and attach some soap at its back, as at *A* in (Fig. ST—7). Place the boat on the surface of water in a pond. As the soap goes into solution, surface tension at the back is decreased and the surface tension in the front pulls the boat forward. Thus the boat goes on moving till the whole surface of water gets covered with soap film.

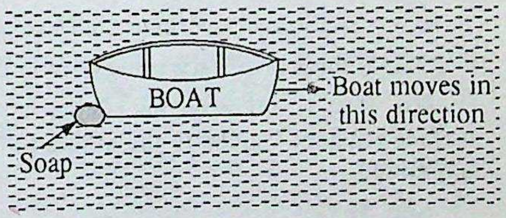


Fig. ST—7

ST—6. SURFACE ENERGY

The potential energy per unit area of the surface film is called the surface energy.

Or

The amount of work done in increasing the area of a surface film through unity under isothermal conditions is called surface energy.

$$\therefore \quad \text{Surface energy} = \frac{\text{Work done is increasing the surface area}}{\text{Increase in area}}$$

Relation between surface energy and Surface Tension or Expression for surface energy.

Take a rectangular wire frame *ABCD* with a horizontal wire *AD* placed across it such that it can move freely up and down. Dip the wire frame in soap solution so that a soap film is formed across *ABCD*. The wire *AD* is pulled inwards by the force of surface tension of the film. To keep the wire in position a force has to be applied downwards equal and opposite to the upward force of surface tension. Let the downward force applied be equal to F. If T be the force of surface tension or force per unit length of the wire and l be the length of the wire AD, then upward force acting on the wire AD = $2l\,T$ because the film has two surfaces and each surface has its own surface tension T.

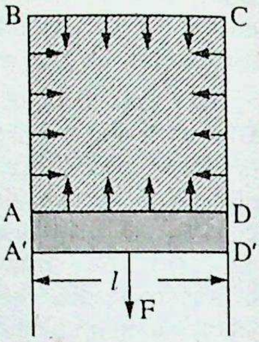


Fig. ST—8

∴ In equilibrium position,

$$F = 2l\,T$$

If the wire is pulled downwards through a small distance x to the position $A'D'$, the area of the film is increased by $l \times x$ on each side.

∴ Work done to increase the area $= F \times x = 2lT \times x$.

Now $2l \times x$ is the total increase in area of the soap film on both sides. The work done in increasing the area of the film is stored in it in the form of surface energy. Therefore, if E be the surface energy of the film, then

$$E = 2l\mathrm{T} \times x = T = 2lx = T \times \text{increase in area}$$

∴ Surface energy = Surface tension × increase in area

The *SI* units of surface energy are joule/m^2 or Jm^{-2}.

Ex. ST—1. If r is the radius of the soap bubble having T the value of surface tension. Calculate the amount of work done if the diameter of the bubble is doubled.

Sol. Here,

Initial radius of soap bubble, $r_1 = r$

Final radius of soap bubble, $r_2 = 2r$

∴ Increase in surface area of the bubble.

$$\begin{aligned} A &= 2 \times 4\pi r_2^2 - 2 \times 4\pi r_1^2 \\ &= 8\pi\,(r_2^2 - r_1^2) = 8\pi\,(4r^2 - r^2) \\ &= 8\pi r^2\,(4 - 1) = 24\pi r^2 \end{aligned}$$

$$\begin{aligned} \text{Now work done} &= \text{Surface tension} \times \text{Increase in area} \\ &= T \times 24\pi r^2 = 24\pi \times Tr^2 \text{ Ans.} \end{aligned}$$

Ex. ST—2. Calculate the energy required to split a drop of water of radius 1 mm into one thousand million droplets of the same size. Surface tension of water = 72 dyne cm^{-1}.

Sol. Here, Radius of water drop, $R = 1\text{ mm} = 10^{-3}\,m$

No. of droplets formed, $n = 1000 \times 10^6 = 10^9$

Surface tension of water, $T = 72\text{ dyne cm}^{-1} = 0{\cdot}072\,Nm^{-1}$

Let radius of droplet be r

Now, Volume of 10^9 droplets = volume of big water drop

$$\therefore \qquad 10^9 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\text{or} \qquad 10^9\, r^3 = R^3 = (10^{-3})^3 = 10^{-9}$$

$$\text{or} \qquad (10^3\, r)^3 = (10^{-3})^3$$

$$\text{or} \qquad r = \frac{10^{-3}}{10^3} = 10^{-6}\, m$$

∴ Increase in surface area, $A = 10^9\, 4\pi r^2 - 4\pi R^2$

$$= 4\pi\, [10^9\, (10^{-6})^2 - (10^{-3})^2] = 4\pi[10^{-3} - 10^{-6}]$$

$$= 4 \times \frac{22}{7}[\cdot 001 - \cdot 000001]$$

$$= 4 \times \frac{22}{7} \times 999 \times 10^{-6}\ \text{m}^2$$

$\therefore$ Total work done $= T \times A$

$$= 0 \cdot 072 \times 3896 \times 10^{-6} \times \frac{22}{7}\, Nm$$

$$= 72 \times 39{\cdot}96 \times 3{\cdot}142 \times 10^{-9}\, Nm$$

$= 9039{\cdot}9 \times 10^{-7}$ joules **(Ans.)**

Ex. ST—3. Calculate the work done in blowing up a soap bubble from an initial surface area of 0·5 cm^2 to final surface area of 1·10 cm^2. The surface tension of the soap solution is 30 *dyne*/cm.

Ans. Given T = 30 *dyne*/cm, initial surface area = 0·5 cm^2

Final surface area = 1·10 cm^2

Since a bubble has two free surfaces

Effective increase in the surface area = A = 2 × (1·10 – 0·50) = 1·20 cm^2

The required work done = W = T × A

W = 30 × 1·20 = 36 erg. **Ans.**

Ex. ST—4. A liquid drop of diameter D breaks up into 27 tiny drops. Find the resulting change in energy. Take surface tension of liquid as T.

Ans. No. of small drops formed = 27

Radius of big drop $R = \frac{D}{2}$

If r is radius of each small drop, then

Volume of 27 small drops = volume of the big drop

$$27 \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$\Rightarrow \qquad r = \frac{D}{6}$$

Initial surface area of big drop $= 4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$

Final surface area of 27 small drops $= 27 \times 4\pi r^2$

$$= 27 \times 4\pi\left(\frac{D}{6}\right)^2 = 3\pi D^2$$

Therefore increase in surface area $= 3\pi D^2 - \pi D^2 = 2\pi D^2$

Change in energy = increase in area × surface tension

Change in energy = $2\pi D^2 \times T$ **Ans.**

ST–7. SHAPE OF LIQUID MENISCUS IN CONTACT WITH A SOLID INSIDES CAPILLARY TUBE) :

Our daily experience shows that the surface of a liquid like water which wets the walls of the container is *concave,* whereas it is *convex* for a liquid like mercury, which **does not wet** the walls of the container.

To explain how it happens, consider a molecule *A* on the *surface of a liquid touching the walls* of a glass tube. It is acted upon by the following three forces.

(*i*) The weight of the liquid molecule acting vertically downwards.

(*ii*) The force of **adhesion** F_1 due to the molecules of the solid, acting along *AB* at right angles to the wall of the container.

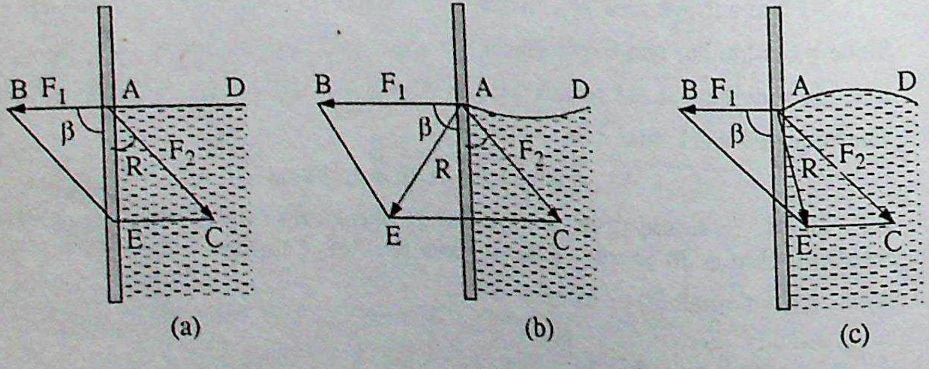


Fig. ST—9

(*iii*) The force of **cohesion** F_2 due to the molecules of the 'liquid, tiny along *AC* making an angle of 45° with the wall of the container.

Since the weight of the liquid molecule is very small, it is generally neglected. The other two forces F_1 and F_2 are inclined to each other at an angle of 135° and their resultant will depend upon the relative magnitude of the two forces. Let the resultant force *R* make an angle β with F_1, then from the parallelogram law of forces.

$$\tan\beta = \frac{F_2 \sin 135^\circ}{F_1 + F_2 \cos 135^\circ} = \frac{F_2 \sin 45^\circ}{F_1 - F_2 \cos 45^\circ}$$

$$= \frac{F_2 / \sqrt{2}}{F_1 - F_2 / \sqrt{2}} = \frac{F_2}{\sqrt{2}\, F_1 - F_2}$$

Free surface of a liquid at rest is always perpendicular to the resultant force acting on it.

Now three cases arise.

(*i*) IF $F_2 = \sqrt{2}\ F_1$, then $\tan\beta = \infty$ or $\beta = 90^\circ$.

Thus the resultant *R* acts vertically downwards and hence the liquid surface *AD* remains **horizontal** as shown in Fig. ST—9 (*a*).

(*ii*) If $F_2 < \sqrt{2}\, F_1$, then tan β is positive or β is an acute angle.

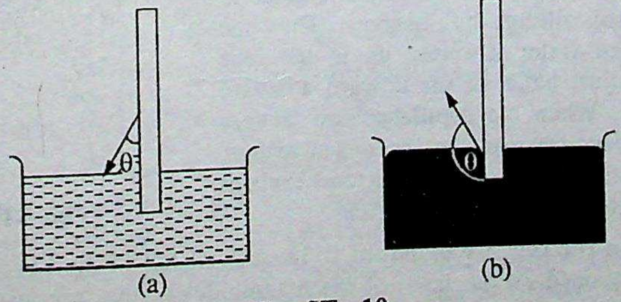
Thus the resultant R is directed outside the liquid as in Fig. ST-9(b). The liquid surface near A curves upwards so as to be perpendicular to the resultant AE and thus assumes a **concave meniscus**. This is true for water and other liquids which wet the walls of the glass tube.

(iii) If $F_2 > \sqrt{2} F_1$, then tan β is negative or angle β is an obtuse angle (but less than 135°) so that the resultant *R* lies inside the liquid, as in Fig. ST—9 (*c*). The liquid surface near *A* curves down wards so as to be **perpendicular to the resultant** *AE* and thus assumes a **convex meniscus.** This is true in the case of mercury which does not wet the walls of the glass tube.

ST—8. ANGLE OF CONTACT.

The angle between the tangent to the liquid surface at the point of contact and the solid surface, inside the liquid is called the **angle of contact** *for that pair of solid and liquid.* It is denoted by θ.

When a liquid comes in contact with a solid, its surface near the point of contact is generally **curved. A glass rod dipped,** in a liquid like water draws it up and forms a curved (*concave*) surface as shown in fig. ST—10 (*a*). But if the same glass rod is dipped in mercury, it gets depressed down and forms a curved (*convex*) surface as shown in fig. ST—10 (*b*).



(a) (b)

Fig. ST—10

If we draw a tangent to the curved surface of the liquid at the point of contact, inside the liquid, the angle θ between the tangent and solid is called the **angle of contact**.

This angle may have any value between 0° and 180°. *It depends upon the nature of the liquid and the solid and is independent of the inclination of the solid.*

The angle of contact θ is **acute** (less than 90°) in the case of liquid which wet the sides of the container.

Example. For *pure water* and *clean glass,* angle of contact is 0° and for ordinary water and glass it is about 18°.

The angle of contact θ is **obtuse** (greater than 90°) in the case of liquids which do not wet the sides of the container.

Example. For *mercury* and *glass* angle of contact is about 138°.

ST—9. CAPILLARITY OR CAPILLARY ACTION

The phenomenon of rise or fall of a liquid in a capillary tube is called capillarity or **capillary action,**

A capillary is a tube of very *fine bore.*

One of the most striking effects of surface tension is to raise a liquid in a capillary tube when it is dipped into the liquid.

When a capillary tube is dipped in a liquid like water which wets it, the liquid immediately rises up (see fig. ST—11) (*a*).

Again when a capillary tube is dipped in a liquid like mercury, which does not wet it, mercury is seen to depress down in the tube (see fig. ST—11(*b*)).

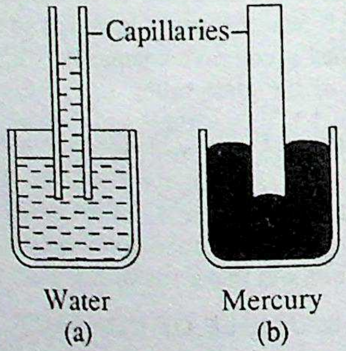


Fig. ST—11

Fig. ST—12 shows that the attractive forces between glass and water molecules are sufficiently great to support the weight of the column of water in the tube.

When water is in a glass (solid) capillary tube, the glass molecules seem to pull up the water molecules. The rest of the water particles, also tend to hang together, and the result is similar to lifting up of iron fillings by magnets. Thus the whole column of water is raised up in the tube until its weight just balances the upward adhesive forces of glass. When the capillary bore is very fine this height becomes many centimetres or metres. This capillary action in fine porous structures explains the lifting of liquids by sponges and trees.

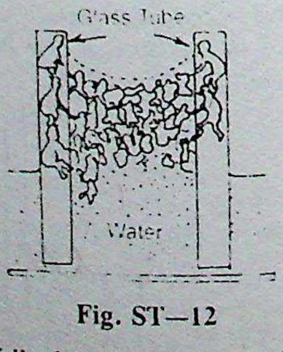


Fig. ST—12

Explanation for the rise of liquid : The rise of fall of the liquid in a capillary tube can be explained on the basis of the magnitudes of the forces of *adhesion* and *cohesion.*

In case of liquids like water, which wets the surfaces, the forces of *adhesion* (for glass and water) is greater than the force of *cohesion*. On dipping the capillary tube in water the force of adhesion makes the molecules to creep up the glass walls making the water surface concave and hence increasing the surface area. Surface tension in its tendency to decrease the surface area to the minimum, flattens the surface of water and the result is that water slightly rises above the level of water in the vessel.

Again the adhesion of water to glass makes the water creep up the glass walls, thus raising the level of water further by a small amount. This process continues and the level of water goes on rising will the downward weight of water in the capillary tube is balanced by the upward force due to surface tension.

In case of liquids like mercury, which do not wet the surface, the force of cohesion between the molecules of mercury is greater than the force of adhesion for mercury and glass, mercury therefore, is depressed in the tube.

Examples of capillarity or capillary action : The following are some familiar examples of capillary action :-

(*i*) The oil rises in cotton wicks through small spaces between the threads which serve as capillary tubes.

(*ii*) Blotting paper absorbs ink by the capillary action of ink in the fine pores.

(*iii*) Water rises in plants up the leaves due to the capillary action in the fine pores of the fibres of wood.

(*iv*) Water comes out off earthen vessel through small pores by capillary action :

(*v*) Ink rises in pens due to the capillary action, as the tip of the nib is split to act as capillary tube.

(*vi*) We use a towel to dry our body after the bath, which soaks water due to capillary action.

(*vii*) A sponge retains water (on being dipped in water) is again due to capillary action.

(*viii*) To avoid damping of mortar and bricks on account of rise of moisture by capillary action, damp proof materials are used in foundations of buildings.

ST—10. EXPRESSION FOR RISE OF LIQUID IN A CAPILLARY TUBE :

Consider a capillary tube open at both ends dipped vertically in a liquid like water which wets, the surface. The liquid resists in the tube upto a certain height and is found to form concave meniscus as shown in fig. ST—13.

Let, θ = the angle of contact

h = height to which liquid rises

r = radius of the tube

T = surface tension of the liquid

The force of surface tension T acts along the tangent at A to the surface of the liquid as shown by AB. It, therefore, pulls the walls of the tubes along AB and according to *Newton's third law of motion* an equal and opposite reaction ($R = T$) acts along AC as shown. This reaction ($R = T$) can be resolved into two rectangular components :-

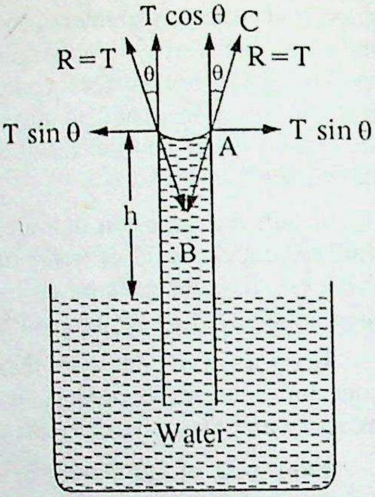


Fig. ST—13

1. T cos θ in the vertically upward direction along the walls.

2. T sin θ in the horizontal direction perpendicular to the walls.

The horizontal component $T \sin\theta$, all round the tube, cancel out and hence plays no part in raising the liquid. The vertical component, $T \cos\theta$, all round the tube, acts parallel to the walls of the tube and pulls the liquid in the upward direction. In equilibrium, it just balances the weight of the liquid column of height h. The force $T \cos\theta$ acts along the whole circumference of the tube. (See fig. ST—13).

Thus the total upward pull

= upward component × circumference

$= T\cos\theta \times 2\pi r$

As r is the radius of the tube, the volume of liquid column of height, h
If ρ is the density of the liquid then mass of the liquid

$$= \pi r^2 h \times \rho$$

Thus the weight of the liquid column which acts in the downward direction

$$= \pi r^2 h \rho g$$

In equilibrium position,
Weight of liquid column of height h = total upward pull

$$\therefore \quad \pi r^2 h\rho g = 2\pi r T\cos\theta$$

$$\text{or} \quad h = \frac{2T\cos\theta}{r\rho g} \quad \ldots(i)$$

In the above equation, θ being less than 90° for liquids, like water, which wet the walls, *cos θ is positive and hence 'h' is positive i.e.*. liquid rises in the capillary tube. But for liquids, like mercury, which do not wet the walls, θ is greater than 90°, therefore, cos θ is negative and hence h is negative *i.e.*, liquid is depressed in the capillary tube.

For pure water in clean glass tube θ = 0 °.

$$\therefore \quad \cos\theta = \cos 0^\circ = 1$$

$$\text{Hence} \quad h = \frac{2T}{r\rho g} \quad \ldots(ii)$$

$$\text{or} \quad T = \frac{r\rho h g}{2} \quad \ldots(iii)$$

Since T and ρ are constant for a given liquid

$$\therefore \quad h \propto \frac{1}{r}.$$

This shows that smaller the radius of capillary tube, greater is the height to which liquid rises.

Ex. ST—3. How high does water rise in a capillary tube whose inner diameter is 44 × 10^{-5} m, if the angle of contact is negligible and surface tension of water being 0·075 N/m.

[g = 9·8 m/s^2, density of water = 10^3 kg/m^3)

Sol. Here,

Radius of the capillary tube, $r = \frac{44\times10^{-5}}{2} = 22\times10^{-5}\,m$

Density of water, $\rho = 10^3 = 1000\ kg/m^3$

Acceleration due to gravity, $g = 9{\cdot}8\ m/s^2$

Surface tension of water, $T = 0{\cdot}075\ N/m$

Angle of contact for water, $\theta = 0°$

Height to which water will rise, $h = ?$

We know,

$$h = \frac{2T\cos\theta}{r\rho g}$$

$$h = \frac{2\times0\cdot075\times1}{22\times10^{-6}\times1000\times9\cdot8}\ \text{metres} \qquad (\because \cos 0° = 1)$$

$$= \frac{2\times75\times10^{-3}}{22\times98\times10^{-3}}\ \text{metres} = \frac{75}{11\times98}$$

$$= \mathbf{0{\cdot}0896\ metre = 8{\cdot}96\ cm.\ (Ans).}$$

Ex. ST—4. Mercury has an angle of contact equal to 140° with soda lime glass. A narrow tube of radius 1·0 mm made of thin glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of experiment is 0·465 N/m. Density of mercury is 13·6 × 10^3 kg/m^3.

Sol. Given, $r = 1{\cdot}0\ \text{mm} = 1\times10^{-3}\ \text{m},\ T = 0{\cdot}465\ \text{N/m}$

$\rho = 13{\cdot}6\times10^3\ \text{kg/m}^3,\ \theta = 140°,\ h = ?,\ g = 9{\cdot}8\ \text{m/s}^2$

We know that

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times0\cdot465\times\cos140°}{1\cdot0\times10^{-3}\times13\cdot6\times10^3\times9\cdot8}$$

$$\therefore \qquad h = -\,5{\cdot}345\times10^{-3}\ \text{m. } \textbf{Ans.}$$

Ex. ST—5. In a capillary of diameter 10^{-3} m, water rises to a height of 0·03 m. Calculate the surface tension of water. Given g = 9·8 m/s^2, density of water = 1000 kg/m^3.

Sol. Given, diameter = 10^{-3} m, radius $= \frac{\text{diameter}}{2} = \frac{10^{-3}}{2}$

$r = 0{\cdot}5\times10^{-3}\ \text{m},\quad g = 9{\cdot}8\ \text{m/s}^2,\quad h = 0{\cdot}03\text{m}.$

$\rho = 1000\ \text{kg/m}^3,\ \theta = 0°$ (for water), $T = ?$

We have the ascent formula, $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad T = \frac{hr\rho g}{2\cos\theta} = \frac{0\cdot 03 \times 0\cdot 5 \times 10^{-3} \times 1000 \times 9\cdot 8}{2 \times \cos 0^\circ}$$

$$\therefore \quad T = 0\cdot 0735 \text{ N/m Ans.}$$

Ex. ST—6. A glass capillary tube of diameter 0·5 mm stands vertically in a vessel containing a liquid which wets the tube and has a density of 800 kg/m^3. The liquid rises up to a height of 2·9 cm. Calculate the surface tension of the liquid.

Sol. Given diameter = 0·54 mm, radius $= \dfrac{0\cdot 54}{2} = 0\cdot 27 \times 10^{-3}$ m

$\rho = 800$ kg/m^3, $h = 2\cdot 9$ cm $= 0\cdot 029$ m, $g = 9\cdot 8$ m/s^2, $\theta = 0°$, T = ?

We have, $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad T = \frac{hr\rho g}{2\cos\theta} = \frac{0\cdot 029 \times 27 \times 10^{-5} \times 800 \times 9\cdot 8}{2 \times \cos 0^\circ}$$

$$T = 0\cdot 0307 \text{ N/m. Ans.}$$

ST—11. EXPERIMENTAL DETERMINATION OF SURFACE TENSION

Surface tension of any liquid which wets the surface can be determined in the laboratory by measuring the rise of liquid in a capillary tube.

To determine surface tension of water take a capillary tube *C* of uniform area. To remove grease etc. clean it with caustic soda solution and then with running, water thoroughly. Fix it on a glass strip *G* with a rubber band *R*. Also fix a thin needle *N* on the same glass strip parallel to the capillary tube. Hold this glass strip vertically in a clamp stand. Now place a beaker containing tap-water and adjust the height of the clamps so that the capillary tube dips in water and water rises in it. Adjust the position of the needle *N*, so that its tip just touches the surface of water (See fig. ST—14).

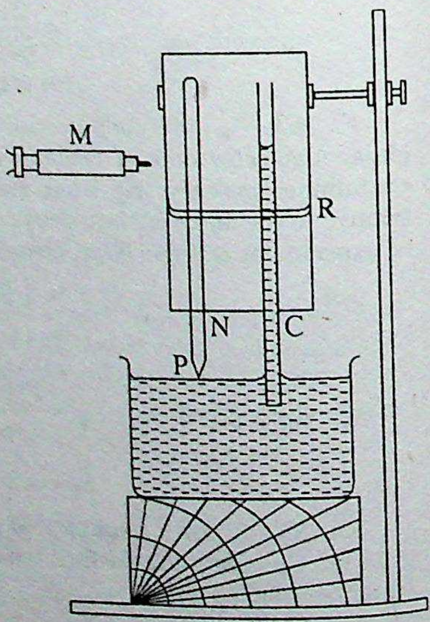


Fig. ST—14

To find the height '*h*' of water in the capillary tube, focus a travelling microscope *M* on the point of contract *P* of water and needle and note the reading of the microscope. Now raise the microscope and focus it on the lower meniscus of water in the capillary tube and not the reading again. The difference of these two readings gives the height of water column in the capillary tube. Note the temperature of water and find its density from tables.

To find the radius of the tube, cut it with a file at the point where water meniscus stands. Hold one of the part of the tube in a horizontal position and focus the microscope on its cross-section, Adjust the horizontal cross-wire of the microscope so as to set it tangentially at A and note the reading. Set the cross wire tangentially again at B and note the reading. The difference of these readings gives the diameter of the tube. Again by adjusting the vertical cross-wires of the microscope take the readings at the positions C and D and find out the diameter. The mean of these diameters and half of it gives the radius of the capillary tube. Then calculate the surface tension by using the relation.

$$T = \frac{r\rho h g}{2\cos\theta} N/m$$

where, T = surface tension in N/m

r = radius of the tube in *metres*

ρ = density of the liquid in kg/m^3

g = acceleration due to gravity in m/s^2

θ = angle of contact.

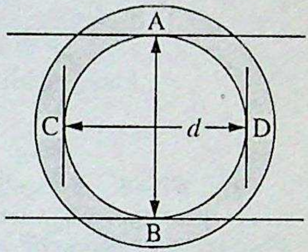


Fig. ST—15

For pure water and clean glass $\theta = 0°$

Surface tension of a liquid decreases with the rise of temperature. Impurities present on the surface of a liquid produce an appreciable change in the value of the surface tension. Therefore, temperature should remain constant during the experiment and there should be no impurities on the surface of water.

Ex. ST—7. A capillary tube of inside radius 0·5 mm is dipped in water of surface tension 75 dynes/cm. To what height is the water raised by capillary action ?

(Angle of contact for water with glass is 0°)

Sol. Here, Radius of tube, $r = 0\cdot5 \times 10^{-3}$ m

Surface tension of water, T = 75 dynes/cm = 0·05 N/m

Density of water, $\rho = 1000$ kg/m^3

Angle of contact, $\theta = 0°$

Let h be the height upto which water rises in the capillary tube.

Using the relation, $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\therefore \quad h = \frac{2\times 0\cdot 075\times 1}{0\cdot 5\times 10^{-3}\times 1000\times 9\cdot 8} m \quad (\theta = 0° \text{ and } \cos 0° = 1)$$

$$= \frac{2\times 75\times 10^{-3}}{5\times 10^{-3}\times 10\times 98} = \frac{2\times 75}{5\times 10\times 98} = 0\cdot 1530\, m$$

$= 15\cdot30$ cm **Ans.**

Ex. ST—8. Calculate the amount of energy evolved when 8 droplets of water of surface tension 72 dynes/cm of radius $\frac{1}{2}$ mm each combine into one.

Sol. Let R be the radius of big drop formed.

Radius of small droplets, $r = 0{\cdot}05$ mm, $0{\cdot}5$ cm

Now volume of 1 big drop = vol. of 8 small drops

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$$R^3 = 8\,r^3$$

$$\therefore \quad R = 2r = 2 \times 0{\cdot}05 = 0{\cdot}1 \text{ cm}$$

Hence, decrease in surface area = surface area of 8 drops surface area of 1 big drop

$$= 8 \times 4\pi r^2 - 4\pi R^2 = 4\pi\,[8 \times (\cdot 05)^2 - (0{\cdot}1)^2]$$

$$= 4\pi\,[0{\cdot}02 - 0{\cdot}01] = 0{\cdot}04\pi \text{ sq. cm.}$$

$\therefore$ Energy evolved = surface tension × decrease in area

$$= 72 \times 0 \cdot 04 \times \frac{22}{7} = 9{\cdot}05 \text{ ergs } \textbf{(Ans).}$$

Ex. ST—9. A capillary tube of inner diameter 0·4 mm is dipped in a liquid of sp. gravity 13·6, surface tension 547 dynes/cm and the angle of contact 135°. Find the depression or elevation of the liquid in the tube.

Sol. We know that $T = \dfrac{rhdg}{2\cos\theta}$

$$\therefore \quad h = \frac{2T\cos\theta}{rdg}$$

Here,

$$r = 0{\cdot}2 \text{ mm} = 0{\cdot}2 \times 10^{-3} \text{ m}$$

$$d = 13{\cdot}6 \text{ gm/cc} = 3{\cdot}6 \times 10^3 \text{ kg/m}^3$$

$$T = 547 \text{ dynes/cm} = 547 \times 10^{-3} \text{ N/m}$$

$$\theta = 135^\circ \text{ and } g = 9{\cdot}8 \text{ ms}^{-2}$$

$$\therefore \quad h = \frac{2 \times 547 \times 10^{-3} \times \cos 135^\circ}{0 \cdot 2 \times 10^{-3} \times 13 \cdot 6 \times 1000 \times 9 \cdot 8}$$

$$= \frac{2 \times 547 \times -\cos 45^\circ}{0 \cdot 2 \times 13 \cdot 6 \times 9 \cdot 8 \times 1000} = \frac{2 \times 547 \times -0 \cdot 7071}{2 \times 136 \times 98}$$

$$= -\,0{\cdot}02902 \text{ m} = -\,2{\cdot}902 \text{ cm } \textbf{(Ans).}$$

Negative sign of h shows that the liquid is depressed in the capillary tube.

Ex. ST—10. Two narrow bores of diameters of 3·0 mm and 6·00 mm are joined together to form a U–shaped tube open at both ends. If U–tube contains water, what is the difference in its levels in two limits of the tube ? Surface tension of water at the temperature of experiment is $7{\cdot}3 \times 10^{-2}$ N/m. Take the angle of contact to be zero & density of water as 1×10^3 kg/m^{-3}.

Sol. Given, $T = 7{\cdot}3 \times 10^{-2}$ N/m, $\theta = 0^\circ$, $\rho = 1 \times 10^3$ kg/m^3

$$g = 9{\cdot}8 \text{ m/s}^2$$

For the tube of diameter 3·0 mm :

Radius of tube $r_1 = \dfrac{3 \cdot 0}{2} = 1 \cdot 5$ mm, $r_1 = 1{\cdot}5 \times 10^{-3}$ m

If h_1 be the height to which water rises, then

$$h_1 = \frac{2T\cos\theta}{r_1 \rho g} = \frac{2\times 7\cdot 3\times 10^{-2}\times\cos 0^\circ}{1\cdot 5\times 10^{-3}\times 1\times 10^{3}\times 9\cdot 8}$$

$$h_1 = 9{\cdot}932\times 10^{-3}\text{ m.}$$

For the tube of diameter 6·0 mm : Radius of tube $= \frac{6\cdot 0}{2} = 3{\cdot}00$ mm

$$r_2 = 3\times 10^{-3}\text{ m.}$$

If h_2 be the height to which water rises, then

$$h_2 = \frac{2T\cos\theta}{r_2 \rho g} = \frac{2\times 7\cdot 3\times 10^{-2}\times\cos 0^\circ}{3\times 10^{-3}\times 1\times 10^{3}\times 9\cdot 8}$$

$$h_2 = 4{\cdot}966\times 10^{-3}\text{ m}$$

Difference in levels of water in two limbs $= h_1 - h_2$

$$h = 9{\cdot}932\times 10^{-3} - 4{\cdot}966\times 10^{-3}$$

$$h = 4{\cdot}966\times 10^{-3}\text{ m } \textbf{Ans.}$$

ST—12. ENGINEERING APPLICATIONS OF SURFACE TENSION.

The following are some of the most familiar engineering applications of surface tension :-

(*i*) **To preserve moisture in fields :** The fields are ploughed and then levelled to increase the size of the pores in the ground. This prevents the rising of water to the surface and thus avoids loss of water by evaporation. If field after ploughing is not levelled, water rises through fine pores tc the surface due to capillary action and evaporates resulting in the drying of the fields.

(*ii*) **Damp proof foundations of buildings :** Bricks and mortar are porous materials and get damped by water of the soil rising through the pores by capillary action. To avoid this, foundations of buildings are made of concrete which is **damp proof.**

(*iii*) **Prevention of loss of water from canals :** If the walls of canals are made of soil or bricks, there is appreciable loss of water due to capillary action. To avoid this, cananl walls are made of concrete which is damp proof and does not allow capillary action.

(*iv*) **Damp proof floors :** Brick floors get damped during rainy season due to the capillary action in the porous material of the bricks. Therefore, floors are made of concrete slabs to avoid dampness.

(*v*) **Action of lubricants :** Liquid lubricants such as oils form a thin layer between the moving parts of machines. In some methods of lubrication, the phenomenon of surface tension is used for the lubricant to rise to the moving parts by capillary action.

QUESTIONS FOR SELF TEST

1. Define the terms :-

(*i*) Cohesive force ; (*ii*) Force of adhesion ; (*iii*) Sphere of influence ;

(*iv*) Angle of contact.

2. What is surface tension ? Give its explanation on the molecular theory.
3. Explain why the surface of mercury in a glass tube is convex and that of water concave.
4. Define angle of contact, when is it acute or obtuse ? Give a laboratory method to determine surface tension of water.
5. Define capillarity. Derive an expression for the height '*h*' through which a liquid of surface tension *T* will rise in a capilliary tube of radius *r*.
6. Mention some engineering applications of surface tension.

PROBLEMS

1. Calculate the work done in spraying a drop of mercury of 1 mm radius into a million drops, all of the same size. Surface tension of mercury = 0·550 Nm^{-1}.
   (**Ans.** 6844×10^{-7} J)
2. What is the energy spent in blowing a soap bubble of radius 10 cm ? What additional work will be performed in further blowing it so that radius becomes 15 cm.[T = 30 dynes/cm] (**Ans.** 75410 ergs, 94260 ergs)
3. Water rises to a height of 10 cm is a capillary tube. If in the same tube level of mercury is depressed by 3·42 cm, compare the surface tensions of water and mercury.
   [Density of *Hg* = 13·6 *g/cc*, angle of contact for water = 0° and for mercury = 135°]. [**Ans.** Tm/Tw = 6·57]
4. Find the height to which water will rise in capillary tube of 1·5 mm diameter. Surface tension of water is 74×10^{-3} Nm^{-1}. [**Ans.** 0·002 m]
   (Pb. July, 1993).
5. Calculate the work done in breaking a drop of water of 2 mm diameter into one thousand million droplets all of the same size. Surface tension of the water is 0·072 N/m. [**Ans.** $9{\cdot}04 \times 10^{-4}$ J]
6. Calculate the amount of energy evolved when one million drops of water of radius 1 mm each coalesce to form a bigger drop. Surface tension of water is 0·075N/m. [**Ans.** 0·93 J]
7. Calculate of the height to which water at 4°C will rise in a capillary tube of 1 mm diameter. Take g = 9·8 m/s^2. Angle of contact = 0° and surface tension of water as 0·072 N/m. [**Ans.** 2·939 cm]
8. Mercury in a capillary tube suffers depression of 13·2 mm. Find the diameter of the tube if angle of contact is 140° and density is 13600 kg/m^3. Surface tension of mercury is 0·54 N/m. [**Ans.** 0·047 cm]
9. A capillary tube of inner diameter 0·5 mm is dipped in a liquid of specific gravity 13·6, surface tension 545 dyne/cm and angle of contact 130°. Find the depression or elevation of liquid in glass tube. [**Ans.** h = – 2·103 cm]
10. Find the difference in levels of mercury in the two limbs of U–tube, if the diameter of bore of one limb is 1 mm & of the other is 4mm. The surface tension of mercury is 0·544 N/m and angle of contact is 130°. [**Ans.** 0·7871 cm]

Chapter HT

Heat and Temperature

Human body is a heat machine, requiring intake of fuel (foods and oxygen), for release of the *heat energy*. This heat energy keeps the body running and enables it to do work.

From experience, we learn that when heat is added to a substance, it increases the *kinetic energy* of the molecules of the substance. The molecules, therefore, are agitated and move with greater speeds. The **temperature** is an indication of the velocities reached by the agitated particles.

Methods of measuring temperature have been developed using various phenomena which are temperature dependent. These phenomena include expansion (of solids, liquids and gases), some electrical effects and changes in colour of substances.

Ordinary mercury thermometer is an example of simple expansion of mercury when heated. Depending upon the temperature (amount of expansion), the mercury rises correspondingly in the tube. Therefore, one can define any temperature change to be proportional to the change in length of the mercury column.

A thermometer can be calibrated by placing it in thermal contact with some natural system that remain at constant temperature (called a *fixed-point temperature*). The fixed-point temperature chosen is that of a mixture of water and ice at atmospheric pressure, which is defined to be zero degree Celsius (0°C). Another fixed point is the temperature of a mixture of water and water vapour (*steam*) at atmospheric pressure. Other thermometers are calibrated by using these fixed points as reference marks.

A temperature scale should be independent of the property of any substance. Such a scale is called an *absolute temperature scale* or *kelvin scale*.

The most useful thermometers for scientific and engineering applications are *platinum resistance thermometer*, *thermocouple* and *pyrometers*.

In this chapter we shall learn the principle, construction and working of various thermometers with special reference to their ranges and applications.

HT—1. HEAT

Heat is a form of energy which produces in us the sensation of warmth or hotness.

This statement does not give clear idea about the heat contents of a body. According to kinetic theory of matter; *heat of a body is the total kinetic energy of all its molecules.*

Therefore, to understand heat phenomenon, the knowledge of kinetic theory of matter is essential.

Kinetic theory of matter : Kinetic theory of matter is based upon the following points ;

(*i*) *Every substance* (solid, liquid or gas) *consists of large number of minute particles called* **molecules.** *Molecules have free existance and retain their chemical properties.*

(*ii*) *The molecules are in a continuous radom motion. They posses all possible velocities in all possible directions.*

(*iii*) *The molecules exert forces of attraction on one another. These forces depend upon intermolecular distances.*

On the basis of kinetic theory, if a molecule has a mass m and velocity v, its kinetic energy is $\frac{1}{2}mv^2$. Since different molecules are in different conditions in a substance, therefore, they move with different speeds and hence have different kinetic energies.

HT—2. CONCEPT OF HEAT

The word heat describes about the energy transfer from one place to another. Thus *heat flow is an energy transfer that takes place as a result of temperature difference only.*

Internal energy of a substance is the energy it possesses at a certain temperature. The higher the temperature of the substance, the greater is its internal energy (*heat*). When heat is added to a substance, its internal energy increases and as a result, its temperature rises. The temperature decreases when heat is removed.

Thus according to kinetic theory, we explain heat phenomenon as under :-

(*i*) Heat is a form of energy.

(*ii*) Heat is convertible into other forms of energy and vice-versa.

(*iii*) When certain quantity of heat disappears, an equivalent quantity of some other form of energy appears.

(*iv*) The internal energy of a substance is the sum of kinetic and potential energies of the molecules of the substance.

(*v*) The rise in temperature means increase in the kinetic energy of the molecules of a substance.

(*vi*) An increase in potential energy means either expansion of the substance or a change in the state.

(*vii*) When heat is supplied to a substance, there is an increase in its internal energy and vice-versa.

HT—3. CONCEPT OF TEMPERATURE

Temperature of a body may be defined as its degree of hotness.

Temperature is that condition of the body which determines the direction of flow of heat when two bodies are placed in contact with each other. The body from which heat flows is said to be at a higher temperature than the body to which the heat flows.

As an illustration place a beaker full of water over the flame of a burner. Touch water in the beaker with your finger after some time. Water in the beaker appears to be hot (at higher degree of hotness). The reason is that heat flows from the flame into water.

Thus temperature of a body is the state or condition which determines the direction of flow of heat when two bodies are placed in contact.

According to modern kinetic theory *temperature of a body represents the average kinetic energy of its molecules.*

$$\text{Thus} \qquad \text{Temperature} \propto \frac{\text{Total kinetic energy of all molecues}}{\text{Number of molecules}}$$

When a body is heated, its molecules move with greater speeds. This results in the increase in the kinetic energy of each molecule. Therefore, total kinetic energy of all molecules and the mean kinetic energy of each molecule is increased. Hence temperature of the body is raised.

HT—4. DIFFERENCE BETWEEN HEAT AND TEMPERATURE

Following are the points which differentiate between heat and temperature :-

(*i*) Heat possessed by a body is the total kinetic energy of all its molecules, whereas temperature is the average kinetic energy of its molecules.

For example, a cup and a large kettle both full of boiling water have the same temperature *i.e.* 100°C. But the total kinetic energy of the molecules in the two cases is not the same. Number of molecules of water in the kettle is much more than in the cup. Therefore, through temperature of both is the same yet kettle possesses much more heat than the cup.

(*ii*) Heat is the cause whereas temperature is the effect. When heat is given to a body its temperature rises.

(*iii*) Heat always flows from a body at a higher temperature to a body at a lower temperature till the temperatures of the two becomes the same.

(*iv*) The rise in temperature of two bodies may be different even when the same amount of heat is supplied to both.

HT—5. TEMPERATURE ON THE BASIS OF KINETIC THEORY

Temperature *of a body is defined as the average kinetic energy of its molecules.*

Mathematically,

$$\text{Temperature} \propto \frac{\text{Total kinetic energy of all molecues}}{\text{Number of molecules}}$$

When a body is heated, its molecules move with greater speeds. This results in the increase in the kinetic energy of each molecule. Therefore, total kinetic energy of all molecules and the mean kinetic energy of each molecule is increased. Hence temperature of the body is raised.

Relation : Consider a body having n molecules, having masses m each.

Let $v_1, v_2, v_3, \ldots\ldots v_n$ be the speeds of molecules 1, 2, 3,.....n respectively at *temperature T_1 (kelvin).*

$\therefore$ Heat in the body $\propto$ Total kinetic energy of all molecules or

$$H = k\left[\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \dots\dots\dots \frac{1}{2}mv_n^2\right]$$

Where k is a constant.

Hence temperature, $T_1 = k.\ \dfrac{\text{Total kinetic energy of all molecues}}{\text{Number of molecules}}$

i.e.

$$T_1 = k\cdot\frac{\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \dots\dots + \frac{1}{2}mv_n^2}{n}$$

If this body is further heated, its molecules will attain greater speeds, say v_1', v_2', v_3', ... v_n',

Where $v_1' > v_1$, $v_2' > v_2$ and so on. If the new temperature is T_2, then

$$T_2 = k\cdot\frac{\frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + \frac{1}{2}mv_3'^2 + \dots\dots + \frac{1}{2}mv_n'^2}{n}$$

Clearly T_2 is greater than T_1.

HT—6. MEASUREMENT OF TEMPERATURE :

The term temperature of an object indicates its degree of **hotness** or **coldness**, when we touch it. Therefore, our senses provide us with a qualitative indication of temperature. But sense of touch is neither reliable nor accurate and is often misleading. For example, when we touch a metal rod and a wooden rod on a day when temperature is 30°C the metal rod feels warmer to the hand even though both are at the same temperature. This is because metal is a better conductor of heat than wood. We need a reliable method for measuring the relative *hotness* or *coldness* of bodies.

Hence, sense of touch was replaced by a thermometer which is an instrument used for measuring the temperature of a body. The science of temperature measurement is called *thermometry*.

Thermometer : *A thermometer is a device used to measure the temperature of a system.*

A thermometer, when put in a substance is in thermal equilibrium with it and measures both the temperature of the substance and its own temperature.

HT—7. BASIC PRINCIPLES USED FOR THE CONSTRUCTION OF A THERMOMETER

In the construction of a thermometer we make use of the change in some physical properties with temperature. *These properties are called* **thermo–metric properties.**

Hence all thermometers are based on the thermometric properties (or temperature dependent properties) of different substances. Some of these thermometric properties chosen for the construction of thermometers are :

1. **Expansion of liquids, gases and solids.** Liquids, gases and solids expand uniformly with change in temperature. This property is used in the construction of may thermometers.

(*i*) *Expansion of liquids.* Liquid thermometers like Mercury thermometer, Alcohol thermometers are based on the principle that liquids expand with rise in temperature.

(*ii*) *Expansion of Gases.* Gas thermometer is based on this principle.

(*iii*) *Expansion of solids.* Bimetallic thermometer is based on the principle that lengths of solids change with temperature.

2. **Change in electrical resistance.** The resistance of a number of substances like platinum etc. changes uniformly with temperature. This property is utilised in the construction of many thermometers. *e.g., platinum–resistance thermometers.*

3. **Thermo–e.m.f. :** Many thermometers are based on the principle of thermo–electric effect (*i.e.* production of *e.m.f.* when the junction of two strips of different conducting substances are kept at different temperatures) *e.g., thermo–electric thermometers* (thermo–couple).

4. **Radiation :** All hot objects emit heat radiations. This fact is utilised for the measurement of temperature of hot bodies *e.g., pyrometers* are based on this principle.

HT—8. SCALES OF TEMPERATURE

For the quantitative measurement of temperature, we must select a *standard temperature interval* (scale) between two easily reproducible fixed temperatures just as we select the standard of length (metre) to be the distance between two fixed marks. The two fixed temperatures selected for this purpose are the meltingn point of ice and the boiling point of water. The interval between these fixed points is called *fundamental interval* and is divided into equal parts by utilising some thermometric property. Each part is called a *degree of temperature* and a scale thus obtained is called **scale of temperature.** The various temperature scales generally used are :

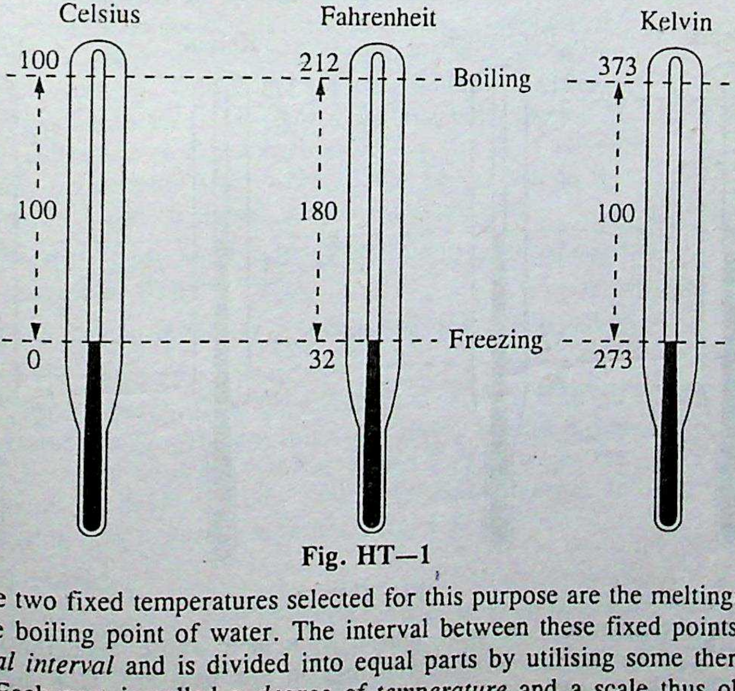


Fig. HT—1

1. Celsius Scale or Centigrade Scale. On the celsius scale, *freezing point of water* (*marked* 0°) is taken as the lower fixed point and *boiling point of water* (*marked* 100°) as the upper fixed point. The interval between the two points is divided into 100 equal parts. Each division is called 1°*C* (*one degree Celsius*).

This scale is also called centigrate (centigrate means 100 degrees) scale because there are 100 divisions.

2. Fahrenheit Scale : On this scale, freezing point of water (*marked* 32°) is taken as the *lower fixed point* and boiling point of water (*marked* 212°) as the *upper fixed point*. The interval between the two fixed points is divided into 180 equal parts. Each division is called 1° *F*. This temperature scale is generally used for meteorological and clinical purposes.

3. Kelvin Scale. On this scale, freezing point of water is marked 273° and boiling point of water as 373°. The interval between them is divided into 100 equal parts. Each division is called 1 *K* (*degree kelvin*). 1 *K* = 1°*C*.

4. Reumer Scale. On this scale, freezing point of water is taken as lower fixed point and is represented by 0°R and boiling point of water is taken as upper fixed point is represented by 80°R. The interval between them is divided into 80 equal parts. Each part is called 1°R.

Relation between the three scales of temperature. To convert the temperature, reading on one scale into another, let us have three exactly similar thermometers graduated in Celsius, Fahrenheit and Kelvin Reumerscales. When the thermometers are placed in ice, the head of mercury column in each thermometer stands at the same level A_1, A_2, A_3, A_4. Again when thermometers are placed in boiling water, the head of mercury column in each thermometer stands at the same level C_1, C_2, C_3, C_4

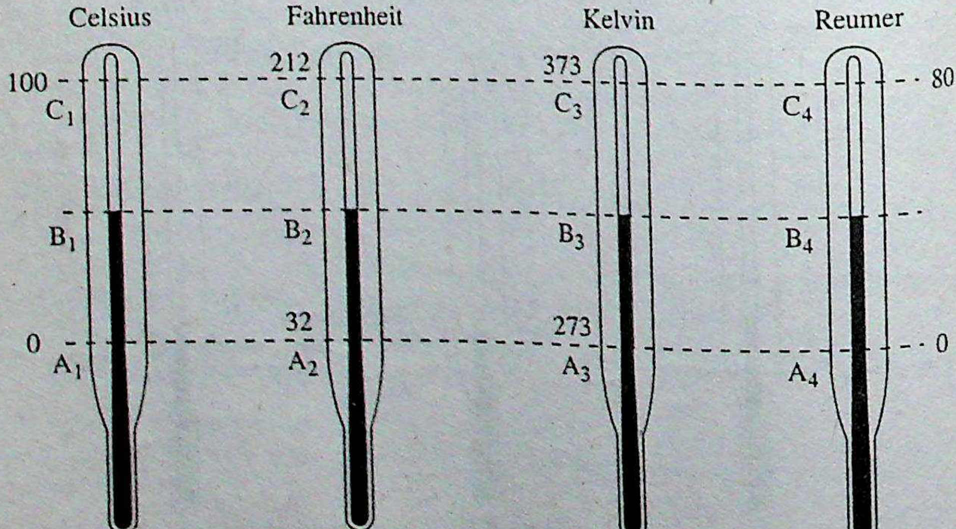


Fig. HT—2

Let the thermometers be now placed in a bath whose temperature is to be measured. Suppose the temperature readings in this bath are *C*, *F* and *K* on the Celsius, Fahrenheit and Kelvin scales respectively. Mercury head in each thermometer will stand at the same level B_1 B_2 B_3, B_4.

Since the thermometers are similar, therefore, geometrically

we have,
$$\frac{A_1 B_1}{A_1 C_1} = \frac{A_2 B_2}{A_2 C_2} = \frac{A_3 B_3}{A_3 C_3} = \frac{A_4 B_4}{A_4 C_4}$$

Thus,
$$\frac{\text{Temp. on any scale} - \text{Lower fixed point}}{\text{Upper fixed point} - \text{Lower fixed point}} = \text{Constant}$$

$$\therefore \quad \frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273}{373-273} = \frac{R-0}{80-0}$$

or
$$\frac{C}{100} = \frac{F-32}{180} = \frac{K-273}{100} = \frac{R}{80}$$

or
$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4}$$

This relation enables us to convert temperature from one scale into temperature on another scale.

HT—9 DEGREE KELVIN AS THE FUNDAMENTAL UNIT OF TEMPERATURE ON SI UNITS OR ABSOLUTE TEMP. SCALE :

According to the kinetic theory of matter, the temperature of a body depends upon the average kinetic energy of its molecules. As the temperature of the body is lowered, the average kinetic energy also diminishes due to the decrease in speeds of the molecules. If the temperature is continuously lowered, a stage would reach when the motion of the molecules ceases. This is the lowest possible temperature which can be attained. At $-273°C$ the molecular motion of every substance stops, therefore, $KE = 0$. This lowest possible temperature (*i.e.*, $-273°C$) has been taken as the zero on the Kelvin scale and is called absolute zero.

Thus the scale which has $-273°C$ as its zero is called **Kelvin scale** or absolute scale.

Each degree on this scale is called degree Kelvin and is represented by K.

Degree Kelvin has been accepted as the unit of temperature on *SI* system as this scale is very helpful in gas laws. It should be noted that Kelvin and Celsius scales use the same fundamental unit (*the centigrade degree*) in expressing temperature measurement *i.e.* $1\ K = 1\ °C$.

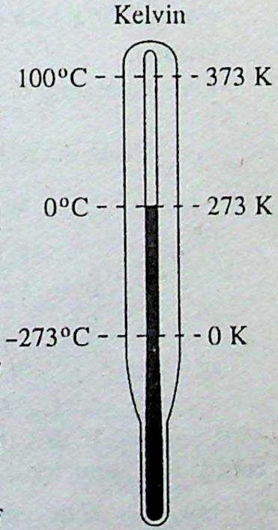


Fig. HT—3

Examples :

(*i*) The melting point of ice on the Kelvin scale is $273\ K = 0°C$.

(*ii*) The boiling point of water on the Kelvin scale is $373\ K = 100°\ C$.

Absolute temperature T corresponding to $t°C$ is given as $T = 273 + t$.

Ex. HT—1. The temperature of a healthy person is 98·4°F. Find the temperature on :

(i) Celsius scale (ii) Kelvin scale.

Sol. Here, Temperature on Fahrenheit scale, $F = 98{\cdot}4°\,F$

(i) Let the temperature of the person be C on the Celsius scale.

We know, $$\frac{C}{5} = \frac{F-32}{9} \quad \text{or} \quad \frac{C}{5} = \frac{98\cdot4-32}{9}$$

$$\therefore \quad C = \frac{5}{9}(98\cdot4-32) = 36\cdot89^{\circ}C \textbf{ (Ans.)}$$

(ii) Let the temperature of the person be K on the kelvin scale,

We know, $$\frac{F-32}{9} = \frac{K-273}{5}$$

or $$\frac{98\cdot4-32}{9} = \frac{K-273}{5} \quad \text{or} \quad \frac{66\cdot4}{9} = \frac{K-273}{5}$$

$$\therefore \quad K = \frac{5}{9}(66\cdot4)+273 = 309\cdot89\,K.$$

Hence required temperature on Kelvin scale = 309·89 K **(Ans.)**

Conver 90°F into degree Kelvin.

Here, $F = 90°\,F$

Let temperature in degree kelvin be K.

$\therefore$ Using the relation, $$\frac{F-32}{180} = \frac{K-273}{100}$$

or $$\frac{90-32}{180} = \frac{K-273}{100}$$

or $$\frac{58}{9} = \frac{K-273}{100} \quad \text{or} \quad 290 = 9K - 2457$$

or $$9K = 2747 \quad \text{or} \quad K = 305\cdot2 \textbf{ (Ans.)}$$

Ex. HT—2. The heights and lowest points of an arbitrary thermometer are 350° and 5°. If this thermometer reads 90°, what will be the temperature on a celsius thermometer ?

Sol. Let, C be temperature on Celsius thermometer.

Now, Lower fixed point on the thermometer = 5°

and, Upper fixed point on the thermometer = 350°

Temperature on this thermometer = 90°

$$\therefore \quad \frac{C-0}{100} = \frac{90-5}{350-5}$$

or $$C = \frac{85}{345}\times100 = 24\cdot6^{\circ}C \textbf{ Ans.}$$

Ex. HT-3. A new thermometric scale proposed to be adopted by a manufacturer of thermometers takes— 50° and 450° as its lower and upper fixed points respectively. The interval is divided equally. What reading will this thermometer give corresponding to average human body temperature of 98·4°F ?

Sol. Here, $F = 98{\cdot}4°\text{F}$

Let x be the reading on the new thermometer :

$\therefore$ $$\frac{x-(-50)}{450-(-50)} = \frac{F-32}{212-32}$$

or $$\frac{x+50}{450+50} = \frac{98 \cdot 4-32}{180} \quad \text{or} \quad \frac{x+50}{500} = \frac{66 \cdot 4}{180}$$

or $$x + 50 = \frac{66 \cdot 4}{180} \times 500 \quad \text{or} \quad x + 50 = 184{\cdot}44$$

$\therefore$ $$x = 184{\cdot}44 - 50 = 134{\cdot}44° \text{ Ans.}$$

Ex. HT-4. There is a certain temperature which has the same reading in both Celsius and Fahrenheit thermometers. Find this temperature.

Sol. Suppose the required temperature is x, so that when the reading is x°C, then the reading on the Fahrenheit thermometer is also x°F.

Using the relation, $$\frac{C}{100} = \frac{F-32}{180}$$

we get, $$\frac{x}{100} = \frac{x-32}{180}$$

or $$180x = 100\ (x - 32)$$

or $$180x = 100 \times 32$$

or $$x = -\frac{100 \times 32}{80} = -\ 40 \text{ Ans.}$$

Thus when the temperature of a body is – 40° on the centigrade thermometer then its temperature is also – 40° on the Fahrenheit thermometer.

Ex HT—5. At what temperature on the Fahrenheit scale will have the reading double of the reading on centigrade scale ?

Sol. We know that, $$\frac{C}{5} = \frac{F-32}{9}$$

Given, $C = t$, and $F = 2t$, so

$$\frac{t}{5} = \frac{2t-32}{9} \quad \text{or} \quad 9t = 10t - 160$$

or $$9t - 10t = -\ 160° \quad \text{or} \quad -t = -\ 160°$$

$\therefore$ $$t = 160° \text{ Ans.}$$

Hence at 160° the reading of Fahrenheit scale will be double.

Ex HT—6. Convert 100° F into centrigrade scale.

Sol. We know that $\frac{C}{5} = \frac{F-32}{9}$ Where, $F = 100°$, $C = ?$

$$\therefore \quad \frac{C}{5} = \frac{100-32}{9}$$

or $$C = 5\left(\frac{100-32}{9}\right) = 5 \times \frac{68}{9} = \frac{340}{9}$$

or $$C = \frac{340}{9} = 37 \cdot 7^{\circ}\text{C Ans.}$$

Ex HT—7. Convert 98·4°F into Kelvin.

Sol. We know that, $\frac{F-32}{180} = \frac{K-273}{100}$ Where $F = 98{\cdot}4°F$, $K = ?$

$$\therefore \quad \frac{98 \cdot 4 - 32}{180} = \frac{K-273}{100}$$

$$\Rightarrow \quad K - 273 = \left(\frac{98 \cdot 4 - 32}{180}\right) \times 100$$

or $$K = \left(\frac{98 \cdot 4 - 32}{180}\right) \times 100 + 273 = 309{\cdot}88 \text{ K Ans.}$$

Ex HT—8. Convert 50°C into Reumer scale.

Sol. We know that, $\frac{C}{100} = \frac{R}{80}$, Where, $C = 50°$, $R = ?$

$$\therefore \quad \frac{50}{100} = \frac{R}{80} \quad \text{or} \quad R = \frac{50}{100} \times 80 = 40^{\circ} \text{ R}$$

Hence 50°C on the Reumer scale is equal to 40° R. Ans.

Ex HT—9. Convert 300 K into Reumer Scale.

Sol. We know that $\frac{K}{100} = \frac{R}{80}$, Where $K = 300$, $R = ?$

$$\therefore \quad \frac{300}{100} = \frac{R}{80}$$

or $$R = \frac{300}{100} \times 80 = 240^{\circ} \text{ R Ans.}$$

HT—10. UNITS OF HEAT : Heat is measured in the following units

(*i*) **Calorie (cal).** The unit of heat on the *CGS* system is **calorie.** *It is defined as the amount of heat required to raise the temperature of 1 gram of water through 1°C (from 14·5°C to 15·5°C).*

(*ii*) **Kilo–Calorie** (kcal). Kilo–calorie is the unit of heat on the *MKS* system.

It is defined as the amount of heat required to raise the temperature of 1 kilogram of water through 1°C.

$\therefore$ 1 kilo calorie = 1000 calories.

(*iii*) **British Thermal Unit (B. Th. U.)** It is the unit of heat on the *F. P. S.* system. It is defined as the quantity of heat required to raise the temperature of 1 *lb* of water through 1°*F*.

1 *B*. Th. *U* = 252 calories,

(*iv*) **Unit of Heat on S.I. System.** Heat is a form of energy, therefore it is measured in energy units *i.e.* **joules or ergs.** The relation between joules and calories is given as,

$$1 \text{ calorie} = 4{\cdot}2 \text{ joules} = 4{\cdot}2 \times 10^7 \text{ ergs and}$$

$$1 \text{ kilo calorie} = 4200 \text{ joules.}$$

HT—11. MERCURY IN GLASS THERMOMETER :

Principle. A mercury in glass thermometer is based on the principle that mercury expands uniformly with rise in temperature.

Construction. It consists of a fine bore glass capillary tube with a bulb *B* at one end and a long stem *S*. The bulb and part of the tube are filled with mercury. A scale is engraved on the stem between the lower fixed point and the upper fixed point.

Working. The bulb of the thermometer is surrounded by the substance whose temperature is to be measured. Mercury expands by gaining heat from the substance and rises in the bore of the tube. When the mercury head is constant, the reading is recorded. This is the temperature of the substance.

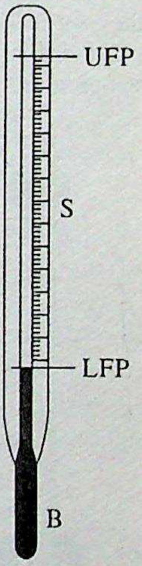


Fig. HT—4

Range. Mercury freezes at –39°*C* and boils at 359°*C*. Therefore, mercury thermometer can be used to measure temperatures between – 30°*C* and 350°*C*. However, by filling the space above mercury with nitrogen, the upper limit can be extended upto 500°*C*.

Advantages :

(*i*) Mercury is opaque and bright and therefore can be easily seen in the glass tube.

(*ii*) It does not wet the glass tube & rises or falls more easily.

(*iii*) Mercury is a good conductor of heat & attains the temperature of body quickly.

(*iv*) It has uniform coefficient of expansion over a wide range of temperatures.

(*v*) It can measure temperatures from –39°C to 367°C.

Disadvantages :

(*i*) Mercury is costly and hence its initial cost is high.

(*ii*) Mercury thermometer is made of glass and hence requires more safety measures.

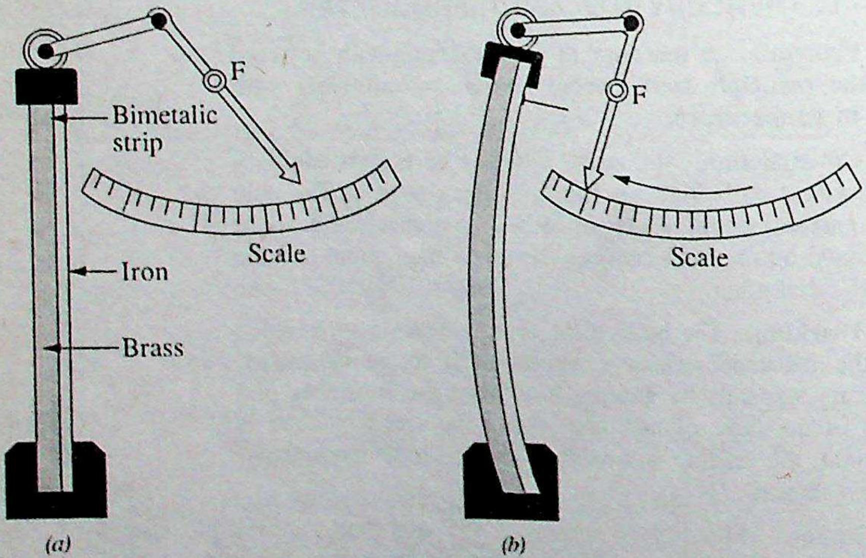
(*iii*) It cannot measure very high temperatures.

(*iv*) It cannot measure temperature of objects from distance.

Bimettalic Thermometer : A bimettalic thermometer is made of two strips of different metals and hence the name (bi means two).

Principle. A bimetallic thermometer is based on the principle that different metals expand by different amounts when heated through the same range of temperature.

Construction. Take bimetallic strips of brass and iron riveted together so that they form a compound bar and fixed at one end. Free end is attached to a lever. The other end of the lever forms a pointer which moves over a scale calibrated to read temperature.



(*a*) (*b*)

Fig. HT-5

At room temperature the compound bar is straight (see fig. HT-5a)

If the bar is heated, it is found to bend (curve) because of unequal expansions of the two strips. As expansion of brass is greater than that of iron, the brass strip lies on the outer side of the curve formed when **heated (see fig. HT-5b).**

In the bent form, the bimetallic strip pushes the lever and as a result the pointer moves over the scale indicating the temperature.

Modified form of bimetallic thermometer consists of a long strip of two metal strips of two metal strips of fairly large difference i their expansions. It is wound in the form of a spiral. The metal strip on the outer side is of larger expansion than that of the inner strip. The two strips are rivetted at the ends. The outer end of the spiral is fixed to a metal block, which is placed in the body whose temperature is to be measured. At the centre of the spiral a pointer is fixed which moves over a scale graduated to read temperature directly.

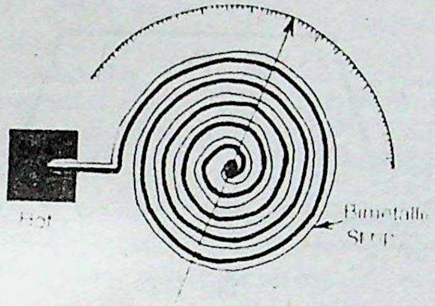


Fig. HT—6

Working. The block is placed in the substance whose temperature is to be measured.

Heat travels from the block through the spiral by conduction. Since the metal on the outer side expands more than that on the inner, the spiral winds inwards and thus moving the pointer on the scale which reads the temperature.

Range. The range of temperature measurement is generally from -40°C to 500°C and depends upon the combination of metal strips. It is used to measure high temperature of furnaces etc. It is not very accurate but very easy to use.

Alcohol thermometer. In order to measure quite low temperatures ($-80°C$) mercury is replaced by alcohol as the thermometric substance. Freezing point of alcohol is $-115°C$ and boiling point is $78°C$. Therefore, it can be used for measuring temp. from $-110°C$ to $60°C$. Alcohol thermometer is more sensitive than mercury thermometer.

HT—13. THERMO–ELECTRIC THERMOMETER (THERMO–COUPLE) :

Principle. It is based on the principle that an *e.m.f.* is produced, if an electric circuit of two different metal wires is made and their junctions are kept at different temperatures. This is known as *thermo-electric effect.*

Consider two unlike metal wires such as copper and iron (**fig. HT—7**). If they are joined together and one of the junctions is heated, *e.m.f.* is produced and current flows in the a circuit. The *e.m.f.* produced is called thermo *e.m.f.* and the arrangement is called *thermo-couple*. The magnitude of thermo *e.m.f.* depends upon the difference of temperature between the *cold junction* and *hot junction*.

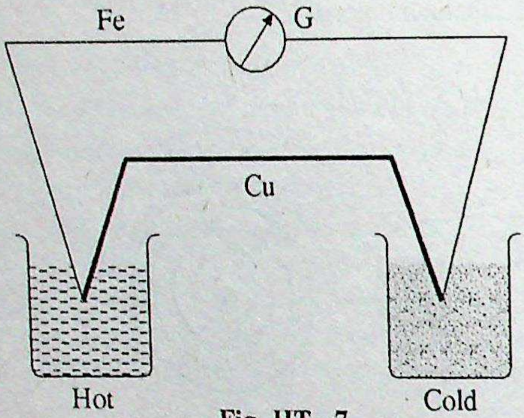


Fig. HT—7

Thus, if one junction is kept at constant temperature, *e.m.f.* produced will be an indication of the higher temperature of the hot junction.

Factors affecting thermo e.m.f. in a themocouple : Thermo *e.m.f.* developed across the two junctions of a thermo couple depends on

(*i*) Nature of materials forming a thermocouple.

(*ii*) Temperature difference between the two junctions.

Thermoelectric series : The list of elements which can be used to make a thermocouple is called thermoelectric series. Thermo electric series is,

Antimony, Iron, Zinc, Silver, Gold, Chromium, mercury, copper, platinium, Cobalt, Nickel, Bismuth. The magnitude of thermo *e.m.f.* developed across the junction of a themocouple is greater if the metals forming a themocouple are distant in the list of thermo electric series. .*e.g* themo *e.m.f.* developed in Antimony-Bismuth themocouple will be maximum.

Variation of themo e.m.f. with temperature : When temperature of both the junctions of a themocouple is same, there is no themo *e.m.f.* produced and the galvanometer (G) shows no deflection. As the temperature of one junction is increased slowly, the thermo *e.m.f* also starts increasing and is maximum at a certain temperature called neutral temperature of themocouple. It is denoted by T_n.

Neutral Temperature. *It is that temperature of the hot junction at which e.m.f. produced is maximum.*

If the temperature of the hot junction is slowly increased. Thermo *e.m.f.* continuously increases. For *small* difference of temperature, the *thermo–e.m.f.* is proportional to temperature. For large difference of temperature, the *e.m.f.* first increases and for a certain temperature of the hot junction it becomes maximum. This temperature is known as **neutral temperature.** It is constant for a given couple and depends upon the pair of metals taken. For *Cu–Fe* thermo-couple, it is 270°C.

As the temperature of hot junction is further increased beyond the neutral temperature the themo *e.m.f.* starts decreasing and ultimately at a certain temperature becomes zero. This temperature is called temperature of inversion. It is denoted by T_i.

Inversion Temperature. If the temperature of the hot junction is further increased, the thermo-*e.m.f.* decreases and finally becomes zero at a certain temperature. This temperature is known as **Inversion Temperature.** It is as much above the neutral temperature as the latter is above the temperature of cold junction. If temperature of cold junction of a *Cu–Fe* thermocouple is 0°*C*, the inversion temperature is 540°*C*. put if the temperature of cold junction is 50°*C*, the inversion temperature is 490°*C*.

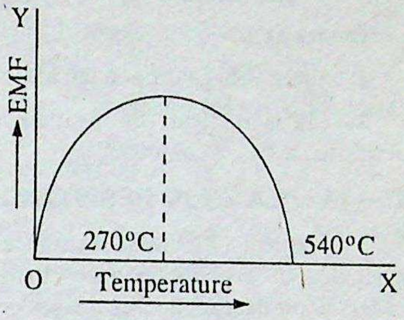


Fig. HT—8

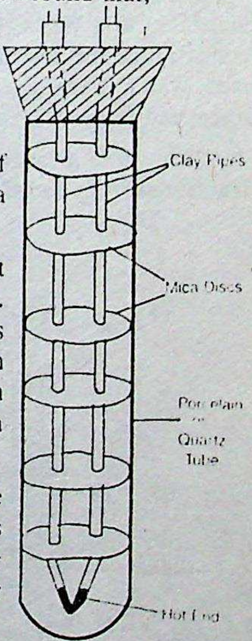
Relation between neutral temperature (T_n) and temperature of inversion (T_i) : If T_o is the temperature of cold junction, T_i is the neutral temperature and T_i is the temperature of inversion for a themo couple then it has been found that,

$$T_n - T_o = T_i - T_n$$

or

$$T_n = \frac{T_o + T_i}{2}$$

i.e. neutral temperature is the mean of the temperature of cold junction (T_o) and temperature of inversion (T_i) of a themocouple.



Construction. It consists of wires *A* and *B* of two different metals joined together to form a junction *H* (**Fig. HT—9**). Generally, wire *A* is of nickel and *B* of Nichrome. This *thermocouple* is placed inside an iron tube ***T*** **insulated** from inside. A high resistance millivoltmeter *mV* is connected between *A* and *B* by very long leads. The millivoltmeter is calibrated with a suitable temperature scale.

Working. The end of tube *T* is placed in contact with the hot body whose temperature is to be determined. An *e.m.f.* is produced in the circuit due to thermo-electric effect. The temperature of the body is directly read from the scale of the millivometer.

Range : Thermo–electric thermometer can be used to measure temperatures in the range of about–250°*C* to 500°*C*. The upper limit can be extended to 2200°*C* by using tungsten and rhenium wires. Thermo–electric thermometers are very useful for measuring varying temperatures and are being widely used to measure temperatures of blast and glass furnaces.

Merits :–

(*i*) The hot junction is generally small and can measure the temperature almost at a point.

(*ii*) The hot junction is robust and can be easily replaced if damaged.

(*iii*) It is cheap and more handy.

(*iv*) It can measure rapidly changing temperatures.

Demerits : –

1. Some voltage drop in the leads cause small error in the temperature measurement.

2. It is difficult to maintain cold junction at a constant (*ice*) temperature, therefore, it causes an error.

imp.

HT—14. PLATINUM RESISTANCE THERMOMETER :

Principle : Platinum resistance thermometer is based on the principle that the resistance of a platinum wire changes with change in temperature. The resistance increases on heating and decreases on cooling the platinum wire.

If the resistance of the wire is R_0 at 0°C, then its resistance R_t at t°C is given by the relation,

$$R_t = R_0 (1 + \alpha_0 t)$$

Where α_0 is constant and is called temperature co-efficient of resistance.

Construction. It consists of a fine platinum wire wound on a mica strip and is connected to a Wheatstone bridge arrangement by means of the leads P_1 P_2. In order to neutralize the resistance introduced by leads P_1 P_2, *compensating* leads C_1 C_2 are used which have the same resistance as of leads P_1 P_2. The leads C_1 C_2 are connected to the Wheatstone bridge opposite to P_1 P_2 at C_1 C_2.

The whole arrangement is placed inside a tube from which air has been removed. The leads P_1 P_2 and C_1 C_2 are kept apart by mica discs. (see fig. HT—10*a*)

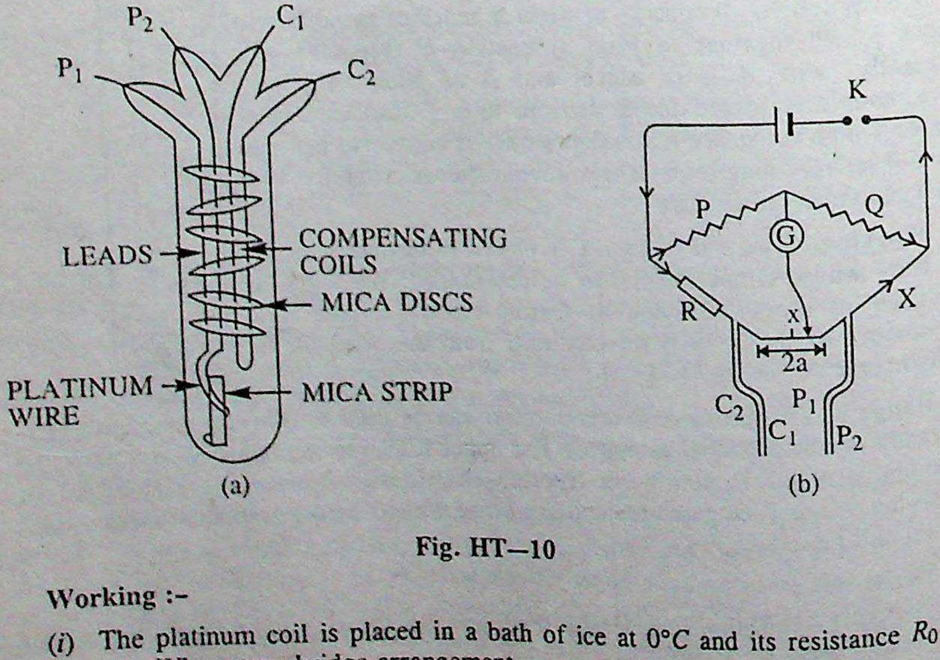


Fig. HT—10

Working :-

(*i*) The platinum coil is placed in a bath of ice at 0°C and its resistance R_0 is measured by Wheatstone bridge arrangement.

(*ii*) The platinum coil is then placed in a steam bath at 100°C and its resistance R_{100} at 100°C is measured.

(*iii*) Now, the platinum wire is placed in a bath whose temperature t°C is to be measured. The resistance of platinum wire is measured at t°C as before. Let it be R_t.

Theory. The resistance of platinum wire is measured with the help of a Wheatstone Bridge (see fig. 1.10 (*b*)). The leads P_1 P_2 are connected in the X arm and compensating leads C_1 C_2 are connected to the R arm. By connecting P_1 P_2 and C_1 C_2 in the opposite arms, the resistance of lead wire is balanced out by that of the compensating leads. The resistance of platinum wire changes by 0·01 ohm per degree centigrade. The steps of such low resistance are not possible in ordinary resistance boxes, therefore, a wire of uniform cross-section of resistance ρ ohm per centimeter and of length $2a$ is connected between R – X arms as shown in Fig. HT-10b

At 0°C the value of R is so adjusted that the balance point is obtained at the mid point of wire at point D. The resistance in the arms P and Q are kept equal, thus

Resistance in X–arm = Resistance in R–arm

$$\therefore\ X + \text{resistance of leads} + R_0 + a\rho = R + \text{Resistance of compensating leads} + a\,.\,\rho.$$

or

$$R_0 = R - X$$

R_{100} and R_t can be determined in the same way when the platinum wire is placed in a bath of unknown temperature t, its resistance increases from R_0 to R_t. Therefore, the balance point shifts towards the side of platinum wire. Thus the length of wire is more in R–arm than in the X–arm. If the balance is shifted to a point distant x from the mid point D, then,

$$R + (a + x)\,\rho = R_t + (a - x)\,\rho + X$$

$$\therefore \qquad R_t = R + 2x\rho - X$$

Thus knowing x and ρ, R_t can be calculated.

Calculations :

If α_0 is the temperature co-efficient of resistance at 0°C, then

$$R_{100} = R_0\,(1 + \alpha_0 \times 100) \quad \ldots(i)$$

and

$$R_t = R_0\,(1 + \alpha_0 \times t) \quad \ldots(ii)$$

From equation (*i*),

$$R_{100} - R_0 = R_0 \times \alpha_0 \times 100 \quad \ldots(iii)$$

From equation (*ii*),

$$R_t - R_0 = R_0\,\alpha_0\,t \quad \ldots(iv)$$

Dividing (*iv*) by (*iii*),

$$\frac{R_0\,\alpha_0\,t}{R_0 \times \alpha_0 \times 100} = \frac{R_t - R_0}{R_{100} - R_0}$$

or

$$\frac{t}{100} = \frac{R_t - R_0}{R_{100} - R_0} \quad \text{or} \quad t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

Since R_t, R_0, R_{100} are known, the required temperature t can be calculated.

Range. The thermometer can be used to measure temperature in the range of −200° C to 1300°C.

Advantages :– (*i*) It has a wide range.

(*ii*) It is very accurate and reliable to 0·01°*C* upto 500°*C* and to 0·1°*C* upto 1300°*C*.

(*iii*) It is free from changes of zero errors.

Disadvantages. *(i)* it is not suitable for measuring rapidly changing temperatures.

(ii) It is difficult to balance the bridge for measuring the resistance.

(iii) Its size is large, therefore, not suitable for measuring temperature at a point.

Ex. HT—10. A platinum resistance thermometer has been calibrated by placing the resistance coil first in a bath of melting ice at 0°C and then in steam bath at 100°C. Coil resistance measured by Wheatstone bridge came out to be 6·284 ohms and 7·321 ohms respectively. When placed in a liquid of unknown temperature it is found to be 6·726 ohms. Find the temperature of the liquid in ;

(*i*) Celsius scale, (*ii*) Kelvin scale.

Sol. Here, Coil resistance at 0°*C*, $R_0 = 6{\cdot}284$ ohms

Coil resistance at 100°*C* $= 7{\cdot}321$ ohms

Coil resistance at unknown temperature *t*, $R_t = 6{\cdot}726$ ohms

(*i*) The unknown temperature *t* of the coil is given by,

$$\frac{t}{100} = \frac{R_t - R_0}{R_{100} - R_0}$$

or

$$\frac{t}{100} = \frac{6\cdot 726 - 6\cdot 284}{7\cdot 321 - 6\cdot 284} = \frac{0\cdot 442}{1\cdot 037}$$

$$\therefore \qquad t = \frac{\cdot 442}{1\cdot 037} \times 100 = \mathbf{42{\cdot}6°C\ (Ans.)}$$

∴ Temperature on Celsius scale is 42·6°*C*

(*ii*) Temperature on kelvin scale, $K = 42{\cdot}6 + 273 = 315{\cdot}6\ K$

Hence temperature on the kelvin scale is 315·6 *K*.

Ex. HT—11. A platinum wire has a resistance of 2·000 ohms at a temperature of melting ice 273·2°C, 2·778 ohms at a temperature of water boiling under normal pressure, and 5·280 ohms at a normal boiling point of sulphur. Calculate the boiling point of sulphur from the given data.

Sol. Let *t* be the boiling point of sulphur.

Here, $R_0 = 2{\cdot}000\ \Omega$, $R_{100} = 2{\cdot}778\ \Omega$ and $R_t = 5{\cdot}280\ \Omega$

$$\therefore \qquad t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{5\cdot 28 - 2\cdot 00}{2\cdot 778 - 2\cdot 000} \times 100$$

$$\text{Hence,} \qquad t = \frac{3\cdot 28}{0\cdot 778} \times 100 = \mathbf{421{\cdot}6°C\ Ans.}$$

Ex HT-12. A silver wire has a resistance of 2·10 ohm at 27·5°C and a resistance of 2·7 ohm at 100°C. Determine the temperature coefficient of resistance of silver.

Sol. Given,

$$R_1 = 2\cdot1 \text{ ohm}, \quad t_1 = 27\cdot5°C$$

$$R_2 = 2\cdot7 \text{ ohm}, \quad t_2 = 100°C.$$

The temperature coefficient of resistance is given by,

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)} = \frac{2\cdot7 - 2\cdot1}{2\cdot1 \times (100 - 27\cdot5)} \, C^{o-1}$$

$$= \frac{0\cdot6}{2\cdot1 \times 72\cdot5} \, °C^{-1}$$

$$\therefore \quad \alpha = 0\cdot039°C^{-1} \text{ Ans.}$$

Ex. HT—13. At what temperature would the resistance of a copper conductor be double its resistance at 0°C ?

Sol. We know that, $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)}$

Given $R_2 = 2R_1, \quad t_1 = 0°C, \quad t_2 = ?$ $\qquad \alpha = \dfrac{1}{273} \, °C^{-1}.$

Hence,

$$\frac{1}{273} = \frac{2R_1 - R_1}{R_1(t_2 - 0)} = \frac{R_1}{R_1(t_2)}$$

or

$$\frac{1}{273} = \frac{1}{t_2}, \quad \Rightarrow \quad t_2 = 273°C.$$

At 273°C the resistance of copper conductor will be double its value at 0°C.

HT—15. PYROMETER

A pyrometer *is a device (thermometer) used to measure very high temperature by observing the radiations from the hot body lying at some distance.*

Principle. Pyrometers are based on the principle of law of radiations. All objects emit radiations. The thermal radiation from a black body depends upon the temperature of the black body. If the temperature of the object is raised, the total amount of radiations emitted is also increased and the colour of the object changes due to change in wave length.

Radiation pyrometers can be used only above red-heat *i.e.*, above 600°*C* and are of two types :-

(*i*) **Total radiation Pyrometers.** These pyrometers respond to the total radiations (heat and light) emitted from the hot body.

(*ii*) **Optical Pyrometers.** They respond to visible light only. In these instruments, the intensity of radiations of certain wavelengths emitted by the body is compared with the intensity of radiations of some wavelength emitted by a standard lamp to obtain the condition of equal brightness.

HT—16. FERRY'S TOTAL RADIATION PYROMETER

This pyrometer measures the total radiations emitted by the body. The temperature is calculated by applying Stefan's law.

Construction. It consists of a highly polished concave mirror *M* having a small opening in the centre, into which an eye piece *E* is fitted.

In front of *M* a small diaphragm *D* is mounted and behind it a metal strip *S* is fixed. The front surface of strip *S* towards *D* is blackened whereas to the back surface of *S* hot junction of a thermocouple *T* is attached. The other junction of *T* and back surface of *S* are protected from direct radiations by suitable screens. The leads from the thermocouple are connected to a millivoltmeter *mV* which measures the *e.m.f.* developed. The mirror *M* can be moved to and form along the axis of the tube for focussing the radiations on *S*.

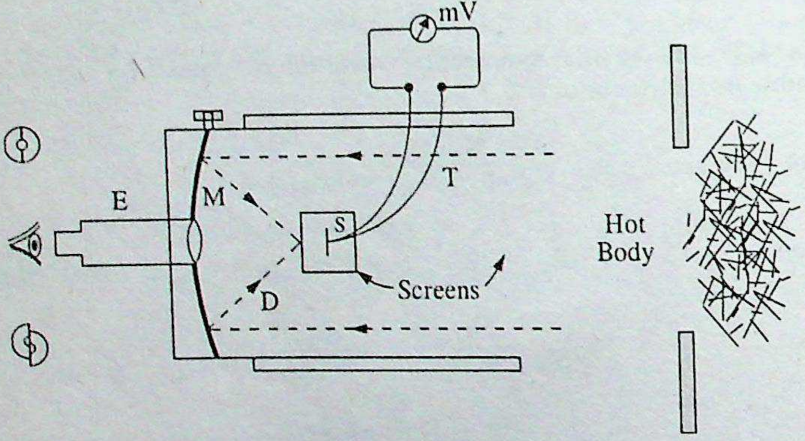


Fig. HT—11

The diaphragm *D* is made up of two semi-circular mirrors inclined to each other at an angle of about 5° with a central hole of about 1·5 *mm* for radiations to pass.

Working. The tube is pointed towards the hot body or the furnace whose temperature is to be measured. The image of the hot body, after reflection from *C* is focused on the blackened face of *S*. The diaphragm *D* is viewed through the eye piece. If the focusing is perfect, the images of the two halves of the diaphragm coincide to form a complete circle (see fig. HT—11). If the focusing is not accurate, the two halves will not coincide but appear displaced. The concave mirror *M* is moved till the relative displacement of the two halves disappears. The temperature of the furnace is read from the millivoltmeter which is calibrated to read directly the temperature of the furnace.

Range. Pyrometers can not be used to measure a temperature lower than 900 *K*. Therefore, pyrometers are suitable for measuring high temperatures *i.e.*, of furnaces.

HT—17. DISAPPEARING FILAMENT OPTICAL PYROMETER

In optical pyrometer, the intensity of heat radiation of a certain wavelength emitted by a hot body is compared with the intensity of emission from a standard lamp, within the same range of wavelengths.

Construction. It consists of a telescope with objective lens *O* at one end and an eye piece at the other end. An electric lamp *L* having a straight filament is placed in the position of cross-wires of the telescope. Diaphragms *DD* are placed on either side of the lamp. The filament of the lamp can be heated to different high temperatures by adjusting the current in the battery circuit with the help of a rheostat *Rh*. The current is measured by a milliammeter connected in the circuit (see fig. HT—12).

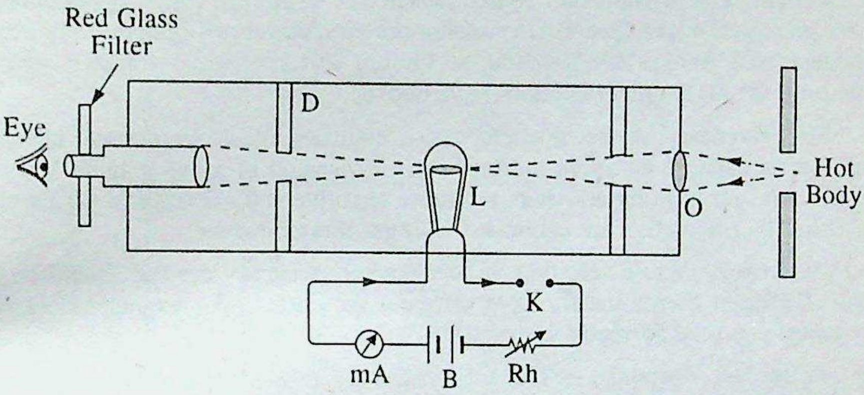


Fig. HT—12.

Working. By adjusting the objective lens *O*, of the telescope, the radiations from the hot body are focussed on the filament of the lamp *L*. The current is passed through the filament which is viewed through the eye piece *E*. The radiations from both, the filament and the hot body pass through a filter of red glass *G* before reaching the eye. If the image of the body is brighter than the filament, the filament appears dark on a bright ground. The temperature of the filament is adjusted by adjusting the current passing through it, until it merges as nearly as possible into the background. The filament in this case is as bright as the image of the hot body and both are at the same temperature. The milliammeter is calibrated directly in degree centigrade. The reading of the milliammeter thus indicates the temperature of the furnace.

Range. Pyrometers are suitable for measuring very high temperatures *i.e.*, temperatures of furnaces. Temperatures upto 3200°*C* can be measured with this thermometer. The range can be extended by using a rotating sectored disc, which cuts out a known fraction of the incident radiations.

Advantages of Pyrometers :

1. Pyrometers can be used to measure the temperature of a body from a distance.

2. They can measure very high temperatures *i.e.*, even more than the temperatures above the melting point of the material of the thermometer.

Disadvantages :

1. Since pyrometers are based on the principle of law of radiations and the laws of heat radiations are true for perfectly black bodies. In actual practice we do not have a perfectly black surface, therefore, the temperatures measured by pyrometers are lower that the actual temperatures.

2. Pyrometers can measure temperatures of bodies which can glow brilliantly and emit radiations. So they are not suitable for low temperatures.

HT—18. CRITERIA FOR THE SELECTION OF A THERMOMETER

While selecting a thermometer for a particular use, the following points should be taken into consideration :

1. **Range.** The thermometer should be selected to suit the range of temperatures to be measured. For example for measuring the temperature of human body clinical thermometer (95°*F* to 110°*F*) should be chosen and not the pyrometer which is suitable only for high temperatures. (above 600°*C*).

2. **Sensitiveness.** A thermometer which indicates even a very small change in temperature is called a sensitive thermometer. Expansion of gases is quite large and uniform, therefore gas thermometers are more sensitive and are suitable for precision work. Other thermometers are calibrated with gas thermometers.

3. **Accuracy.** While selecting a thermometer, accuracy desired should be kept in view. Different thermometers have different accuracies. For example Beckman's thermometer can read correctly upto 0·01°*C*.

4. **Time of Response.** *The time taken by a thermometer to indicate the temperature is called the time of response of the thermometer.*

For a good thermometer time of response should be minimum.

A thermometer takes certain time to attain the temperature of the substance when put in it. Some thermometers have greater time of response and some have smaller time of response and indicate the temperature quickly, therefore, are suitable for measuring rapidly changing temperatures.

5. **Minimum absorption of Heat.** When a thermometer is put in a hot substance, whose temperature is to be measured, heat flows from substance to the thermometer till the temperatures of the two become the same. This is a stage of thermal equilibrium and measurement takes place only at this stage. Therefore, the thermometer absorbs some heat from the substance and its temperature falls. Thus the temperature measured is less than the true temperature. For accurate temperature measurement, thermometer should absorb very small quantity of heat from the substance *i.e.*, it should have low heat capacity.

6. **Accessibility.** In some cases, the body whose temperature is to be measured is not accessible. For example, pyrometers are used to measure the temperature of a furnace from a certain distance.

7. **Cost.** Very accurate and sensitive thermometers are very costly and ordinary thermometers are cheap. Ordinary cheap thermometers are quite suitable for rough measurements but for precise and accurate measurements, costly thermometers may be preferred.

8. **Ruggedness.** Ruggedness means toughness or ability to bear heavy shocks. If the place of use of the thermometer is subjected to disturbances and heavy shocks due to some movements, the thermometer used in such cases should be robust and tough to bear heavy shocks.

HT—19. VARIOUS THERMOMETERS WITH PRINCIPLES AND THEIR RANGES :

Various thermometers with their principles and ranges are given in the table below :-

Sr.No.	Type of thermometer	Thermometric substance	Principle	Range	
				Lower	Higher
1.	Mercury thermometer	Mercury	Expansion of mercury	– 30°C	350°C
2.	Alcohol thermometer	Alcohol	Expansion of Alcohol	– 100°C	60°C
3.	Constant volume gas thermometer	Hydrogen or Nitrogen	Variation of pressure of gas with temperature at constant volume	– 200°C	1600°C
4.	Bimetallic thermometer	Strips of different metals of metals	Differential expansion	– 50°C	500°C
5.	Platinum resistance thermometer	Platinum wire	Variation of Resistance with temperature	-200°C	1300°C
6.	Thermo-electric thermometer (thermocouple)	Nichrome and Nickel wires	production of thermo *e.m.f.*	-200°C	1600°C
7.	Total radiation pyrometer	……	Measurement of total radiations	…..	upto 2000°C
8.	Optical pyrometer	……	Matching brightness	…..	upto 3200°C

QUESTIONS FOR SELF TEST

1. Differentiate between heat and temperature on the basis of kinetic theory.
2. Heat is a form of energy. Justify.
3. What are the different scales of temperature measurement ? Derive the relation between them.
4. What are the basic principles of thermometers. Discuss thermocouple thermometer.
5. Explain the principle, construction and working of a bimetallic thermometer.
6. Discuss a thermometer you will choose to determine the temperature of a furnace.
7. Discuss the criteria for the selection of a thermometer for a particular use.
8. What are advantages and disadvantages of mercury thermometer.
9. What is alcohol themo meter ?
10. What is themoelectric effect ?
11. Define neutral temperature and temperature of inversion for a themocouple.
12. Derive a relation between neutral temperature and temperature of inversion.
13. What is themoelectric series ?
14. What is pyrometer ? How it measure temperatures.
15. What is platinium resistance themometer ? Explain.
16. What is relation between the centigrate, Fahrenheit and Reumer scale of temperatures ? Explain.

PROBLEMS

1. Find the value of absolute zero on the Fahrenheit scale. (–459·4° F)
2. M.P. of gold is 1063°C. Find its value in degree F and degree kelvin. (1945·4°F, 1336 K)
3. Centigrade and Fahrenheit thermometers placed into two different hot liquids read 65°C and 118°F. What will they read when interchanged ? (149°F, 45°C)
4. A platinum resistance thermometer has resistance of 5·5 Ω at 0°C and 7·624 Ω at 100°C respectively. The resistance is found to be 14·45 Ω when put in a hot body. Calculate the temperature of the body. (421·37°C)
5. At what temperature the reading given by Fahrenheit thermometer is double as given by centigrade thermometer ? (160°C).
6. What is the mathematical relation between °C, °F, K and °R ?
7. Oxygen boils at –183°C, Find this temperature approximately in Fahrenheit scale. **(Ans. – 297°F)**
8. Change of temperature of a body is 100°C. What will be its changes in kelvin scale. **(Ans. 100 K)**
9. On Fahrenheit scale what is the value of 73 K. (F = – 328°F)
10. If temperature of a person is 40°C. Find its value in kelvin. **(Ans. 313 K)**

Chapter EOS

Expansion of Solids

The heat phenomenon that enters our lives in many interesting ways is *thermal expansion*. As the temperature of most of the substances rises, their molecules are speeded up. In their increased agitation (speeds), they demand a little more space. This results in expansion. The resulting expansion is found to occur in varying amounts in solids, liquids and gases.

The clicking of rail road wheels on the rails results from the little *gaps* left between the rail joints to avoid buckling under hot summer temperatures. Structures such as bridges, buildings and concrete pavements which are exposed to wide temperature variation have expansion joints separated by suitable distances.

To remove tight glass stopper from a bottle, is done by heating the mouth of the bottle from outside. The mouth expands more than the glass stopper and loosens. The problem for the successful production of light bulbs and radio tubes was solved by developing metals and glasses which when sealed together would expand by the same amount so as to tolerate varying temperatures without breaking.

In working with expansion we are usually interested in either *linear expansion* or *volume expansion*. In this chapter we shall study various applications of expansion of solids in engineering.

EOS—1. CONCEPT OF EXPANSION OF SOLIDS

Most of the bodies expand as their temperature rises. This phenomenon plays an important role in a number of engineering applications. For example, thermal expansion joints must be included in buildings, concrete floor bridges to compensate for changes in dimensions with temperature variations.

A crystalline solid consists of a regular array of atoms held together by elastic forces. A mechanical model of these forces can be obtained by imagining that the atoms are connected by a set of stiff springs (see fig. EOS—1). Inter atomic forces are taken to be elastic in nature. At ordinary temperature, the atoms vibrate about their mean positions with small amplitudes. The average spacing between the atoms is of the order of 10^{-10} *m*.

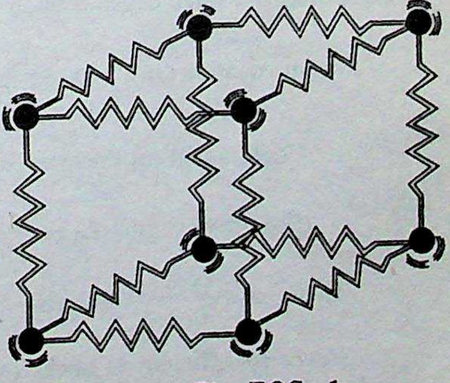


Fig. EOS—1

As the temperature of the solid increases, the atoms vibrate with larger amplitudes and the average space between them increases. This results in the expansion of the solids with the rise in temperature. If the expansion of the body is very small as compared to its initial dimensions, then the change in any dimension (length, width or thickness) is, approximately a linear function of the temperature.

EOS—2. LINEAR EXPANSION

The increase in any linear dimension of a solid such as the length, width or height on heating is called **linear expansion.**

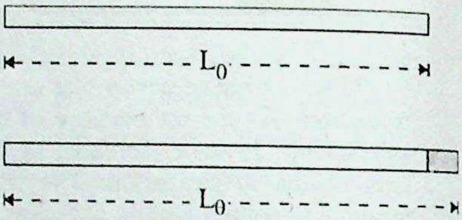


Fig. EOS—2

When a solid rod is heated, its length increases. The increase in length depends upon :-

(*i*) Original length.

(*ii*) Rise in temperature of the rod.

(*iii*) Nature of the rod material.

Let, L_0 = Length of rod at 0°C

L_θ = length of rod at θ°C

∴ Increase in length $= L_\theta - L_0$

It has been found experimentally that :

$$L_\theta - L_0 \propto L_0 \qquad \textit{(original length)}$$

$$L_\theta - L_0 \propto \theta \qquad \textit{(rise in temperature)}$$

Combining the two,

$$L_\theta - L_0 \propto L_0\,\theta$$

or
$$L_\theta - L_0 = \alpha\, L_0\, \theta \qquad \ldots(i)$$

Where α is the constant of proportionality and is called *co-efficient of linear expansion*. Its value depends upon the nature of the material of the rod.

From equation (*i*), $$\alpha = \frac{L_\theta - L_0}{L_0\,\theta}$$

∴ Co-efficient of linear expansion $= \dfrac{\text{Change in length}}{\text{Original length} \times \text{Rise of temp.}}$

If original length, $L_0 = 1$ and rise in temp., $\theta = 1°C$.

Then, $\alpha = \dfrac{L_1 - 1}{1 \times 1}$ = Increase in length per unit length per °C rise in temperature.

Hence **co-efficient of linear expansion** *of a solid may be defined as the increase in length per unit length for 1°C rise in temperature.*

From equation (*i*), $L_\theta - L_0 = \alpha\, L_0\, \theta$

or $$L_\theta = L_0 + \alpha L_0 \theta = L_0(1 + \alpha\theta)$$

$$\therefore \qquad L_0 = L_0\,(1 + \alpha\theta)$$

The value of α is very small, therefore, initial temperature of the rod may not be $0°C$.

Thus if L_1 is the length of the rod at $\theta_1 °C$ and L_2 is its length at $\theta_2\ °C$, then

$$L_2 = L_1\,[1 + \alpha\,(\theta_2 - \theta_1)]\ i.e.,$$

Expanded length = Original length [1 + Co-effi. of linear expansion) × [Rise in temp.)]

Units of α : We know, $\alpha = \dfrac{\text{Change in length}}{\text{Original length} \times \text{Rise of temp.}} = \dfrac{\text{Metres}}{\text{Metres} \times °C} = /°C$

Therefore, unit of α is $/°C$

EOS—3. EXPERIMENTAL DETERMINATION OF CO-EFFICIENT OF LINEAR EXPANSION : Imp.

A simple method for the determination of co-efficient of linear expansion in the laboratory is given below :-

Apparatus : It consists of a rod *AB* about 50 cm long and made of the material whose co-efficient of linear expansion is to be determined. It is enclosed in a wide metal jacket supported on two supports fixed on the base of the apparatus. The ends of the jacket are closed by means of rubber corks through which rod *AB* is passed. The end *A* of the rod touches an adjustable screw.

The jacket is provided with two small side tubes *I* and *O* near its ends which act as the inlet and outlet for steam respectively. The jacket is also provided with a small central tube for fixing the thermometer *T*.

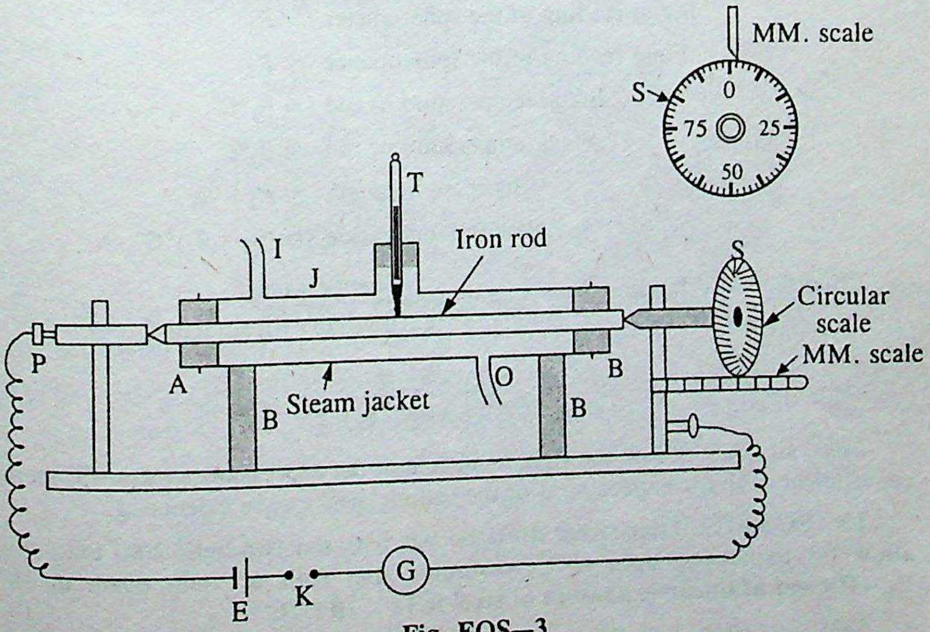


Fig. EOS—3

The flat end B faces the pointed end of the screw of spherometer S which is fixed on a metal pillar in such a way that the screw and the rod are in the same straight line. In order to know as to when, the screw of the spherometer just touches the end B of the rod, a galvanometer G and a cell E are connected through a key K between terminals P and Q.

Working :

(*i*) The length of rod AB is accurately measured with a metre scale at the room temperature. The rod is then placed in position in the jacket and the temperature of the thermometer is noted.

(*ii*) The key K is inserted and the spherometer screw is gradually moved forward till it just touches the rod. This is indicated by a sudden deflection of the galvanometer. The reading of the spherometer R_1 is noted. The screw is then moved back to allow space for the expansion of the rod.

(*iii*) The inlet I is connected to the steam generator and steam is allowed to enter the jacket. The temperature of rod AB indicated by the thermometer rises gradually and finally becomes steady.

(*iv*) When the temperature remains constant for about 5 minutes, the spherometer screw is again moved forward till the galvanometer shows deflection. The reading of spherometer R_2 is noted as before. The constant temperature of thermometer is also recorded.

(*v*) The difference in the two readings of the spherometer gives the increase in length of rod AB. Also the difference between the two temperature readings gives the rise in temperature.

Calculations : Let, Original length of the rod $= L$

Initial reading of the spherometer $= R_1$

Final reading of the spherometer $= R_2$

Initial temperature of rod $= \theta_1 °C$

Final temperature of rod $= \theta_2 °C$

Increase in length $= R_2 - R_1$

Rise in temperature $= (\theta_2 - \theta_1) °C$

$$\text{Co-efficient of linear expansion} = \frac{\text{Change in length}}{\text{Original length} \times \text{Rise of temp.}}$$

$$\therefore \qquad \alpha = \frac{R_2 - R_1}{L \times (\theta_2 - \theta_1)} \text{ per } °C$$

Since all quantities on the right hand side of the expression are known, therefore, co-efficient of linear expansion α of the rod material can be calculated.

Ex. EOS—1. What space must be left between two rails 20m long so as to allow for expansion upto 40°C, assuming that rails are laid down at 15°C ? Co-efficient of linear expansion of steel is $12 \times 10^{-6}\ °C^{-1}$.

Sol. Here, Length of rail at 15°C, $L_{15} = 20\ m$

Rise in temperature, $(\theta_2 - \theta_1) = 40 - 15 = 25°C$

Co–efficient of linear expansion, $\alpha = 0{\cdot}000012/°C$

The length L_{40} at 40°C is given by the relation,

$$L_{40} = L_{15}[1 + \alpha(\theta_2 - \theta_1)] = 20[1 + {\cdot}000012(25)]$$
$$= 20[1 + {\cdot}000300] = 20[1{\cdot}0003] = 20{\cdot}006\ m$$

∴ Space that must be left between two consecutive rails

$$= 20{\cdot}006 - 20 = \mathbf{0{\cdot}006\ m} = 0{\cdot}6 \text{ cm (Ans.)}$$

Ex. EOS—2. A steel tyre 1·5m in diameter is to be shrunk on to a cart–wheel of which the average diameter is 1 mm greater than the inside diameter of the tyre. Assuming the co–efficient of expansion of the material of the tyre to be 12 × 10^{-6} °C^{-1}, calculate the necessary rise of temperature of the tyre in order that it may easily slip on to the wheel.

Sol. Let the initial temperature of the tyre be $\theta_1°C$ and final temperature of tyre be $\theta_2°C$

Diameter of tyre $= 1{\cdot}5\ m$

and Diameter of wheel $= 1{\cdot}5\ m + 1\ mm$

$= 1{\cdot}5\ m + 0{\cdot}001\ m = 1{\cdot}501\ m.$

∴ Circumference of tyre, $L_1 = \pi \times \text{diameter} = \pi \times 1{\cdot}5\ m$

Circumference of wheel. $L_2 = \pi \times \text{diameter} = \pi \times 1{\cdot}501\ m$

In order that the tyre may slip on to the wheel, it should be heated till its circumference becomes equal to the circumference of wheel *i.e.* $1{\cdot}501\ \pi\ m$. Let θ be the rise in temperature.

∴ Rise in temperature, $\theta = (\theta_2 - \theta_1)$

Hence, $L_2 = L_1\{1 + \alpha(\theta_2 - \theta_1)\}$

or $\pi \times 1{\cdot}501 = \pi \times 1{\cdot}5\{1 + 12 \times 10^{-6} \times \theta\}$

or $1{\cdot}501 = 1{\cdot}5 + 1{\cdot}5 \times 12 \times 10^{-6}\ \theta$

or $18 \times 10^{-6}\ \theta = 0{\cdot}001$

or $\theta = \dfrac{0\cdot001}{18\times10^{-6}} = \dfrac{10^3}{18} = 55\cdot5°C$ **(Ans).**

Ex. EOS—3. An iron bridge is 250 m long. If the temperature of the bridge rises from 10°C to 50°C, how much will its length increase, if $\alpha = 12 \times 10^{-6}$ °C^{-1}.

Sol. Here, length of bridge at 10°C, $L_{10} = 250\ m$

Final temperature $= 50°C$

Co–efficient of linear expansion, $\alpha = 12 \times 10^{-6}/°C$

Let, length of bridge at 50°C $= L_{50}$

∴ Increase in length, $(L_{50} - L_{10}) = \alpha \times L_{10} \times (50 - 10)$

$= 12 \times 10^{-6} \times 250 \times 40m = 120000 \times 10^{-6}\ m$

$= 0{\cdot}12\ m = 12$ cm **(Ans.)**

Ex. EOS—4. The co-efficient of linear expansion of brass being 1·5 times that of iron, find the length of brass rod which will expand through the same length as the expansion of a rod of iron 1·5 *m* long for any increase in temperature.

Sol. Let, l_1 = length of iron rod = 1·5 *m*

α_1 = co-efficient of linear expansion of iron = α

Again l_2 = length of brass rod = ?

α_2 = co-efficient of linear expansion of brass = $1 \cdot 5\ \alpha$

Let, θ be the rise in temperature

$\therefore$ Increase in length of iron rod $= l_1\, \alpha_1 \times \theta$ ...(*i*)

and Increase in length of brass rod $= l_2\, \alpha_2 \times \theta$ ...(*ii*)

From (*i*) & (*ii*), we have

$$l_1\, \alpha_1\, \theta = l_2\, \alpha_2\, \theta \qquad \text{(increase in length is equal)}$$

or

$$l_2 = \frac{l_1 \alpha_1 \theta}{\alpha_2 \times \theta} = \frac{1 \cdot 5 \times \alpha \times \theta}{1 \cdot 5\alpha \times \theta} = 1m \text{ (Ans).}$$

Ex. EOS—5. A square metal plate of each side 100 cm long at 0°C, has a circular hole of diameter 40 cm in the middle of it. At what temperature will the sides be 101 cm long and what will then be the diameter of the hole ? (Co-efficient of linear expansion of the metal = 0·0000125/°C).

Sol. Here, side of square at $0°C$, $L_0 = 100$ cm

Let rise in temperature be $\theta°C$

$\therefore$ Side of square = 101 cm at $\theta°C$

$\therefore$ $L_\theta = 101$ and $\alpha = 0 \cdot 0000125/°C$

Now, $L_\theta - L_0 = L_0\, \alpha(\theta - 0)$

$\therefore$ $101 - 100 = 100 \times 0 \cdot 0000125 \times \theta$

or

$$\theta = \frac{1}{0 \cdot 00125} = \frac{100000}{125} = 800° C \text{ (Ans)}$$

In the case of hole,

Diameter at $0°C$, d_1 = 40 cm

Rise in temperature, θ = $800°C$

Co-eff. of linear-expansion α = $0 \cdot 0000125/°C$

Let diameter at $800°C$ = d_2

$$\text{Circumference of hole at } 0°C = 2\pi \frac{d_1}{2} = \pi d_1$$

$$\text{Circumference of hole at } 800°C = 2\pi \frac{d_2}{2} = \pi d_2$$

Now $\pi d_2 = \pi d_1\, (1 + \alpha\theta)$

or $$d_2 = d_1 (1 + \alpha\theta) = 40 (1 + \cdot 0000125 \times 800)$$
$$= 40 (1 + \cdot 01) = 40 \times 1\cdot 01 = 40\cdot 4 \text{ cm } \textbf{(Ans.)}$$

Ex. EOS—6. A steel cylinder has an aluminum alloy piston and at a temperature of 20°C, when the external diameter of the piston is exactly 10 cm, there is an alround clearance of 0·05 mm between the piston and the cylinder wall. At what temperature will the fitting be perfect ?

(α for steel = $1\cdot 2 \times 10^{-5}$ and α for Al = $1\cdot 6 \times 10^{-5}$ per °C respectively).

Sol. Here, External diameter of piston = 10 cm

Clearance between piston and cylinder = 0·05 mm = 0·005 cm

∴ Internal diameter of cylinder = 10 + 2 × 0·005 = 10·01 cm

Initial temperature = 20°C

Let θ be the temperature at which fitting is perfect.

On heating, both steel cylinder and aluminum piston expand. At θ°C the internal diameter of cylinder is exactly equal to the external diameter of the piston, therefore, the circumference of the two are equal.

Now circumference of steel cylinder at θ°C

$$= \pi \times 10\cdot 01 \, [1 + 0\cdot 000012 \, (\theta - 20)] \qquad \ldots(i)$$

Circumference of aluminium piston at θ°C

$$= \pi \times 10 \, [1 + 0\cdot 000016 \, (\theta - 20)] \qquad \ldots(ii)$$

From (i) and (ii), we have

$$\pi \times 10\cdot 01 \, [1 + 0\cdot 000012 \, (\theta - 20)] = \pi \times 10 \, [1 + 0\cdot 000016 \, (\theta - 20)]$$

or $$10\cdot 01 + 10\cdot 01 \times \cdot 000012 \, (\theta - 20) = 10 + \cdot 00016 \, (\theta - 20).$$

$$0\cdot 01 = (0\cdot 00016 - 0\cdot 00012) \, (\theta - 20)$$

or $$\theta - 20 = \frac{0 \cdot 01}{\cdot 00004} = 250$$

∴ $$\theta = \mathbf{250 + 20 = 270°C} \textbf{ (Ans.)}$$

EOS—4. SUPERFICIAL OR SURFACE EXPANSION :

The increase in the surface area of a solid on heating is called **Superficial expansion.**

When a solid body is heated, its surface area increases, this increase in surface area depends upon :

(i) Original surface area.

(ii) Rise in temperature.

(iii) Nature of the material of the body.

Let, S_0 = Surface area of the body at 0°C

S_θ = Surface area at θ°C

∴ Increase in surface area = $S_\theta - S_0$

It is found experimentally that :

$$S_\theta - S_0 \propto S_0 \text{ (original surface area)}$$

$$S_\theta - S_0 \propto \theta \text{ (rise in temperature)}$$

Combining the two,

$$S_\theta - S_0 \propto S_0 \times \theta$$

or $$S_\theta - S_0 = \beta S_0 \theta \qquad \ldots(i)$$

where β is the constant of proportionality and is called *co–efficient of superficial expansion*. Its value depends upon the nature of material of the body.

From equation (*i*), $\beta = \dfrac{S_\theta - S_0}{S_0 \times \theta}$

i.e., Co–efficient of superficial expansion

$$= \frac{\text{Increase in surface area}}{\text{Original surface area} \times \text{Rise of temp.}}$$

If original surface area, $S_0 = 1$, and rise in temperature, $\theta = 1°C$

Then, $\beta = \dfrac{S_1 - 1}{1 \times 1}$ = increase in surface area per unit area per °C rise in temperature.

Hence **Co–efficient of superficial expansion** *of a solid is defined as the increase in its surface area per unit area for 1°C rise in temperature.*

Also from equation $S_\theta - S_0 = \beta S_0 \theta$

or $S_\theta = S_0 + S_0 \theta$

∴ $S_\theta = S_0 (1 + \beta\theta)$

We get,

EOS—5. CUBICAL EXPANSION

The increase in volume of a solid on heating is called **cubical expansion.**

When a body is heated, it expands in all directions and hence its volume increases. The increase in volume depends upon :–

(*i*) Original volume of the body.

(*ii*) Rise in temperature of the body.

(*iii*) Nature of the material of the body.

Let, V_0 = Volume of the body at 0°C

V_θ = Volume at θ°C

∴ Increase in volume $= V_\theta - V_0$

It is found experimentally that :

$$V_\theta - V_0 \propto V_0 \text{ (original volume)}$$

$$V_\theta - V_0 \propto \theta \text{ (rise in temperature)}$$

P+T+O

Combining the two,

$$V_\theta - V_0 \propto V_0 \theta$$

or $$V_\theta - V_0 = \gamma V_0 \theta \qquad \ldots(i)$$

Where γ is the constant of proportionality and is called *co-efficient of cubical expansion*. Its value depends upon the nature of material of the body.

From equation (*i*), $$\gamma = \frac{V_\theta - V_0}{V_0 \times \theta}$$

i.e., Co-efficient of cubical expansion, $$\gamma = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{Rise in temp.}}$$

If original volume, $V_0 = 1$ and rise in temperature, $\theta = 1°C$

Then, $\gamma = \frac{V_1 - 1}{1 \times 1}$ = Increase in volume per unit volume per °*C* rise in temperature.

Hence, **Co-efficient of cubical expansion** *of a solid may be defined as the increase in its volume per unit original volume for 1°C rise in temperature.*

Also from equation (*i*), $V_\theta - V_0 = \gamma V_0 \theta$

or $$V_\theta = V_0 + \gamma V_0.0 = V_0 (1 + \gamma_0)$$

$\therefore$ $$V_\theta = V_0 (1 + \gamma\theta)$$

EOS—6. DEFINITIONS AND RELATION BETWEEN α, β AND γ.

imp.

Co-efficient of Linear expansion. *Co-efficient of linear expansion may be defined as the increase in length of solid per unit original length, per degree* **centigrade** *rise in temperature*. It is denoted by α.

Co-efficient of superficial expansion. *Co-efficient of superficial expansion may be defined as the increase in surface area of the solid per unit original area, per degree centigrade rise in temperature*. It is denoted by β.

Co-efficient of cubical expansion. *Co-efficient of cubical expansion may be defined as the increase in volume per unit original volume, per degree centigrade rise in temperature*. It is denoted by γ.

Relation between α and β. Let there be a rectangle whose length and breadth at $\theta°C$ are x_0 and y_0 respectively. On heating through $\theta°C$, length and breadth become x_θ and y_θ. Therefore, we have,

$$x_\theta = x_0 (1 + \alpha\theta) \qquad \ldots(i)$$

and $$y_\theta = y_0 (1 + \alpha\theta) \qquad \ldots(ii)$$

Where, α is the co-efficient of linear expansion of the material.

Now surface area at $0C$, $S_0 = x_0 y_0$ and surface area at $\theta°C$, $S_\theta = x_\theta y_\theta$

$\therefore$ $$S_\theta = S_0 (1 + \beta\theta) \qquad \ldots(iii)$$

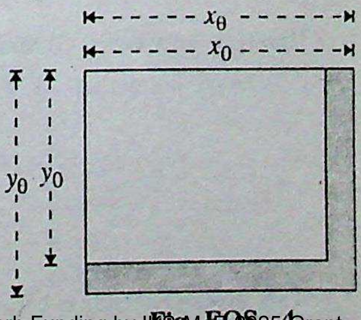


Fig. EOS—4

Where β is the co-efficient of superficial expansion.

Multiplying (*i*) and (*ii*), we get

$$x_0 y_0 = x_0 y_0 (1 + \alpha\theta)^2 \qquad \ldots(iv)$$

Comparing equation (*iv*) with (*iii*), we get

$$S_0 = S_0 (1 + \alpha\theta)^2 \qquad \ldots(v)$$

From (*iii*) and (*v*), we have

$$1 + \beta\theta = (1 + \alpha\theta)^2 = 1 + 2\alpha\theta + \alpha^2 \theta^2$$

Since α is small, therefore, $\alpha^2 \theta^2$ can be neglected

$$\therefore \quad 1 + \beta\theta = 1 + 2\alpha\theta$$

$$\text{or} \quad \beta = 2\alpha$$

Relation between α and γ. Suppose there is a rectangular parallelopiped having length, breadth and thickness at 0°*C* as x_0, y_0 and z_0 respectively. On heating through θ°*C*, each dimension expands to x_θ, y_θ and z_0 respectively.

$$\therefore \quad x_\theta = x_0 (1 + \alpha\theta) \qquad \ldots(i)$$

$$y_\theta = y_0 (1 + \alpha\theta) \qquad \ldots(ii)$$

$$z_0 = z_0 (1 + \alpha\theta) \qquad \ldots(iii)$$

$$\therefore \quad \text{Volume at } 0°C, \; V_0 = x_0 y_0 z_0$$

$$\text{and} \quad \text{Volume at } \theta°C, \; V_\theta = x_\theta y_\theta z_\theta$$

. We know that, $V_\theta = V_0 (1 + \gamma\theta)$ ...(*iv*)

Where γ is the co-efficient of cubical expansion.

Substituting the values of V_θ and V_0 in (*iv*), we have,

$$x_\theta y_\theta z_\theta = x_0 y_0 z_0 (1 + \gamma\theta) \qquad \ldots(v)$$

Also multiplying (*i*), (*ii*) and (*iii*), we get

$$x_\theta y_\theta z_0 = x_0 y_0 z_0 (1 + \alpha\theta)^3 \qquad \ldots.(vi)$$

Comparing equations (*v*) and (*vi*), we obtain

$$1 + \gamma\theta = (1 + \alpha\theta)^3 = 1 + 3\alpha\theta + 3\alpha^2\theta^2 + \alpha^3\theta^3$$

Since α is small, therefore, neglecting $\alpha^3\theta^3$ and $3\alpha^2\theta^2$, we get,

$$1 + \gamma\theta = 1 + 3\alpha\theta$$

$$\text{or} \quad \gamma = 3\alpha \qquad \ldots(ii)$$

Relation between α, β and γ.

We know that, β = 2α and γ = 3α

$$\text{or} \quad \alpha = \frac{\beta}{2} \text{ and } \alpha = \frac{\gamma}{3}$$

$$\therefore \quad \frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}.$$

This gives the relation between the three co-efficients of expansion.

Ex. EOS—7. A thin square sheet at 0°C has each side 1 m. Each side becomes 1·01 m at 100°C. Find

(*i*) **Co–efficient of linear expansion**

(*ii*) **Co–efficient of superficial expansion.**

Sol. Here, Length of each side of square at $0°C = 1\ m$

Length of each side of square at $100°C = 1{\cdot}01\ m$

Rise in temperature of square plate $= 100 - 0 = 100\ °C$

(*i*) Co–eff. of linear expansion, $\alpha = \dfrac{\text{Change in length}}{\text{original length} \times \text{rise in temperature}}$

$$\therefore \qquad \alpha = \frac{L_{100} - L_0}{L_0 \times \theta} = \frac{1 \cdot 01 - 1}{1 \times 100} = 0{\cdot}0001/°C \text{ (Ans.)}$$

(*ii*) Co–eff. of superficial expansion, $\beta = 2\alpha = 2 \times 0{\cdot}0001 = 0{\cdot}0002/°C$ **(Ans.)**

Ex. EOS—8. A brass cube has every edge 10 cm long at 15°C. What will be the area of each face at 60°C ?

Sol. Length of each edge at $15°C$, $L_{15} = 10$ cm

Final temperature, $= 60°C$

α for brass $= 18 \times 10^{-6}/°C$

Let, each edge at $60°C = L_{60}$

Now $L_{60} = L_{15}\{1 + \alpha\,(60 - 15)\} = 10\,\{1 + 18 \times 10^{-6} \times 45\}$

$= 10{\cdot}0081$ cm.

$\therefore$ Area of each face at $60°C = 10{\cdot}0081 \times 10{\cdot}0081$

$= 100{\cdot}1610656$ sq. cm. **(Ans).**

Ex. EOS—9. If volume of a block of metal changes by 0·12 percent when it is heated through 20°C, what is the co–efficient of linear expansion of the metal ?

Sol. Change in volume $= 0{\cdot}12$ per cent

$\therefore$ Let, original volume $= 100$ units

Rise in temperature $= 20 - 0 = 20°C$

We know that, $\gamma = \dfrac{\text{Change in volume}}{\text{Original Volume} \times \text{Rise in temp.}}$

or $\gamma = \dfrac{0 \cdot 12}{100 \times 20} = \dfrac{0 \cdot 12}{2000} = 0 \cdot 00006$

Now $\gamma = 3\alpha$

$$\therefore \qquad \alpha = \frac{\gamma}{3} = \frac{0 \cdot 00006}{3} = 0{\cdot}00002/°C \text{ (Ans.)}$$

Ex. EOS—10. Determine the change in volume of a block of cast iron having a volume of 4916 cm^3, when the temperature changes from 25°C to 57°C. Co-efficient of linear expansion of cast iron is $10 \times 10^{-6}/°C$.

Sol. Volume at $25°C = 4916 \text{ cm}^3$

Final temperature $= 57°C$

Rise in temperature $= 57 - 25 = 32°C$

$\alpha = 10 \times 10^{-6}/°C$

Co-efficient of cubical expansion, $\gamma = 3\alpha = 3 \times 10 \times 10^{-6} = 30 \times 10^{-6}/°C$

Now,
$$\gamma = \frac{\text{Change in volume}}{\text{Original volume} \times \text{rise in temp.}}$$

∴ Change in volume of cast iron = γ × original vol. × rise in temp.

$$= 30 \times 10^{-6} \times 4916 \times 32 = \mathbf{4{\cdot}719 \text{ cm}^3} \textbf{ (Ans.)}$$

EOS—7. EXPANSION OF MEASURING SCALES

Metals expand on heating. Therefore, metal scales, such as steel tapes, brass scales expand with rise in temperature and contract with fall in temperature. The readings taken with such scales are correct at one temperature, say θ_1 the temperature at which they are graduated (see fig. EOS—5(*ii*). When the temperature of the scale is greater than θ_1, the distance between any two of its divisions increases, and its reading is therefore, less than the actual value (see fig. EOS—5 (*iii*). When the temperature is below θ_1, the distance between any two of its divisions decrease (due to contraction), and its reading is more than the true value (see fig. EOS—5)

Therefore, corrections are required to be made for temperature changes in the measurements while using metal scales.

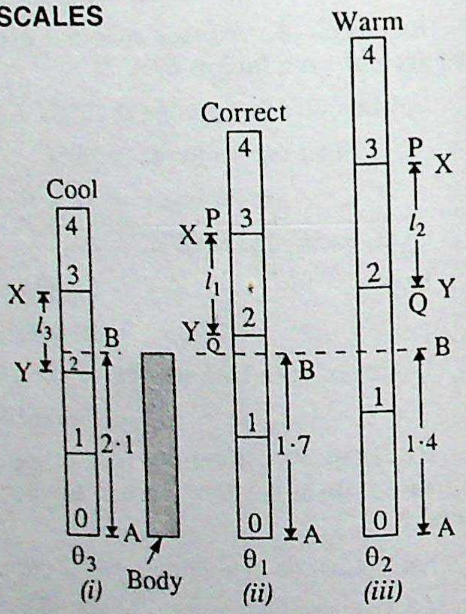


Fig. EOS—5

Correction. Let, the distance between any two points X, Y, on the scale is l_1 at θ_1, the temperature of its calibration. When the temperature rises to θ_2, the true length, l_2 between the points X, Y, is given as,

$$l_2 = l_1 [1 + \alpha (\theta_2 - \theta_1)] \qquad \ldots(i)$$

Where, α is the co-efficient of linear expansion of the material of the scale. In terms of number of divisions on the scale, the distance between X, and Y at temperature θ_2 will still be only l_1, Thus,

True length at any temperature

= Scale value [1 + α (change in temperature)]

Ex. EOS—11. The height of the mercury column in a barometer provided with a brass scale correct at 0°C is observed to be 749·0 mm on an occasion when the temperature is 15°C. Find the true height of the mercury column at 15°C. Co-efficient of cubical expansion of brass is 0·000054 per degree C.

Sol. Here, co-efficient of cubical expansion for brass,

$$\gamma = 0 \cdot 000054 \text{ per degree } C$$

$\therefore$ Co-eff. of linear expansion, $\alpha = \frac{1}{3}\gamma = \frac{1}{3} \times 0 \cdot 000054 = 0 \cdot 000018$ per deg C.

$$\therefore \qquad \theta = 15°C$$

$\therefore$ True height of mercury at $15°C$, $h_{15} = h_0 (1 + \alpha\theta)$

$$= 749 \cdot 0 \, (1 + 0 \cdot 000018 \times 15) = 749 \cdot 0 \times 1 \cdot 00027$$

$$= \mathbf{749 \cdot 2 \text{ mm. (Ans.)}}$$

Ex. EOS—12. A steel tape measures the length of a copper rod as 90 cm when both are at 10°C, the calibration temperature for the tape. What would the tape read for the length of the rod when both are at 30°C ? α for steel is $1 \cdot 1 \times 10^{-5}$ C^{-1} and α for copper = $1 \cdot 7 \times 10^{-5}$ C^{-1}.

Sol. Here, length of copper rod at $10°C = 90 \cdot 0$ cm

Co-eff. of linear expansion for Cu, $\alpha = 0 \cdot 000017/°C$

Length of copper rod at $30°C = 90\,[1 + 0 \cdot 000017 \times 20]$

$$= 90\,[1 + \cdot 000017 \times (30 - 10)]$$

$$= 90\,[1 \cdot 00034] = 90 \cdot 0306 \text{ cm}$$

Co-efficient of linear expansion for steel, $\alpha = 0 \cdot 000011/°C$. Steel tape expands with rise in temperature. To find the reading on the steel tape corresponding to the length 90·0306 cm of copper rod at $30°C$, we are to calculate the length of steel tape which becomes 90·0306 cm after expansion. Let the reading on the steel tape at $30°C$ be x.

$$\therefore \qquad 90 \cdot 0306 = x\,(1 + \alpha 0)$$

or $$x = \frac{90 \cdot 0306}{(1 + \alpha\theta)} = 90 \cdot 0306\,(1 + \alpha\theta)^{-1}$$

$$= 90 \cdot 0306\,[1 - \alpha\theta] \qquad \left[\begin{array}{l}\text{Using Binomial Theorm} \\ (1+x)^n = 1 + nx + \dots\dots\dots\end{array}\right]$$

$$= 90 \cdot 0306\,[1 - 0 \cdot 000011 \times 20]$$

$$= 90 \cdot 0306 \times 0 \cdot 99978 = 90 \cdot 01079$$

Hence reading of steel tape at $30°C = 90 \cdot 01079$ cm **(Ans.)**

Ex. EOS—13. Part of a steel tape used by a surveyer is 20·00 m at 12°C. What is the over all length measured by using this part of the tape one hundred times on a warmer day corresponding to 22°C. (α for steel = 11×10^{-6}/K).

Sol. Length of the part of steel tape at $12°C = 20 \cdot 00\ m$

Co-efficient of linear expansion for steel $= 11 \times 10^{-6}/°C$

Temperature of the day $= 22°C$

Rise in temperature, $\theta = 22 - 12 = 10°C$

$\therefore$ Length of chain at 22°C, $L_{22} = L_{12}\,[1 + \alpha\,(22 - 12)]$

$$= 20{\cdot}00\,[1 + {\cdot}000011 \times 10]$$

$$= 20{\cdot}00 \times 1{\cdot}00011 = 20{\cdot}0022\ m.$$

Thus length measured when chain is used for 1 time = 20.0022 m

$\therefore$ Total length measured when chain is used for 100 times

$$= 20{\cdot}0022 \times 100 = 2000{\cdot}22\ m \text{ (Ans.)}$$

Ex. EOS—14. A brass scale is correct at 0°C. The length of an iron rod measured by it at 50°C is 10 meters. What is the actual length of iron rod ? α for brass = 19×10^{-6}/°C, α for iron = 12×10^{-6}/°C.

Sol. Brass scale is correct at 0°C. It expands when the temperature rises to 50°C. The actual length corresponding to 1 m on brass scale at 50°C is given as :-

Length, $L_2 = L_1\,[1 + \alpha\,(\theta_2 - \theta_1)]$

$$= 1\,[1 + 19 \times 10^{-6} \times (50 - 0)] = 1\,[1 + 0{\cdot}00095]$$

$$= 1{\cdot}00095\ m$$

$\therefore$ 10 m length of iron rod on brass scale at 50°C is actually

$$= 1{\cdot}00095 \times 10 = 10{\cdot}0095\ m$$

$\therefore$ Actual length of iron road at 50°C = 10·0095 m (Ans.)

EOS-8 CHANGE OF DENSITY WITH TEMPERATURE

The density of a substance is its mass per unit volume.

i.e., $$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

When a substance is heated, its volume increases but the mass remains the same. Therefore, density of the substance decreases with the rise in temperature and increase when temperature falls.

Relation. Let the volume and density of a substance of mass m be V_0 and ρ_0 respectively at 0°C. Suppose the substance is heated to a temperature θ°C. Let its volume and density then become V_θ and ρ_θ respectively. Then,

$$\text{Mass} = \text{Volume} \times \text{Density} = \text{Constant}$$

$\therefore$ $$m = V_0\rho_0 \qquad \ldots(i)$$

Also $$m = V_\theta\rho_\theta \qquad \ldots(ii)$$

$\therefore$ $$V_0\rho_0 = V_\theta\rho_\theta$$

But $$V_\theta = V_0\,(1 + \gamma\theta)$$

$\therefore$ $$V_0\rho_0 = V_0\,(1 + \gamma\theta)\,\rho_\theta$$

or $$\rho_0 = \rho_\theta\,(1 + \gamma\theta)$$

or $$\gamma = \frac{\rho_0 - \rho_\theta}{\rho_\theta \times \theta}$$

Where, γ = Co–efficient of cubical expansion.

If the temperature rises from $\theta_1°C$ to $\theta_2°C$ and ρ_1 and ρ_2 are the densities at $\theta_1°C$ and $\theta_2°C$ respectively, then,

$$\rho_0 = \rho_1(1+\gamma\theta_1)$$

also

$$\rho_0 = \rho_2(1+\gamma\theta_2)$$

$$\therefore \quad \rho_1(1+\gamma\theta_1) = \rho_2(1+\gamma\theta_2)$$

or

$$\rho_1 = \rho_2(1+\gamma\theta_2)(1+\gamma\theta_1)^{-1} \qquad \left[\begin{array}{l}\text{Using Binomial Theorm}\\ (1+x)^n = 1+nx+\ldots\ldots\ldots\end{array}\right]$$

$$= \rho_2(1+\gamma\theta_2)(1-\gamma\theta_1)$$

$$= \rho_2(1-\gamma\theta_1+\gamma\theta_2-\gamma^2\theta_1\theta_2)$$

or

$$\rho_1 = \rho_2[1+\gamma(\theta_2-\theta_1)] \text{ neglecting } \gamma^2\theta_1\theta_2.$$

Ex. EOS—15. The density of a substance at 0°C is $8{\cdot}92 \times 10^3$ kg m^{-3} and at 80°C it is $8{\cdot}80 \times 10^3$ kg m^{-3}. Calculate the co-efficient of its linear expansion.

Sol. Here, Density at 0°C, $\rho_0 = 8{\cdot}92 \times 10^3\ kg\ m^{-3}$

Density at 80°C, $\rho_{80} = 8{\cdot}80 \times 10^3\ kg\ m^{-3}$

Rise in temperature, $\theta = 80 - 0 = 80°C$

Now, co-eff. of cubical expansion, $\gamma = \dfrac{\rho_0-\rho_{80}}{\rho_{80}\times\theta}$

$$= \frac{8\cdot 92\times10^3-8\cdot 80\times10^3}{8\cdot 80\times10^3\times80} = \frac{0\cdot 12}{8\cdot 80\times80} = 0{\cdot}00017/°C$$

We know, $\gamma = 3\alpha$

$\therefore$ Co-efficient of linear expansion, $\alpha = \dfrac{1}{3}\gamma = \dfrac{1}{3}\times 0\cdot 00017 = 0{\cdot}000056/°C$ **(Ans.)**

Ex. EOS—16. A piece of a metal weights 50 gm in air, 45·50 gm when immersed in a liquid at 5°C and 45·58 gm at 35°C. Find the co-efficient of cubical expansion of the liquid if the co-efficient of linear expansion of the metal is $2 \times 10^{-5}\ °C^{-1}$.

Sol. Loss of weight of metal in liquid at 5°C = 50 – 45·50 = 4·50 *g*

Loss of weight of metal in liquid at 35°C = 50 – 45·58 = 4·42 *g*

According to Archimede's Principle,

Loss in weight = Volume of metal × density of liquid × *g*

$$\therefore \quad 4{\cdot}50 = V_5 \times \rho_5 \times g \qquad \ldots(i)$$

and

$$4{\cdot}42 = V_3 \times \rho_3 \times g \qquad \ldots(ii)$$

Dividing (*i*) by (*ii*), we get,

$$\frac{4\cdot 50}{4\cdot 42} = \frac{V_5\times\rho_5\times g}{V_{35}\times\rho_{35}\times g} = \frac{V_5\times\rho_5}{V_{35}\times\rho_{35}}$$

or

$$\frac{\rho_5}{\rho_{35}} = \frac{4\cdot 50}{4\cdot 42}\times\frac{V_{35}}{V_5} = \frac{4\cdot 50}{4\cdot 42}\times\frac{V_5[1+\gamma(35-5)]}{V_5}$$

or $$\frac{\rho_5}{\rho_{35}} = \frac{4\cdot 50}{4\cdot 42}[1+3\times\alpha(30)] = \frac{4\cdot 50}{4\cdot 42}[1+3\times 0\cdot 00002\times 30]$$

$$= \frac{4\cdot 50}{4\cdot 42}\times 1\cdot 0018$$

Now, $$\rho_5 = \rho_{35}(1+\gamma\times 30)$$

or $$\frac{\rho_5}{\rho_{35}} = 1+30\gamma$$

$$\therefore \quad 1+30\gamma = \frac{4\cdot 50}{4\cdot 42}\times 1\cdot 0018$$

or $$30\gamma = \frac{4\cdot 50\times 1\cdot 0018}{4\cdot 42} - 1 = \frac{4\cdot 50\times 1\cdot 0018 - 4\cdot 42}{4\cdot 42}$$

$$\therefore \quad \gamma = \frac{4\cdot 50\times 1\cdot 0018 - 4\cdot 42}{4\cdot 42\times 30} = 0\cdot 000515/^\circ\text{C}$$ **(Ans.)**

EOS—9. THERMAL STRESSES

The forces developed in solid bodies when expansion or contraction is opposed are called **thermal stresses.**

If the ends of a metal rod are rigidly fixed between two supports so that expansion or contraction is opposed when heated or cooled, then compressive or tensile stresses are developed, in the rod. These stresses are called *thermal stresses*.

Some times these stresses become so large that the rod may bend when expansion is opposed or may break when the contraction is opposed. Thus in the design of structures which are subjected to changes in temperature, provision is always made for expansion or contraction.

For example, when rails are laid there is always a gap (clearance) left between the rail ends (see fig. EOS—6(*a*). In case of long steam pipes expansion is compensated by inserting expansion joints or a section of the pipe is bent in the form of Ω (see fig. EOS—6 (*b*).

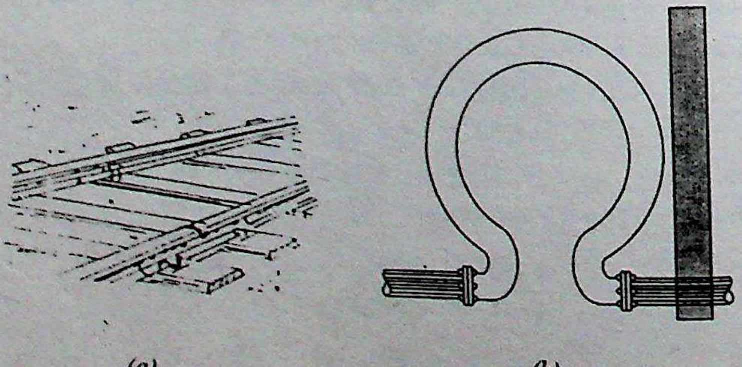

(*a*) (*b*)

Fig. EOS—6

Similarly in case of bridges, one end is rigidly fixed while the other end rests on rollers (see fig. EOS—7).

In some cases forces arising due to thermal expansion and contraction are made use of in engineering. For example, a heated steel tyre is placed over a wooden cart wheel. When it is cooled, it contracts and tightly fits the wheel. Again to hold the metal parts together firmly, red hot rivets are put through the holes and the ends of rivets are spread by hammering. On cooling, the rivets contract and hold the metal parts very tightly.

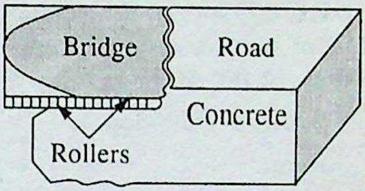


Fig. EOS—7

EOS—10. EXPRESSION FOR THE THERMAL STRESS

Consider a metal rod between two rigid supports P and Q. Let l_1 be the length of the rod and θ_1, the temperature at which the rod just fits between the supports. If the rod is heated to a temperature θ_2, it will try to expand to a greater length l_2, but cannot expand due to the supports. The value of l_2 is given as,

$$l_2 = l_1 [1 + \alpha (\theta_2 - \theta_1)]$$

$\therefore$ Expansion of rod is, $l_2 - l_1 = l_1 \alpha (\theta_2 - \theta_1)$

Where α is the co-efficient of linear expansion of the material of the rod.

The force which opposes the expansion is the force which would compress a rod of natural length l_2 by an amount equal to $(l_2 - l_1)$. The magnitude F of this force depends on the area of cross-section A of the rod and the Young's modulus of elasticity Y of the material of the rod.

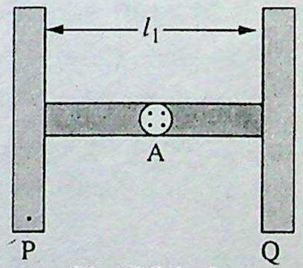


Fig. EOS—8

$$\therefore \qquad \text{Stress} = \frac{\text{Force}}{\text{area}} = \frac{F}{A}$$

$$\text{and} \qquad \text{Strain produced} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l_2 - l_1}{l_1}$$

$$= \frac{l_1 \alpha (\theta_2 - \theta_1)}{l_1} = \alpha (\theta_2 - \theta_1)$$

According to Hooke's Law,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F / A}{\alpha (\theta_2 - \theta_1)}$$

$$\text{or} \qquad \frac{F}{A} = Y\alpha (\theta_2 - \theta_1)$$

$$\therefore \qquad \text{Thermal stress} = \frac{F}{A} = Y\alpha (\theta_2 - \theta_1)$$

$$= Y \times \alpha \times \text{Rise in temperature}$$

Now Force developed, $F = YA\alpha(\theta_2 - \theta_1)$

i.e., Force developed = Elasticity × Area of rod × co–eff. of linear expansion × rise in temp.

Ex. EOS—17. Calculate the magnitude of the force developed in a rod of steel of area of cross–section 4 sq. cm for a temperature difference $(\theta_2 - \theta_1)$ of 100°C. (α for steel = $12 \times 10^{-6}/°C$ and Y = 2×10^{12} dynes per sq. cm.

Sol. Force developed, $F = YA\alpha(\theta_2 - \theta_1)$

Here, $Y = 2 \times 10^{12}$ dynes per sq. cm

$A = 4$ sq. cm

$\alpha = 12 \times 10^{-6}/°C$.

and $\theta_2 - \theta_1 = 100°C$

$\therefore$ $F = 2 \times 10^{12} \times 4 \times 12 \times 10^{-6} \times 100$ dynes

$= 96 \times 10^8$ dynes = 9·6 tons *wt.* (Ans.)

Ex. EOS—18. Girders of railway line are welded together at 10°C. Calculate the stress produced in the rails when heated by sun to 40°C. Given that co–efficient of linear expansion for steel is $12 \times 10^{-6}/°C$ and Young's modulus of elasticity for steel is 20×10^{10} N/m^2.

Sol. Here, Co–efficient of linear expansion, $\alpha = 12 \times 10^{-6}/°C$

Young's modulus of elasticity for steel, $Y = 20 \times 10^{10}\ N/m^2$

Rise in temperature, $\theta_2 - \theta_1 = 40 - 10 = 30°C$

Let L be the length of rails at $10°C$ and l be the increase in length on heating.

$$\therefore \quad \text{Strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{l}{L}$$

$$\text{But,} \quad \frac{\text{Increase in length}}{\text{Original length}} = \text{Co-efficient of linear expansion} \times \text{rise in temp.}$$

$$\therefore \quad \text{Stream,} \ \frac{l}{L} = \alpha \times (\theta_2 - \theta_1) = 12 \times 10^{-6} \times 30 = 360 \times 10^{-6}$$

Now, thermal stress (force per unit area) produced,

$$\frac{F}{A} = \text{Elasticity} \times \text{strain}$$

$$= 20 \times 10^{10} \times 360 \times 10^{-6}$$

$$= \mathbf{72 \times 10^6\ N/m^2}$$

Hence the stress produced is $72 \times 10^6\ N/m^2$. **(Ans.)**

Ex. EOS—19. An electric steel wire 2 mm in diameter is just stretched between two fixed points at a temperature of 25°C. Calculate the tension in the wire when the temperature falls to 15°C. Given that co–efficient of linear expansion for steel is $12 \times 10^{-6}/°C$ and Young's modulus of elasticity for steel is 20×10^{10} N/m^2.

Sol. Here, Young's modulus of elasticity, $Y = 20 \times 10^{10}\ N/m^2$

Co-efficient of linear expansion, $\alpha = 12 \times 10^{-6}/°C$

Diameter of the wire, $d = 12\ \text{mm} = 0{\cdot}002\ m$

$\therefore$ Area of cross-section of the wire $A = \pi \dfrac{d^2}{4} m^2$

$$= \frac{22}{7} \times \frac{(0 \cdot 002)^2}{4} m^2 = 3{\cdot}143 \times 10^{-6}\ m^2$$

Fall in temperature, $\theta = 25 - 15 = 10°C$

Tension (force) in the wire developed is given by,

$$F = YA\,\alpha\,\theta$$

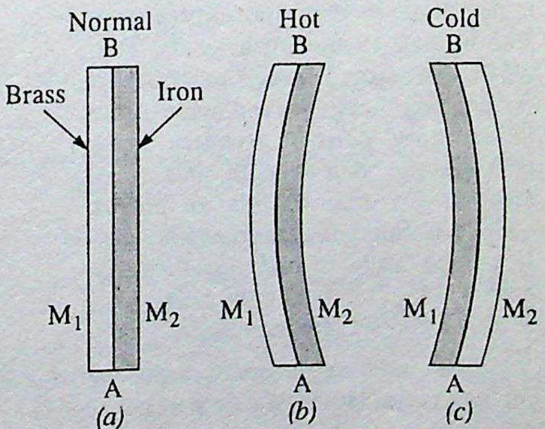
$$\therefore \quad F = 20 \times 10^{10} \times 3{\cdot}143 \times 10^{-6} \times 12 \times 10^{-6} \times 10\ N$$

$$= \mathbf{75{\cdot}432}\ N\ \textbf{(Ans.)}$$

EOS—11. DIFFERENTIAL EXPANSION OF METALS

Different metals expand through different amounts when heated through the same range of temperature. This is called differential expansion of metals.

Let there be two strips of different metals M_1 and M_2 (brass and iron), welded together along AB. Such a strip made of two different metals is called bimetallic strip. At normal temperature the strip remains straight (see fig. EOS—9 (*a*). The metal M_1 (brass) has greater co-efficient of linear expansion than metal M_2 (iron). Therefore, when the bimetallic strip is heated, M_1 (brass) will expand more than M_2 (iron), and the bimetallic strip will bend with M_1 on the outer side (see fig. EOS—9 (*b*). When the strip is cooled, brass contracts more than iron, therefore, the strip bends such that brass lies to the inner side (see fig. EOS—9(c).



(*a*) (*b*) (*c*)

Fig. EOS—9

The bending of bimetallic strips on heating are used as a device for controlling temperatures and for automatic fire alarms. It can also be used for automatic record of temperature variations on a thermograph. The device for controlling temperature is known as a thermostat.

In watches, the compensated balance wheel is made of two metals, the outer metal being more expensible, It is made in two, three or more segments (see fig., EOS—10).

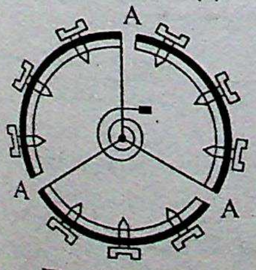


Fig. EOS—10

EOS—12. THERMOSTAT

A thermostat is a device for maintaing constant temperature at a given value.

Principle. *A thermostat works on the principle that different metals expand by different amounts when heated through the same range of temperature.*

The principle of a thermostat is illustrated in fig. EOS—11. *AB* is a bimetallic strip of brass and iron placed inside an enclosure in which temperature is to be controlled. The strip carries a contact *K*, which presses against another contact *K'* on the end of an adjustable screw *S*. When the strip gets heated, it tends to bend away from K^1. Th temperature at which the contact is broken can be set by adjusting the screw *S*. When the contact points open, they switch off the heating current and the heating stops (see fig. EOS—11). The temperature begins to fall. As the temperature falls, the bimetallic strip straightens and the contact at $K K^1$ is again made. The current begins to flow and the temperature is kept constant.

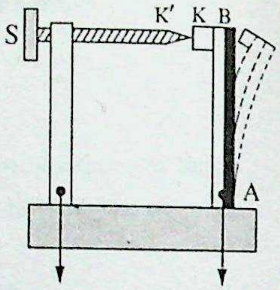


Fig. EOS—11

Applications of thermostats. The following are the important applications of thermostats :

1. Automatic temperature control in Domestic electric Press. Thermostats are used for automatic control of temperature in domestic electric iron. Due to their fibrer structures, different clothes require different temperatures for pressing them. Silk and nylon clothes being very delicate require lower temperature than woolen and cotton clothes. To achieve this an electric contact is fitted on the bimetallic strip. It can be made to make or break the electric circuit carrying the heating coil *H* at the pre-set temperature.

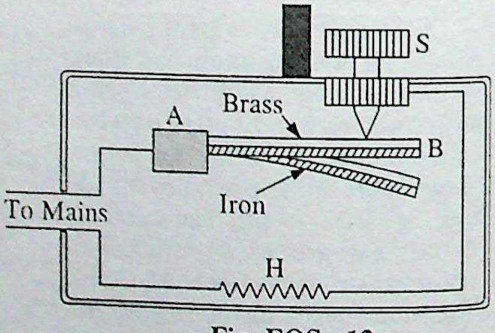


Fig. EOS—12

The operating temperature can be adjusted by adjusting the normal gap between the two contacts with the help of a screw S.

As the bimetallic strip is heated beyond a certain pre-set temperature, it bends downwards, there by breaking the circuit. When cooled below a given temperature the strip straightens upwards, thereby closing the circuit again the thus restoring heating.

2. Automatic fire Alarms : The use of a bimetallic strip in automatic fire alarms is illustrated in fig. EOS—13. It is fitted in such a way in a building or a workshop that at

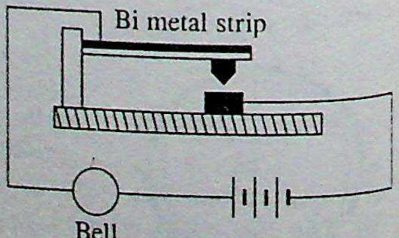


Fig. EOS—13

normal temperature the contact points do not touch each other and the circuit is open. see fig. When the temperature in a building rises beyond a certain value (due to fire etc.) the bimetallic strip bends over and electric circuit containing a bell is completed. The bell starts ringing and the alarm is raised.

3. **Thermograph :** A thermograph is a device used for automatic record of temperature variations on a certain day. It consists of a bimetallic strip fixed at one end. The free end is attached to one end of a lever. The other end of the lever touches a graph paper wrapped round a rotating drum. The drum completes one revolution in a specified number of hours say 24. The strip bends upwards or downwards depending upon the variations in temperature. If the end of the lever is fed with ink, it will trace a temperature graph on the paper.

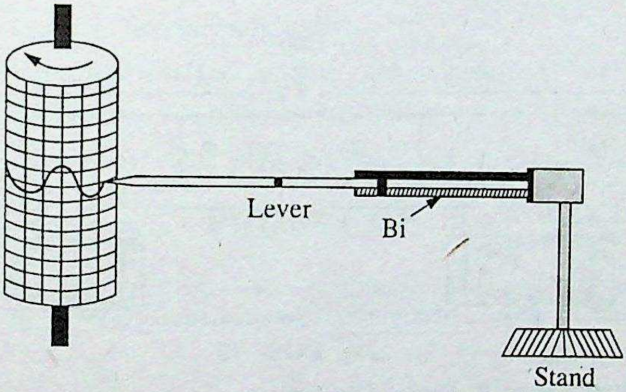


Fig. EOS—14

EOS—13. PRACTICAL AND ENGINEERING APPLICATIONS OF EXPANSION OF SOLIDS.

The expansion of solids is utilisied in several ways, in some as welcome aid while in others as harmful effects to be overcome.

Some of the important applications where it serves a useful purpose are :

(*i*) **Metallic Thermostat :** It consists of a bimetallic strip and is used to maintain constant temperature at a given value. The principle of differential expansion is used in its construction. The differential expansion makes it convenient to automatically switch off and switch on the heating current. With such a device temperature up to 300°C can be maintained steady within ± 1°C for a long period.

(*ii*) **Sealing a wire in glass :** Platinum expands to practically the same amount as glass. Therefore, platinum wires can be sealed into glass without any risk of cracks being developed as it happens in case of copper or iron. Invar, an alloy of steel and nickel has the same expansion as glass. Therefore, a wire of invar can be sealed into glass.

(*iii*) **Prevention of bulging walls from collapsing :** Walls of buildings which tend to bulge outwards can be prevented from collapsing by passing, iron rods through them across the building. By fixing the extremities of the rods tightly by nuts and bolts, while the rods are being heated in the centre. When the rods cool and contract, the walls are pulled together and assume their proper positions.

(*iv*) **Riveting of Boiler plates :** Boiler plates are joined with red hot rivets, which on cooling draw the plates together, thereby forming steam proof joints.

(*v*) **Fitting of metal tyres on cart wheels :** Iron tyres of cart wheels are fixed in position by heating them until their inner diameters are large enough to slip over the wooden–rims of the wheels. On cooling they contract and bind the wheels firmly.

The ways in which expansion of solids becomes an obstacle to be removed, are discussed below :

1. Laying of railway lines. When railway lines are laid, small gaps must be left between the adjacent rails in order to prevent the line from **buckling** and even breaking due to changes of temperature.

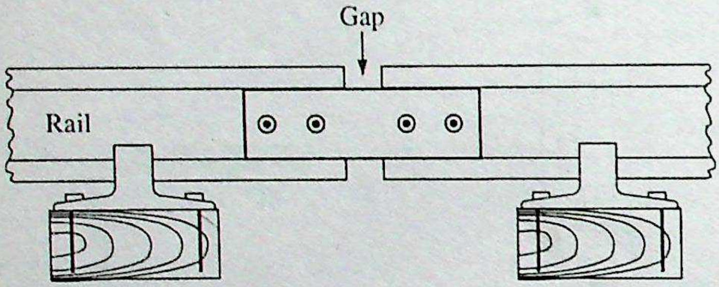


Fig. EOS—15

2. Steel bridges on Rollers. The girders of a steel bridge are not firmly fixed into the pillers of bricks on which they rest, but are supported on rollers. The rollers allow them to expand or contract without affecting the pillers.

3. Steam Pipes. In case of hot water and steam pipes, the expansion or contraction is allowed by providing *expansion bends or loops at* suitable distances. Thus the thermal stresses are prevented and no damage is done to the entire pipeline.

4. Concrete Roads and floors. Concrete roads and floors are never laid in one piece. These are always laid in sections and expansion channels are provided between the sections. These spaces behave as expansion joints. If the concrete slabs are not laid in sections, then during summer, great thermal stresses may be developed and buckling may result.

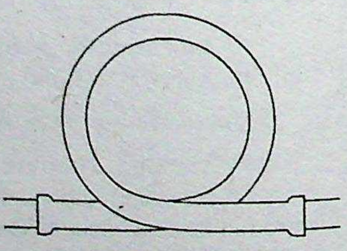


Fig. EOS—16

5. Overhead cables. Electric and telephone wires between two poles are given enough sag to allow for contraction during winter. If the sag is not provided, then during contraction, enormous thermal stresses will be developed which may damage the poles.

QUESTIONS FOR SELF TEST

1. Define co–efficient of linear expansion of a solid. Describe an accurate method for the experimental determination of co–efficient of linear expansion of a metallic rod.

2. Define co-efficient of linear, superficial and cubical expansion of a solid. Establish the relation between them.
3. Derive an expression for the thermal stress set up in a metallic rod when it is heated or cooled through θ°C.
4. What is the effect of temperature on the density of a substance ? Derive a relation for it.
5. Explain the principle of bimetallic strip utilized in thermostatic controls.
6. Mention some engineering applications of expansion of solids.
7. What do you mean by differential expansion of metals.
8. What is thermostat ? Give its application.
9. Explain why some gap is left while lying railway lines ?
10. Why electric and telephone cables contract during winter ? Explain.
11. Why steam pipes are provided loops or expansion bends at suitable distance ?

PROBLEMS

1. The co-efficient of linear expansion of brass is 18×10^{-6} °C^{-1}. Find the length of a rod of brass which will increase in length by 2 mm on heating it through 90°C.

   (Ans. 123·4 cm)

2. The area of a one metre square plate of a metal increases by 10sq. cm when heated through 50°C. Find the co-efficient of cubical expansion of the material of the plate. **(Ans.** 0·00003)

3. An iron and a zinc rod are each 2 m long at 0°C and both are heated equally. At 50°C, the zinc rod is found to be longer by 0·181 cm. Find the co-efficient of linear expansion of iron when that of zinc is $29{\cdot}3 \times 3 \times 10^{-6}$ °C^{-1}.

   (Ans. 0·0000117)

4. Given that brass expands 1·5 times iron, find the volume of a piece of brass expanding through the same volume as does 100 CC of iron. **(Ans.** 66·67 CC)

5. A steel wire, of diameter 1·0 mm, supports a load which is sufficient to keep it taut, the temperature being 20°C. If the temperature falls to 0°C. What additional load would be required to restore the length of the steel wire to its initial value. Youngs modulus for steel = 2×10^{12} dynes/cm^2 and co-efficient of expansion of steel = 10^{-5} °C^{-1}. **(Ans.** 3·2 kg.)

6. The density of metal at 30°C is $7{\cdot}8 \times 10^3$ kg/m^2 and at –20°C, it is $7{\cdot}814 \times 10^3$ kg/m^3. Find the co-eff. of linear expansion of the metal. **(Ans.** 12×10^{-6} °C^{-1}).

Chapter TOH

Transfer of Heat

Heat flow is an energy transfer that takes place as a consequence of temperature difference only. It is important to understand the rate at which heat is transferred between a system and its surroundings and the mechanisms responsible for the heat transfer. A thermos bottle or some other thermally insulated vessel is used to store hot coffee for a certain length of time. The vessel is used to store hot between the outside air and the hot coffee. Ultimately the coffee reaches the air temperature, because the vessel is not a perfect insulator. In general, there is no heat transfer between a system and its surroundings when they are at the same temperature.

The most common heat transfer process is *heat conduction*. In this process heat transfer takes place as an exchange of kinetic energy between the molecules, where the less energetic molecules gain energy by cooling with the more energetic molecules.

Two other important heat transfer processes are convection and radiation. *Convection* is heat transfer as a result of the actual movement of the heated particles from one place to another. *Radiation* is heat transfer by waves. All bodies radiate energy continuously in the form of electromagnetic waves. The surface of the sun emits radiant energy. New techniques are being developed to convert free solar radiation into useful forms of energy.

A body radiates and also absorbs radiations from its surroundings. The body that is hotter than its surroundings radiates more energy than it absorbs, whereas a body that is cooler than its surroundings absorbs more energy than it radiates. A black body is an *ideal radiator*. It also absorbs all energy incident on it. An ideal radiator is also a good absorber of raidations.

In this chapter we shall study different modes of heat transfer with special reference to applications of various laws of heat radiation in science and technology.

TOH—1. TRANSFER OF HEAT

The transference of heat from one body to another or from one part of a body to another is called the transfer of heat. The transfer of heat always takes place in a definite direction *i.e.* from bodies at higher temperature to bodies at lower temperature.

Heat transfer takes place not only from one body to another, but also from one part to another part of the same body. For example, when we heat one end of a brass rod in the fire, its other end which we hold in our hand gradually gets heated.

TOH—2. MODES OF TRANSFER OF HEAT

Imp.

There are three modes of transfer of heat :

(*i*) Conduction.

(*ii*) Convection,

(*iii*) Radiation.

(*i*) **Conduction.** *The process of transfer of heat in a material medium in which heat energy is transferred from molecule to molecule, in the direction of fall of temperature, without the actual motion of the molecules of the medium is called* **conduction.**

When one end of an iron rod is placed in fire and the other end is held in hand, then the heat reaches the other end in due course of time. It means heat has passed from the end at higher temperature to the end at lower temperature. The particles of the rod do not move from their mean positions, but heat is handed over from particle to particle.

Explanation of conduction on the basis of kinetic theory. The molecules of the rod at the hot end have more kinetic energy due to higher temperature, while the molecules at the cold end have lesser kinetic energy due to lower temperature. Therefore, molecules at hot end begin to vibrate with larger amplitudes and hand over the excess kinetic energy to the neighbouring molecules. The neighbouring molecules now having excess energy, transfer it to the next molecules. In this way energy is transferred from molecule to molecule, in the rod from hot end to the cold end. Thus, in heat transfer by *conduction,* only the molecular energy is transferred and there is no transfer of the molecules of the medium. Through solids, heat is transferred by conduction.

(*ii*) **Convection.** *The process of transfer of heat in which heat is transferred from one point to another by the actual motion of the heated particles away from the source of heat is called* **convection.**

When a flask partly filled with water is heated on a narrow bunsen burner flame, water is seen to move upwards from the centre of the flask and downwards along the sides. This is because water at the bottom gets heated, expands, becomes lighter and rises up. Cold water from the top, being heavier, moves downwards along the sides to take the place of heated water.

In this example the heated (high energy) water molecules themselves carry heat from the bottom (source) to the top, while the cold (low energy) molecules move down, near to the source of heat, gets heated and move upwards. Thus convection currents are set up.

Liquids and gases transfer heat by convection.

The hot gases rising up from a chimney is an example of convection.

Heating of buildings by hot air, cooling of engines and cooling of transformers are some applications of convection.

(*iii*) **Radiation.** *Radiation is the process of heat transfer in which heat is transferred from one place to another in straight lines, in the form of heat waves, with the velocity of light without heating the medium through which it passes.*

Heat from the Sun comes to us on the earth by radiation. Similarly when we sit near an open fire, heat comes to us by radiation.

The molecules of a hot body vibrate rapidly about their mean positions. These vibrating molecules emit waves (energy) which travel in straight lines with the velocity of light. When these waves carrying radiant energy fall on low speed (cold) molecules, their temperature rises.

TOH—3. THERMAL CONDUCTIVITY

Imp.

The ability of a material to conduct heat through it is called thermal conductivity. Different materials have different thermal conductivities. Thermal conductivity of a material depends upon the molecular structure. In case of metals, the molecules actively participate in the transfer of heat energy. Thus metals have high thermal conductivity.

Factors on which Heat conduction depends :

Consider a cube of a material having side equal to d and area of the face A. Suppose its face P is at higher temperature $\theta_1 K$ and the other face Q at lower temperature $\theta_2 K$. If there is no loss of heat from any face of the cube, then all the heat will flow from hot face to cold face. The total quantity of heat Q flowing from hot face to cold face depends upon the following factors :

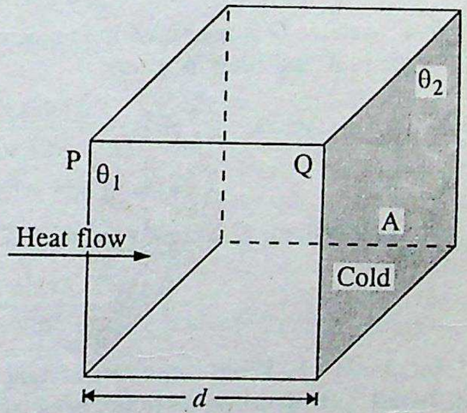


Fig. TOH—1

(*i*) The face area A of the cube. The greater the area A, greater is the flow of heat *i.e.* $Q \propto A$

(*ii*) The temperature difference $(\theta_1 - \theta_2)$. The greater the temperature difference, greater is the heat flow between the faces *i.e.* $Q \propto (\theta_1 - \theta_2)$

(*iii*) The *distance between two faces* of the cube. The greater the distance, lesser is the quantity of heat flow *i.e.,* $Q \propto 1/d$

(*iv*) The time t in seconds. The greater the time for which heat is allowed to flow, greater is the heat flow *i.e.,* $Q \propto t$.

(*v*) Nature of the material of the cube.

Combining the first four factors, we get ;

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{d}$$

or

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} \qquad \ldots(i)$$

Where K is the proportionality constant and is called the *co-efficient of thermal conductivity* of the material. Value of K depends upon the nature of the material.

In equation (*i*),

If $A = 1\ m^2, \quad d = 1\ m$

$\theta_1 - \theta_2 = 1\ K, \quad t = 1$ second

Then,
$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = \frac{K \times 1 \times 1 \times 1}{1}$$

$$\therefore \quad Q = K$$

Thus co-efficient of thermal conductivity of a material may be defined as the quantity of heat that flows in one second through an area of 1 m² of the material of thickness 1 m, when its opposite faces are maintained at a temperature difference of 1 K.

Units of Thermal Conductivity.

We know,
$$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$$

$$\therefore \quad K = \frac{Q \times d}{A(\theta_1 - \theta_2)t}$$

In *SI* system, Q is measured in joules, d in metres, A in square metres, temperature difference in K and time in seconds.

$$\therefore \quad K = \frac{\text{joules} \times m}{m^2 \times K \times \text{second}} = \frac{\text{joules}}{m \times K \times \text{second}}$$

$$= J\ m^{-1}\ s^{-1}\ K^{-1} \text{ or watt } m^{-1}\ K^{-1}.$$

In *MKS* system, Q is measured in k–cal

$$\therefore \quad K = \frac{k\,\text{cal} \times m}{m^2 \times {}^\circ C \times \text{second}} = k\ \text{cal}/m/s/{}^\circ C.$$

Ex. TOH—1. What is the rate of heat transfer through a copper rod 0·01 m long and 2×10^{-4} m² in cross–sectional area when one end of the rod is kept at 373 K and the other end at 273 K ? Thermal conductivity of Copper = 3851 J/m/s/K.

Sol. Here,

Area of cross–section, $A = 2 \times 10^{-4}\ m^2$

Thickness of the rod *i.e.* length, $d = 0{\cdot}01\ m$

Temperature difference, $\theta_1 - \theta_2 = 373 - 273 - 100\ K$

Thermal conductivity, $K = 3851\ J/m/s/K$

Time, $t = 1$ second.

The quantity of heat Q transferred is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = \frac{3851 \times 2 \times 10^{-4} \times 100 \times 1}{0 \cdot 01} \text{ joules}$$

$$Q = \mathbf{7702\ J\ (Ans.)}$$

Hence, 7702 joules of heat energy is transferred through this copper rod in 1 second.

Ex. EOH–2. **A person has wrapped his body by a woollen blanket 0·4 cm thickness. The temperature of the body is 37°C, while temperature of outer atmosphere is 17°C. Calculate how much heat in joules per second will be lost from per square metre area of his body ?**

$$\text{K for wool} = 10^{-4} \text{ cal cm}^{-1} {}^{\circ}\text{C}^{-1} \text{ s}^{-1}.$$

Sol. Here, Area of surface, $A = 1\ m^2$

Thermal conductivity of wool, $K = 10^{-4}\ cal\ cm^{-1}\ {}^{\circ}C^{-1}\ s^{-1}$

$= 10^{-5}\ k\ cal\ m^{-1}\ {}^{\circ}C^{-1}\ s^{-1}$

Temperature difference, $(\theta_1 - \theta_2) = 37 - 17 = 20°C$

Time, $t = 1\ s$

Thickness of blanket, $d = 0{\cdot}4\ cm = 0{\cdot}004\ m$

Quantity of heat Q lost per second is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = \frac{10^{-5} \times 1 \times 20 \times 1}{0 \cdot 004} k.cal.$$

$$= 0{\cdot}05\ k.\ cal = 0{\cdot}05 \times 4186\ J$$

$$= 209{\cdot}3 \text{ joules } \textbf{(Ans.)}$$

TOH—4. TEMPERATURE GRADIENT

The change of temperature in the material with distance is called **temperature gradient.** It is measured in kelvin per metre.

Thus if there is a temperature fall of $(\theta_1 - \theta_2)$ over a distance x metre in a material then,

$$\text{Temperature gradient} = \frac{\theta_1 - \theta_2}{x} \text{ kelvin per metre.}$$

If a small change in temperature $d\theta$ occurs over a small distance, dx,

$$\text{Then, temperature gradient} = \frac{d\theta}{dx}$$

We know,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = KAt\frac{\theta_1 - \theta_2}{d}$$

Here, $\frac{\theta_1 - \theta_2}{d}$ is the temperature gradient *i.e.,* change of temperature with distance.

Thus heat transfer equation may be written as :

$$Q = KAt \times temp.\ gradient.$$

From the above heat transfer equation, the rate of flow of heat is written as,

$$\frac{Q}{t} = KA \times \frac{d\theta}{dx}$$

Gandhi Memerial College Of Education Bantalab Jammu

Where, $\frac{d\theta}{dx}$ = temperature gradient

To define K, let $A = 1$ and $\frac{d\theta}{dx} = 1$, then

$$\frac{Q}{t} = K \quad \text{or} \quad K = \frac{Q}{t}$$

Thus co–efficient of thermal conductivity may be defined as the quantity of heat which flows per second through a unit area of the material where a unit temperature gradient is maintained.

Ex. TOH—3. A steam pipe at 200°C is converted with some material 0·05 m thick. The outer surface is at 50°C. A loss of 20 *K*. cal/minute/m length is observed. If the circumference of the pipe is 0·5 m, find the coefficient of conductivity of the material.

Sol. Here, temperature difference, $\theta_1 - \theta_2 = 200 - 50 = 150°C$

Quantity of heat lost per minute per meter length,

$$Q = 20\ k.\ \text{cal}$$

$\therefore$ Time, $t = 1$ minute $= 60\ s$

Area of conducting surface,

$$A = \text{circumference} \times \text{length}$$
$$= 0{\cdot}5\ m \times 1\ m = 0{\cdot}5\ m^2$$

Thickness, $d = 0{\cdot}05\ m$.

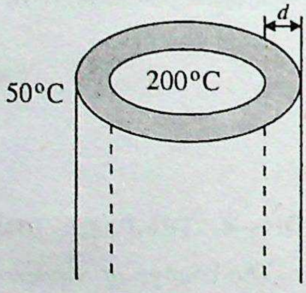


Fig. TOH—2

Quantity of heat flowing is given by the relation, $Q = \frac{KA(\theta_1 - \theta_2)t}{d}$

$\therefore$ Co–efficient of thermal conductivity,

$$K = \frac{Q \times d}{A(\theta_1 - \theta_2)t}$$

$$= \frac{20 \times 0 \cdot 05}{0 \cdot 5 \times 150 \times 60} = 0 \cdot 00022 \text{ (Ans.)}$$

TOH—5. DETERMINATION OF CO–EFFICIENT OF THERMAL CONDUCTIVITY OF A GOOD CONDUCTOR BY SEARLE'S METHOD :

Searle's apparatus for measuring thermal conductivity consists of a rod AB of the material whose thermal conductivity is to be determined. The end A is heated by means of a steam jacket S. A thin copper tube is wound round the end B. A steady flow of water is maintained through this tube and the temperatures of incoming and out going water are noted by the thermometers T_3 and T_4. Two other thermometers T_1 and T_2 are placed in two holes drilled in the rod AB at a known distance apart. Some mercury is poured in each hole for ultimate contact of the thermometer bulbs with the rod. The whole apparatus is covered with bad conductor *like felt or cork* to reduce loss of heat from the surface of the rod *AB* by convection or radiation.

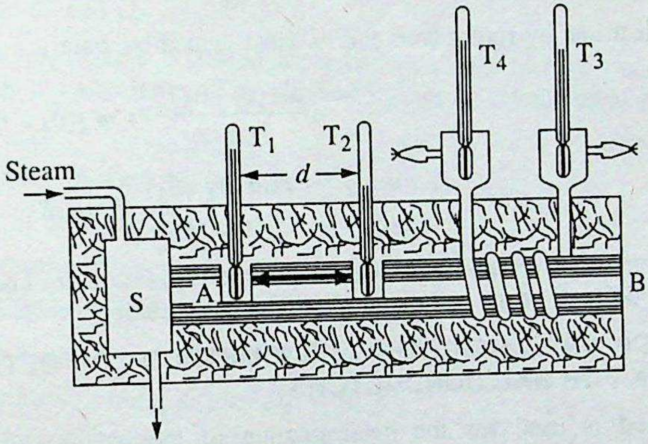


Fig. TOH—3

Heat is conducted through the material of the rod from hot end *A* to the cold end *B*. This heat flowing from end *A* to end *B* heat up the water flowing in the copper tube. The flow of water in the tube is so regulated that *steady state* is reached.

When steady state is reached *i.e.*, when all the temperature readings are stationary, it means the temperature of every point of bar *AB* remains constant. Under such conditions, heat flowing through any section of the rod is the same. In other words the rod *AB* does not loose any heat due to radiation. Therefore, heat flowing through the rod is taken up entirely by the circulating water and none by apparatus.

At this steady state, water is collected in a weighed beaker for a known length of time. The temperature readings of the thermometer are also noted. The calculations for the value of *K* are made as below :

Let, Area of cross-section of rod, $AB = A\ m^2$

Temperature reading of thermometer, $T_1 = \theta_1 °C$

Temperature reading of thermometer, $T_2 = \theta_2 °C$

Temperature reading of thermometer, $T_3 = \theta_3 °C$

Temperature reading of thermometer, $T_4 = \theta_4 °C$

Mass of water collected $= m\ kg$

Time for which water is collected $= t$ seconds

Distance between thermometers T_1 and $T_2 = d\ m$

Rise of water temp. $= \theta_4 - \theta_3$

$\therefore$ Heat gained by water $=$ mass × sp. heat capacity × rise in temp.

$= m \times s \times (\theta_4 - \theta_3) = ms\,(\theta_4 - \theta_3)$ joules

Let *K* be the thermal conductivity of the rod material.

Then, quantity of heat that flows through the rod AB in t seconds is,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$$

But, Heat lost by rod at cold end = Heat gained by water,

$$\therefore \quad = \frac{KA(\theta_1 - \theta_2)t}{d} = m\,s(\theta_4 - \theta_3)$$

or

$$K = \frac{msd(\theta_4 - \theta_3)}{At(\theta_1 - \theta_2)} \; J/m/s/K$$

Since all quantities on the right hand side of the expression are known, therefore, thermal conductivity K of the rod material can be calculated.

TOH—6. LEE'S AND CHARLTON'S DISC METHOD FOR DETERMINATION OF K FOR BAD CONDUCTORS

This method is used for the determination of thermal conductivity of poor conductors (*e.g.* rubber, glass, ebonite etc.) The heat transferred through a body is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$$

Since poor conductors conduct a very small amount of heat, therefore it is difficult to measure this value in experiments. In order to increase it to measurable quantity, we should either increase area A or increase temperature difference $\theta_1 - \theta_2$ or decrease the thickness d. But there is a limit to the increase in temperature difference. Hence Q is increased by increasing A and decreasing d. Therefore, while finding thermal conductivity of a poor conductor, *a thin sheet of large area of the material* should be used.

The apparatus consists of two thin discs D_1 and D_2 of the material whose thermal conductivity is to be determined. These discs are *sandwiched* between four copper discs C_1, C_2, C_3 and C_4 as shown in fig. TOH—4.

There is a heating element between the copper discs C_1 and C_2. The element is heated by passing current from a battery outside. The temperature on both sides of the discs D_1 and D_2 are measured accurately with thermocouples θ_1, θ_2, θ_3 and θ_4. The whole apparatus is suspended in a constant temperature bath by means of a string.

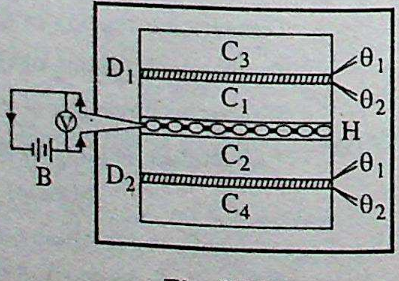


Fig. TOH—4

Method :

(*i*)A steady current is passed through the heating element. The current flowing through the element produces heat. This heat produced is conducted through C_1 and C_2 and then passes through D_1 and D_2. Copper discs C_1 and C_2 ensure uniform distribution of heat.

(*ii*)The value of current in the element is so adjusted that steady state is reached and all temperature readings are constant. Under such conditions, all heat produced by heating element is conducted through discs D_1 and D_2 and is then radiated from the outer surfaces of C_3 and C_4.

(*iii*)At steady state, current is passed for a known length of time t seconds. The temperature readings are also noted. The value of K, can be calculated as below :-

Let,

Voltage across the heating filament = V volts

Current through heating filament = I amp.

Temperature of inner side of D_1 = $\theta_2°C$

Temperature of outer side of D_1 = $\theta_1°C$

Temperature of inner side of D_2 = $\theta_3°C$

Temperature of outer side of D_2 = $\theta_4°C$

Thickness of disc D_1 = d_1 m

Thickness of disc D_2 = d_2 m

Area of cross-section of each disc (D_1 or D_2) = A m^2

Time, = t seconds

Temperature difference of the sides of D_1 = $\theta_2 - \theta_1$

Temperature difference of the sides D_2 = $\theta_3 - \theta_4$

Let K be the thermal conductivity of the material of discs D_1 and D_2 in $J/m/s/K$.

∴ Heat transferred through disc D_1, $Q_1 = \dfrac{KA(\theta_2 - \theta_1)t}{d_1}$ joules

Heat transferred through disc D_2, $Q_2 = \dfrac{KA(\theta_3 - \theta_4)t}{d_2}$ joules

Total heat transferred by both discs, $Q = Q_1 + Q_2$

$$= \frac{KA(\theta_2 - \theta_1)t}{d_1} + \frac{KA(\theta_3 - \theta_4)t}{d_2} \text{ joules}$$

Electrical energy consumed = $V\,I\,t$ joules

At steady state all heat produced electrically is conducted away by the discs D_1 and D_2. Therefore, two heats are equal *i.e.*,

$$= \frac{KA(\theta_2 - \theta_1)t}{d_1} + \frac{KA(\theta_3 - \theta_4)t}{d_2} = V\,I\,t$$

or $$KA\left[\frac{(\theta_2 - \theta_1)}{d_1} + \frac{(\theta_3 - \theta_4)}{d_2}\right] = VI$$ (Cancelling t throughout) or $$K = \frac{VI}{A\left[\frac{(\theta_2-\theta_1)}{d_1}+\frac{(\theta_3-\theta_4)}{d_2}\right]} \ J/m/s/K$$

Since all quantities on the right hand side of the expansion are known, therefore, thermal conductivity K of the material of discs D_1 and D_2 can be calculated.

TOH—7. HEAT CONDUCTION THROUGH COMPOUND MEDIA :

(i) Heat conduction through bodies in series : *The bodies are said to be in series thermally if the rate of transfer of heat is the same through all the bodies.*

Consider a compound media consisting of two materials of different thicknesses and conductivities.

Let the heat flows from medium 1 medium to medium 2. The two bodies are then in series thermally.

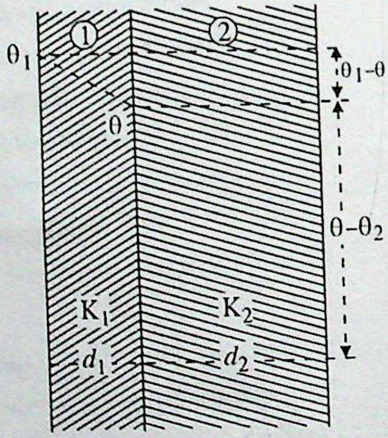


Fig. TOH—5

For medium 1. Let,

Thermal conductivity $= K_1$

Thickness $= d_1$

Temperature of hot face $= \theta_1\ K$

Temperature of interface $= \theta\ K$

Temperature difference between the two faces $= \theta_1 - \theta$.

Then the rate of flow of heat q *i.e.* heat flow per second is given by,

$$q = \frac{K_1 A(\theta_1-\theta)}{d_1} \quad \text{...(i)}$$

For medium 2. Let,

Thermal conductivity $= K_2$

Thickness $= d_2$

Temperature of the cold face $= \theta_2\ K$

Temperature difference $= \theta - \theta_2$

Then the rate of flow of heat is the same as through medium 1.

$$\therefore \quad q = \frac{K_2 A(\theta-\theta_2)}{d_2} \quad \text{...(ii)}$$

Since the two bodies are in series, therefore, the rate of flow of heat is the same in both cases *i.e.*,

$$\therefore \quad \frac{K_1 A(\theta_1-\theta)}{d_1} = \frac{K_2 A(\theta-\theta_2)}{d_2}$$

or $$\frac{K_1 A\theta_1}{d_1}-\frac{K_1 A\theta}{d_1}=\frac{K_2 A\theta}{d_2}-\frac{K_2 A\theta_2}{d_2}$$

or $$\frac{K_2 A\theta}{d_2}+\frac{K_1 A\theta}{d_1}=\frac{K_1 A\theta_1}{d_1}+\frac{K_2 A\theta_2}{d_2}$$

or $$\theta A\left[\frac{K_2}{d_2}+\frac{K_1}{d_1}\right]=A\left(\frac{K_1\theta_1}{d_1}+\frac{K_2\theta_2}{d_2}\right)$$

∴ Temperature of interfere, θ is given as,

$$\theta=\frac{\dfrac{K_1\theta_1}{d_1}+\dfrac{K_2\theta_2}{d_2}}{\dfrac{K_2}{d_2}+\dfrac{K_1}{d_1}}=\frac{\Sigma\dfrac{K\theta}{d}}{\Sigma\dfrac{K}{d}}$$

To find the quantity of heat transferred we have,

From equation (*i*), $\theta_1-\theta=\frac{q\,d_1}{K_1 A}$ ...(*iii*)

From equation (*ii*), $\theta-\theta_2=\frac{q\,d_2}{K_2 A}$ ...(*iv*)

Adding (*iii*) and (*iv*), we get,

$$\theta_1-\theta_2=\frac{q}{A}\left(\frac{d_1}{K_1}+\frac{d_2}{K_2}\right) \quad \text{or} \quad q=\frac{A(\theta_1-\theta_2)}{\dfrac{d_1}{K_1}+\dfrac{d_2}{K_2}}$$

or $$q=\frac{\theta_1-\theta_2}{\dfrac{d_1}{K_1 A}+\dfrac{d_2}{K_2 A}} \quad \text{or} \quad q=\frac{\theta_1-\theta_2}{\Sigma\dfrac{d}{KA}}$$

i.e., Rate of heat transfer,

$$q=\frac{\text{Difference in temperature between the faces of compound media}}{\Sigma\dfrac{d}{KA}}$$

∴ Heat transferred for t seconds, $Q=t\times\frac{\theta_1-\theta_2}{\Sigma\frac{d}{KA}}$.

Ex. TOH—4. Two metallic plates of similar areas of cross section having thicknesses d_1 and d_2 and co–efficient of thermal conduction K_1 and K_2 respectively are placed in perfect thermal contact. Prove that equivalent co–efficient of conduction is :

$$\mathbf{K}=\frac{\mathbf{d_1+d_2}}{\dfrac{\mathbf{d_1}}{\mathbf{K_1}}+\dfrac{\mathbf{d_2}}{\mathbf{K_2}}}$$

Sol. When two metallic plates are placed in thermal contact, the rate of heat flow through each plate is the same.

Let θ_1 be the temperature of the hot face, θ the temperature of interface and θ_2 the temp. of the cold face.

If K_1 and K_2 are the co-efficients of conduction and d_1, d_2 the thicknesses of the plates respectively having equal areas of correction, then,

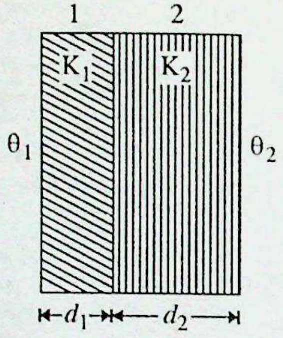


Fig. TOH—6

The rate of heat flow q through plate 1 is given by,

$$q = \frac{K_1 A(\theta_1 - \theta)}{d_1} \qquad \ldots(i)$$

and rate of heat flow q through plate 2 is given by,

$$q = \frac{K_2 A(\theta - \theta_2)}{d_2} \qquad \ldots(ii)$$

Now, from equation (*i*), $\theta_1 - \theta = \frac{q d_1}{K_1 A}$ ...(*iii*)

Also from equation (*ii*), $\theta - \theta_2 = \frac{q d_2}{K_2 A}$ ...(*iv*)

Adding (*iii*) and (*iv*), we get,

$$\theta_1 - \theta_2 = \frac{q d_1}{K_1 A} + \frac{q d_2}{K_2 A} = \frac{q}{A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2}\right)$$

$$\therefore \quad q = \frac{A(\theta_1 - \theta_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \qquad \ldots(v)$$

Let K be the equivalent co-efficient of conduction of the compound plate.

∴ Rate of heat flow is given by, $q = \frac{KA(\theta_1 - \theta_2)}{d_1 + d_2}$ ...(*vi*)

Since rate of heat flow is the same, therefore, from (*v*) and (*vi*) we have,

$$\frac{KA(\theta_1 - \theta_2)}{d_1 + d_2} = \frac{A(\theta_1 - \theta_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

or

$$\frac{K}{d_1 + d_2} = \frac{1}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

$$\therefore \qquad K = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}.$$

Ex. TOH—5. **If two metallic plates of equal areas of cross-section and thickness, but having co-efficient of thermal conductivity as K_1 and K_2 are placed in perfect thermal contact. Prove that the equivalent co-efficient of thermal conductivity,** $$\mathbf{K = \frac{2K_1 K_2}{K_1 + K_2}}.$$

Sol. Proceed as in Ex. 4 up to equation (*v*), *i.e.*

$$q = \frac{A(\theta_1 - \theta_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

In this case the thickness are the same

$$\therefore \qquad \text{put, } d_1 = d_2 = d \text{ each}$$

$$\therefore \qquad q = \frac{A(\theta_1 - \theta_2)}{d\left(\frac{1}{K_1} + \frac{1}{K_2}\right)}.$$

Let K be the equivalent co-efficient of conduction of the compound plate.

$$\therefore \qquad q = \frac{KA(\theta_1 - \theta_2)}{2d}$$

Hence $$\frac{KA(\theta_1 - \theta_2)}{2d} = \frac{A(\theta_1 - \theta_2)}{d\left(\frac{1}{K_1} + \frac{1}{K_2}\right)} = \frac{A(\theta_1 - \theta_2)}{d\,\frac{K_1 + K_2}{K_1 K_2}}$$

or $$K = \frac{2}{\frac{K_1 + K_2}{K_1 K_2}} = \frac{2K_1 K_2}{K_1 + K_2}$$

Hence the result.

(*ii*) **Heat conduction through bodies in parallel :** *The bodies are said to be in parallel thermally if the same temperature difference exists between their faces.*

Consider a compound media consisting of two materials of different thickness shown in fig. TOH—7. Let the heat flows as shown when, the two bodies are in parallel thermally.

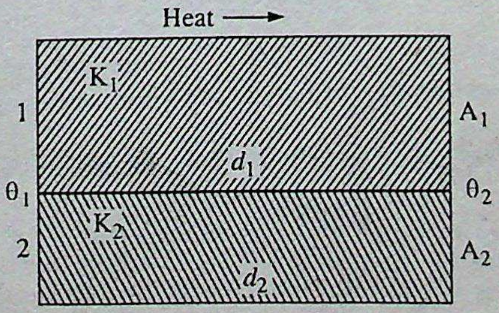


Fig. TOH—7

Rate of heat flow through medium 1, $q_1 = \dfrac{K_1 A_1 (\theta_1 - \theta_2)}{d_1}$

Rate of heat flow through medium 2, $q_2 = \dfrac{K_2 A_2 (\theta_1 - \theta_2)}{d_2}$

Total heat flow through the compound medium per second,

$$q = q_1 + q_2$$

or

$$q = \frac{K_1 A_1 (\theta_1 - \theta_2)}{d_1} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{d_2}$$

$$= (\theta_1 - \theta_2)\left[\frac{K_1 A_1}{d_1} + \frac{K_2 A_2}{d_2}\right]$$

$\therefore$

$$q = (\theta_1 - \theta_2)\left[\Sigma \frac{KA}{d}\right]$$

Total heat transferred in t seconds.

$$Q = q \times t = (\theta_1 - \theta_2)\left[\Sigma \frac{KA}{d}\right] \times t\,.$$

Ex. TOH—6. The outer faces of a rectangular slab made of equal thicknesses of iron and brass are maintained at 100°C and 0°C respectively. Find the temperature of the interface. The conductivities of iron and brass are 0·2 and 0·3.

Sol. Suppose θ_1 is the temperature of the hot face and θ_2 that of cold face.

Let θ be the temperature of the interface.

Since the two metals are arranged in series, therefore, rate of heat flow through each plate is the same.

Now rate of heat flow through iron slab is given as, $q = \dfrac{K_1 A (\theta_1 - \theta)}{d}$ ...(i)

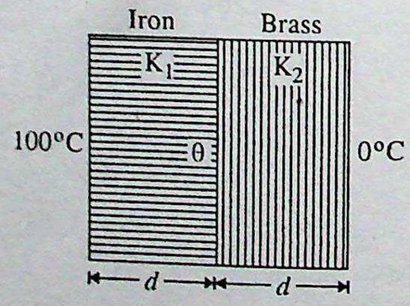


Fig. TOH—8

Also rate of heat flow through brass slab, $q = \dfrac{K_2 A (\theta - \theta_2)}{d}$ ...(ii)

Since rate of heat flow is the same, therefore, form (i) and (ii), we have

$$\frac{K_1 A (\theta_1 - \theta)}{d} = \frac{K_2 A (\theta - \theta_2)}{d}$$

or $\quad K_1 (\theta_1 - \theta) = K_2 (\theta - \theta_2)$

or $\quad K_1\theta_1 - K_1\theta = K_2\theta = K_2\theta_2$

or $\quad (K_1 + K_2)\,\theta = K_1\,\theta_1 + K_2\theta_2$

Here, $K_1 = 0{\cdot}2$ and $K_2 = 0{\cdot}3$

Also $\theta_1 = 100°C$ and $\theta_2 = 0°C$

$$\therefore \quad \theta = \frac{K_1\theta_1 + K_2\theta_2}{K_1 + K_2} = \frac{0\cdot 2\times100+0\cdot3\times0}{0\cdot2+0\cdot3} = \frac{20}{0\cdot5} = 40^{\circ}C \text{ (Ans.)}$$

Ex. TOH—7. The walls of a room consist of parallel layers of cement, bricks and wood of thicknesses 2, 30 and 1 cm respectively. Find how much heat passes put by conduction per square metre of the wall per minute if the temperature of air in contact with all is 5°C outside and 30°C inside. The value of K for cement, brick and wood are 0·0007, 0·006 and 0·004 respectively.

Sol. Here,

Temperature difference between the compound media

$$(\theta_1 - \theta_2) = 30 - 5 = 25°C$$

Time, $t = 1 \text{ minute} = 60 \text{ seconds}$

Area, $A = 1\ m^2 = 10000 \text{ sq. cm.}$

Here, heat condition takes place through these materials in series

$$\therefore \quad \text{Heat conducted, } Q = \frac{(\theta_1-\theta_2)t}{\Sigma\frac{d}{KA}}$$

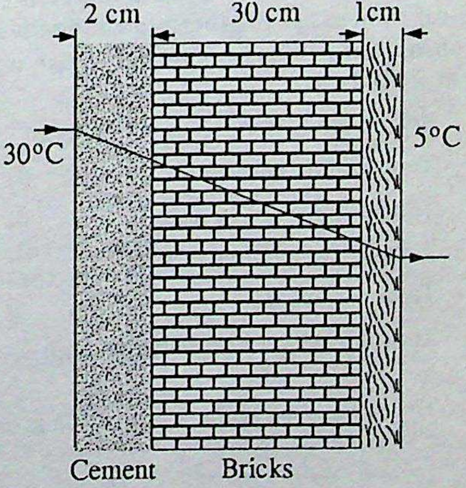


Fig. TOH—9

Now, $\Sigma\frac{d}{KA}$

$$= \frac{d_1}{K_1A} + \frac{d_2}{K_2A} + \frac{d_3}{K_3A}$$

$$= \frac{1}{A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)$$

$$= \frac{1}{10000}\left(\frac{2}{0\cdot0007} + \frac{30}{0\cdot006} + \frac{1}{0\cdot004}\right)$$

$$= 0\cdot7786$$

$$\therefore \quad Q = \frac{25\times60}{0\cdot7786} = \textbf{1450 cals. (Ans.)}$$

Thus 1450 calories of heat pass out per minute by conduction through each square metre of wall.

Ex. TOH—8. A steel boiler of plate thickness 0·02 has a calcareous deposite of 0·005 m thick. If the temperature of the outer surface of the boiler plate and innermost surface of deposit be 423 K and 373 K, calculate temperature of the interface, K for steel = 462 J/m/s/K, K for deposit = 9·24 J/m/s/K.

Sol. Here,

Thermal conductivity for steel, $K_1 = 462\ J/m/s/K$

Thermal conductivity for deposit, $K_2 = 9{\cdot}24\ J/m/s/K$

Thickness of steel plate, $d_1 = 0{\cdot}02\ m$

Thickness of deposit, $d_2 = 0{\cdot}005\ m$

Temperature of outer surface of boiler, $\theta_1 = 423\ K$

Temperature of innermost surface of deposit, $\theta_2 = 373\ K$

Let the temperature of interface be $\theta\ K$

∴ The temperature of interface is given as,

$$\theta = \frac{\dfrac{K_1\theta_1}{d_1} + \dfrac{K_2\theta_2}{d_2}}{\dfrac{K_2}{d_2} + \dfrac{K_1}{d_1}} = \frac{\dfrac{462 \times 423}{0 \cdot 02} + \dfrac{9 \cdot 24 \times 373}{0 \cdot 005}}{\dfrac{9 \cdot 24}{0 \cdot 005} + \dfrac{462}{0 \cdot 02}}$$

$$= \frac{9771300 + 689304}{1848 + 23100} = \mathbf{419{\cdot}3\ K\ (Ans.)}$$

Ex. TOH—9. A pond of water is covered with a layer of ice 4 cm thick and 20 m^2 in area. If the thermal conductivity of ice is 0·003, find the amount of heat transmitted per hour from the water up through ice, the upper surface of which is at – 40°C.

Sol. Here, Thermal conductivity of ice, $K = 0{\cdot}003$

Temperature of water, $\theta_1 = 0°C$

Temperature above ice, $\theta_2 = -40°C$

Area of conducting surface, $A = 20\ \text{m}^2$

Thickness of ice, $d = 4\ \text{cm} = 0{\cdot}04\ \text{m}$

Time for which heat is transmitted, $t = 1\ \text{hour} = 3600\ s$

The amount of heat transmitted is given by the relation, $Q = \dfrac{KA(\theta_1 - \theta_2)t}{d}$

$$\therefore \quad Q = \frac{0 \cdot 003 \times 20 \times [0 - (-40)] \times 3600}{0 \cdot 04}$$

$$= \frac{0 \cdot 003 \times 20 \times 40 \times 3600}{0 \cdot 04} = 216000 \text{ k cal } \textbf{Ans.}$$

TOH—8. RADIATION

Radiation is the transfer of heat from hot body to a cold body in a straight line with the velocity of light without heating the intervening medium.

Or

Radiation may be defined as the transfer of heat by waves.

All bodies above absolute zero emit heat radiations. The higher the temperature, the greater is the rate of heat radiations.

TOH—9. DETECTION OF HEAT RADIATION :

The commonly used instruments for detecting heat radiations are :

(i) **Differential air thermoscope.** It consists of two glass bulbs connected by a narrow U tube containing a coloured light liquid. The bulbs contain air. One of the bulbs is coated with lamp black. When both the bulbs are at the same temperatures, the liquid level in the two arms of U tube is the same. When heat radiations fall on the blackened bulb, these are absorbed and liquid level in it falls indicating heat radiations.

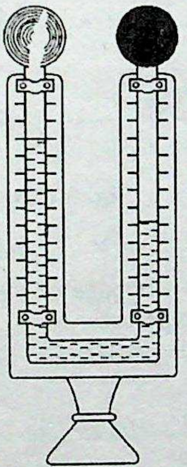


Fig. TOH—10

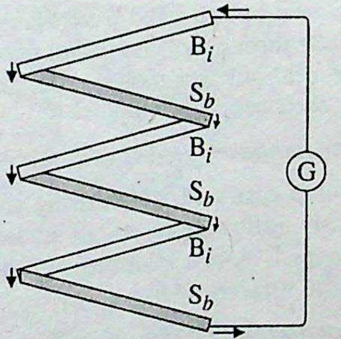


Fig. TOH—11

(ii) **Thermopile.** A thermopile consists of a number of thermocouples. The hot junction is exposed to radiations. The deflection in the galvanometer is the indication of radiations falling on the thermopile.

TOH—10. PROPERTIES OF HEAT RADIATIONS

The important properties of heat radiations are given below :–

1. *Heat radiations travel in straight lines :* Place a hot ball on one side of three screens having holes in the same straight line (fig. TOH—12.) Place the blackened bulb of a differential air thermoscope in front of the holes on the other side. The

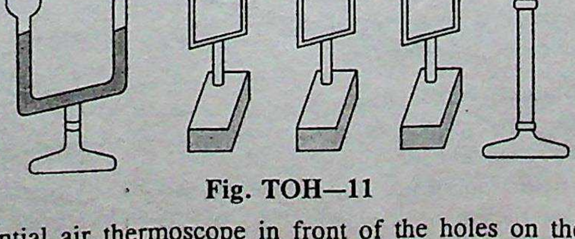


Fig. TOH—11

thermoscope shows temperature difference. Now disturb one of the screens so that the holes do not remain in straight line. It will be seen that thermoscope does not indicate any difference of temperature. This proves that heat radiations have been stopped by the screen. This shows that heat radiations travel in straight lines.

2. *Heat radiations require no medium for their propagation and can travel through vacuum.*

Enclose a wire filament in a glass bulb which can be exhausted as shown in fig. TOH—13. Place the thermometer inside the bulb and evacuate the bulb. Now pass current through the filament. Thermometer will indicate a rise in temperature thereby showing that heat has travelled from filament to thermometer through vacuum. *This shows that heat radiations require no material medium for their propagation.*

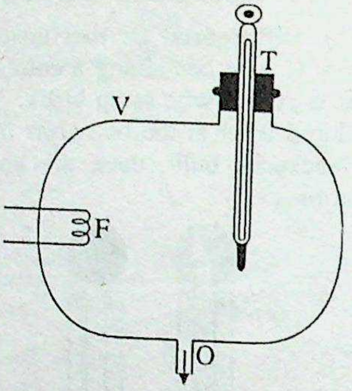


Fig. TOH—13

3. *Heat radiations travel with the velocity of light i.e., 3×10^8 m/s.*

This fact is easily verified during solar eclipse. The stoppage of light by moon is simultaneously followed by a fall in temperature. In other words, as soon as light rays are stopped, heat radiations are also stopped. *This shows that heat radiations travel with the velocity of light.*

4. *Heat radiations can be reflected from polished surfaces like light and obey the laws of reflection.*

Place two tubes infront of a polished surface (like mirror) so that tubes are inclined equally to the normal on the polished surfaces as shown in fig. TOH—14. Place a hot ball in front of one tube and a thermoscope in front of the other tube. The thermoscope will indicate a change in temperature as angle of incidence is equal to the angle of reflection. If the inclination is made unequal with normal, the thermoscope will not indicate any change of temperature, thereby showing that heat radiations are not reading the thermoscope. *This shows that heat radiations obey the laws of reflection.*

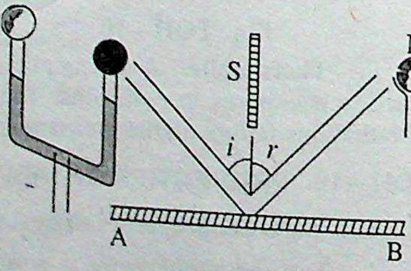


Fig. TOH—14

5. *Heat radiations can be refracted like light and obey the laws of refraction.*

Focus heat radiations from the sun by a lens on a black paper as shown in fig. TOH—15. The paper will catch fire. This shows that heat radiations are brought to focus by means of a lens and *hence they obey the laws of refraction.*

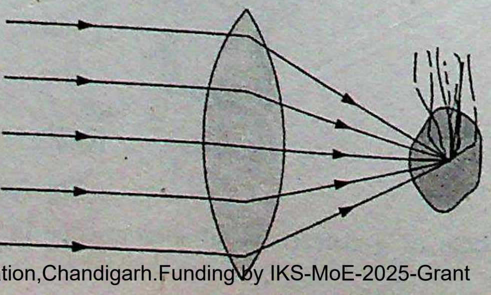


Fig. TOH—15

6. *Heat radiations do not heat the medium through which they pass.*

In the above case, if we touch the lens, it will be seen that it is as cold as before. In otherwords, heat radiations passing through the lens have not heated it. *This shows that heat radiations do not heat the medium through which they pass.*

7. *Heat radiations like light obey inverse square law i.e., intensity of heat radiations varies inversely as the square of the distance from the hot source.*

Take a large flat sided tank and fill it with boiling water. Place a thermoscope at a certain distance from it as shown in fig. TOG–16. Note down the difference in levels of liquid. Now take the thermoscope away to double the distance. It will be seen that the difference in the levels becomes $\frac{1}{4}$th. This proves inverse square law.

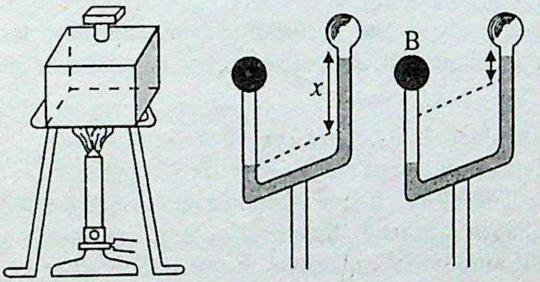


Fig. TOH—16

8. *In a homogeneous medium heat radiations are transmitted equally in all directions.*

A hot sphere appears equally bright on all sides. If a thermopile is taken on all sides at equal distances, the galvanometer indicates the same reading every where. This shows that heat radiations travel equally in all directions.

TOH—11. PREVOST'S THEORY OF HEAT EXCHANGES

In 1792 Prevost proved that thermal radiation was an exchange process. According to this theory.

"All bodies at all temperatures above absolute zero emit radiations to their surroundings and also simultaneously receive radiations from their surroundings".

The exchange process goes on according to the following assumptions :-

(i) *The amount of radiation emitted increases with temperature.*

(ii) *The rate of emission depends on the temperature of the body and the nature of the surface.*

(iii) *The rate of emission of heat radiation is affected by the surroundings.*

(iv) *There is continuous exchange of heat between the body and the surroundings.*

(v) *The rise or fall in temperature of a body is the net result of the exchange of heat between the body and the surroundings.*

Thus when two bodies are placed near each other, then both emit as well as receive heat radiations. The hot body being at higher temperature emits more heat than it receives from the cold body. On the other hand, cold body receives more heat than it emits. The result is that the cold body gains kinetic energy whereas the hot body suffers a loss of energy. Therefore, temperature of cold body is increased and the temperature of hot body is decreased. This continues till both the bodies are at the same temperature. At this stage, the two bodies emit and receive the same amount of heat and there is no further change in temperature.

It should be noted that the exchange is a dynamic process *i.e.*, bodies emit and receive heat at all temperatures. Even if the temperatures of the two bodies are same, the emission does not stop but *dynamic equilibrium* is set up *i.e.*, the two bodies receive and emit the same amount of heat. Therefore, no change in temperature results. *Prevost's theory of heat exchange leads to the fact that good, absorbers are good radiators and bad absorbers are bad radiators.*

Consider two spheres A and B suspended in a constant temperature enclosure A being blackened and sphere B being polished. The spheres will be in thermal equilibrium with the enclosure.

Sphere A is a black body therefore, it should absorb most of the radiations incident on it. But its temperature remains constant. Therefore, it must emit radiations at the same rate at which it absorbs. Sphere B being polished absorbs very small amount of radiations. Since temperature remains constant, therefore, it must emit very small radiations. Thus we can conclude that good absorbers are good radiators and bad absorbers are poor emitters.

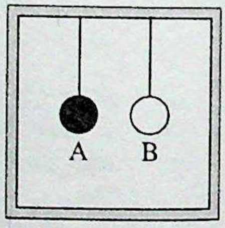


Fig. TOH—17

TOH—12. EMISSION OF RADIATION

The amount of heat radiations emitted by a body depends upon its temperature and nature of surface of the body.

Consider a body of surface area A at a temperature θ_1 K. Let its surroundings be at lower temperature θ_2 K. The hot body emits heat radiations. Let the amount of radiations emitted in a given time be θ.

It has been found experimentally that,

(i) Amount of heat radiations emitted depends upon the surface area A of the body. The greater the surface area, greater is the heat energy emitted *i.e.*,

$$Q \propto A$$

(ii) Amount of heat radiations emitted by the body depends upon the temperature difference between the body and the surroundings. The greater the temperature difference, greater is the heat energy emitted i.e.,

$$Q \propto (\theta_1 - \theta_2)$$

(iii) Amount of heat radiations emitted depends upon time. More the time for which heat is radiated, more is the heat energy emitted by the body *i.e.*,

$$Q \propto t$$

(iv) Amount of heat radiations emitted by a body depends upon the nature of surface of the body.

In other words,
$$Q \propto A$$
$$Q \propto (\theta_1 - \theta_2)$$
$$Q \propto t$$

Combining the three, $Q \propto A\,t\,(\theta_1 - \theta_2)$
$$Q = e\,A\,t\,(\theta_1 - \theta_2)$$

Where e is a constant of proportionality and is called *emissivity or emissive power* of the surface,

If $A = 1\ m^2$, $t = 1$ second, $\theta_1 - \theta_2 = 1\ K$

Then, $Q = e \times 1 \times 1$

$\therefore$ $Q = e$

Hence emissivity of a surface is defined as the amount of heat radiations emitted by 1 m^2 area of the surface in 1 second when the difference of temperature between the surface and the surroundings is 1 K.

TOH—13. ABSORPTION OF RADIATIONS

Whenever radiations fall on a body then,

(i) A part of it is absorbed by the body.

(ii) A part of it is sent back in all directions due to irregular reflection.

(iii) The rest is transmitted.

Let us suppose that Q is the amount of heat radiations falling on a surface of certain area in a certain time. If this surface absorbs Q_1, reflects and scatters Q_2 and transmits Q_3, then

$$Q = Q_1 + Q_2 + Q_3$$

Absorbing Power : Then ratio $\frac{Q_1}{Q}$ is called the absorbing power and is represented by a, *i.e.*, Absorbing power, $a = \frac{Q_1}{Q}$

Hence **absorbing power** *of a surface is defined as the ratio of heat radiations absorbed by the surface of certain area in a certain time to the total radiations falling on the same surface in the same time.* Black bodies have greater absorbing power.

Reflecting power : The ratio $\frac{Q_2}{Q}$ is called reflecting power of the surface and is represented by r, *i.e.*, Reflecting power, $e = \frac{Q_2}{Q}$

Hence **reflecting power** *of a surface is defined as the ratio of heat radiations reflected and scattered by the surface of certain area in a certain time to the total radiations falling on the same surface in the same time.* The reflecting power of a polished surface like mirror is very large.

Transmitting power : The ratio $\frac{Q_3}{Q}$ is called transmitting power of the surface and is represented by t *i.e.*, transmitting power, $t = \frac{Q_3}{Q}$

Hence **transmitting power** *of a surface is the ratio of heat radiations transmitted by the surface of certain area in a certain time to the total radiations falling on the same surface for the same time.*

Now, $$a + r + t = \frac{Q_1}{Q} + \frac{Q_2}{Q} + \frac{Q_3}{Q} = \frac{Q_1 + Q_2 + Q_3}{Q} = \frac{Q}{Q} = 1$$

i.e., $$a + r + t = 0$$

This shows that sum of these three powers of a given surface for given incident radiations is equal to one. Therefore, if a surface has high *absorbing* power, it will have *lesser reflecting* and *transmitting* power. Thus a good absorber will be a bad reflector and vice versa.

TOH—14. BLACK BODY

A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it.

A perfectly black body (surface) neither reflects nor transmits any radiant energy incident on it but absorbs all. Therefore, absorbing power of such a surface is 1 or 100%. Thus the surface appears black for all colours of incident radiations. The chief characteristic of such a body is that when heated to a suitable high temperature it emits total radiations (all possible wavelengths). This radiation is called black body radiation. As black body (surface) is a perfect absorber; it is also a perfect radiator.

There is no perfectly black surface in practice. The nearest approach to a perfectly black surface is a surface coated with lampblack. Such a surface absorbs 96% of incident radiation.

TOH—15. BLACK BODY IN PRACTICE

No surface absorbs all radiations incident upon it. Therefore, there is no perfectly black body (surface) in practice. However, in practice, black body conditions can be closely realised by having a small opening O in a closed double walled hollow sphere (Fig. TOH—18.) There is a conical projection P opposite to the opening which protects direct reflection of any radiation in the opening from the surface opposite to it. The sphere is painted with lamp black from inside. Radiations entering the opening O suffer multiple reflections at the inner walls. After a few reflections almost the entire radiation gets absorbed. At each reflection about 80% of radiant energy is reflected and 20% is absorbed. After two reflections, 64% of original radiant energy is reflected and 36% has been absorbed. Thus after 10 reflections, about 99% of the original incident energy will be absorbed. *Hence the opening behaves like a perfect absorber.*

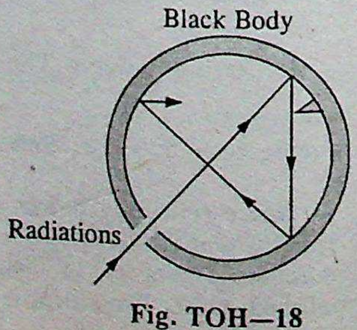


Fig. TOH—18

When such a sphere is heated, the opening becomes a source of thermal radiation *i.e.*, the opening emits radiation. Thus it is the opening which is to be considered as a black body and not the total sphere .

TOH—16. KIRCHHOFF'S LAW OF HEAT RADIATION

According to this law, *the ratio of the emissive power to the absorptive power for radiations corresponding to a certain wavelength is constant for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.*

Mathematically, $$\frac{e\lambda}{a\lambda} = \text{constant} = E\lambda$$

Where, $e\lambda$ = emissive power of the surface corresponding to wave length λ

$a\lambda$ = absorptive power of the surface corresponding to wave length λ

and E = emissive power of a perfectly black body corresponding to wavelength λ.

Thus according to Kirchoff's law, it is clear that if $e\lambda$ is large, $a\lambda$ must also be large. Therefore, if a body emits strongly the radiations of certain wavelength, it must also absorb the same radiation strongly. Thus good radiators are good absorbers.

Proof. Consider a body placed in a uniformly heated enclosure at a constant temperature, T.

Let dQ be the amount of heat energy, lying between wavelength λ and $\lambda + d\lambda$, incident per unit area of the surface of the body per unit time. If $a\lambda$ is the absorptive power of the surface of the body corresponding to wavelength λ, then heat energy absorbed by the body per unit area per unit time = $a\lambda dQ$.

The energy which is not absorbed *i.e.*, the remaining energy is either reflected or transmitted.

$\therefore$ Heat energy reflected or transmitted per unit area per unit time $= dQ - a\lambda\, dQ$

$$= (1 - a\lambda)\, dQ$$

If $e\lambda$ is the emissive power of the body corresponding to wave length λ, then heat energy radiated by the body per unit area per unit time between wavelengths λ and $\lambda + d\lambda = e\lambda d\lambda$

$\therefore$ Sum of the heat energy emitted by the body per unit area per unit time

$$= (1 - a\lambda)\, dQ + e\lambda\, d\lambda.$$

Since the body is in thermal equilibrium with the enclosure, therefore, the sum of heat energy emitted by the body must be equal to the amount of incident radiations.

$$\therefore\ (1 - a\lambda)\, dQ + e\lambda d\lambda = dQ$$

$$\text{or}\quad dQ - a\lambda \cdot dQ + e\lambda d\lambda = dQ$$

$$\text{or}\qquad e\lambda d\lambda = a\lambda d\lambda \qquad \ldots(i)$$

For a perfectly black body, absorbing Power, $a\lambda = 1$.

$$\frac{e\lambda}{a\lambda} = E\lambda \quad \text{or} \quad e\lambda = E\lambda$$

Hence $$E\lambda d\lambda = 1 \times dQ = dQ \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get,

$$\frac{e\lambda}{E\lambda} = a\lambda \quad \text{or} \quad \frac{e\lambda}{a\lambda} = E\lambda \qquad \text{...(iii)}$$

$$\therefore \quad \frac{e\lambda}{a\lambda} = \text{constant for all surfaces.}$$

It is clear from the relation *(iii)* that if $e\lambda$ is larger, then $a\lambda$ should also be correspondingly larger *i.e.*, good radiators are good absorbers.

Experimental verification of Kirchhoff's Law :

Kirchhoff's Law for heat radiations can be verified by Ritchie's apparatus (see fig. TOH—19).

The apparatus consists of two metal cylinders A and B connected to the ends of a glass U tube containing a light, coloured liquid. Another cylinder C of the same diameter as A or B but greater in length is fixed in between A and B in such a way that the three cylinders are co-axial. The cylinder C can rotate about its axis.

The face P_1 of the cylinder A is polished whereas the face B_1 is coated with lamp black. In the same way, face P_2 of cylinder B is polished and face B_2 is coated with lamp black. Also face P of the cylinder C is polished but face B is coated with lamp black.

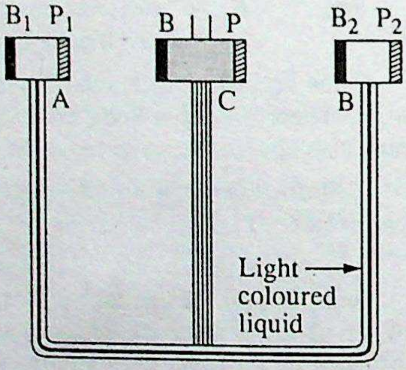


Fig. TOH—19

Now the polished face P of cylinder C is turned towards the coated face B_2 while the coated face B of cylinder C is turned towards the polished face P_1. Boiling water is put in the and cylinder C. It is observed that level of the liquid in U-tube remains unchanged.

It means that amount of heat absorbed by P_1 from B is the same as that absorbed by B_2 from P. *This is possible only if good absorbers are good emitters.*

If the cylinder C is now rotated so that its blackened face B faces the black surface B_2 of B and its polished face P faces the polished surface P_1 of A, then level of liquid under the cylinder B falls and that of liquid under the cylinder A rises. This is possible only if black surface is good radiator as well as good absorber whereas polished surface is bad radiator and bad absorber.

Thus good radiators are good absorbers. *This verifies Kirrchhoff's law.*

TOH—17. STEFAN'S LAW OF HEAT RADIATION

Stefan's law states that the total amount of heat energy radiated by a perfectly black body per second per unit area is directly proportional to the fourth power of its absolute temperature.

This law is also known as Stefan's fourth power law.

If E is the *total energy radiated per second per unit area of the surface of a black body* at absolute temperature T, then

$$E \propto T^4$$

or

$$E = \sigma T^4$$

Where σ is a constant of proportionality and is called Stefan's constant. Its value in *SI* units is $5 \cdot 67 \times 10^{-8}\ J\,s^{-1}\,m^{-2}\,K^{-4}$ or $5 \cdot 67 \times 10^{-8}$ watt $m^{-2}\,K^{-4}$.

Stefan's law does not refer to the net loss of heat by a body but it simply deals with the amount of heat energy radiated by the body by virtue of its temperature, irrespective of what it gains from the surroundings.

The law can be extended to represent the net loss of heat energy by a body after exchange with the surroundings.

Let there be a black body A at absolute temperature T_1 surrounded by another black body B at absolute temperature T_2.

∴ Rate of loss of heat energy by body A per unit area, $E_1 = \sigma T_1^4$

Rate of gain of heat energy by body A from body B

per unit area, $E_2 = \sigma T_2^4$

∴ Net loss of heat energy per unit area per second from body A,

$$E = E_1 - E_2$$

or

$$E = \sigma T_1^4 - \sigma T_2^4 = \sigma (T_1^4 - T_2^4)$$

If the body is not a perfectly black body, then

$$E = e\,\sigma (T_1^4 - T_2^4)$$

Where e is called the emissivity of the surface. The value of e depends upon the nature of the surface.

Ex. TOH—10. Calculate the rate of amount of heat energy radiated from a body at 400 K, given Stefan's constant equal to $56 \cdot 72 \times 10^{-9}$ J/s/m²/K⁴.

Sol. Here,

Temperature of the body $= 400\ K$.

Stefan's constant, $\sigma = 56 \cdot 72 \times 10^{-9}\ J/s/m^2/K^4$

The rate of heat energy radiated is given as,

$$E = \sigma T^4$$

∴

$$E = 56 \cdot 72 \times 10^{-9} \times (400)^4\ J/s/m^2$$

$$= \mathbf{1452 \cdot 032\ J/s/m^2.}\ \textbf{Ans.}$$

Ex. TOH—11. If the Sun's surface radiates heat at the rate of $6 \cdot 3 \times 10^7$ watt/m², Calculate the temperature of the Sun's surface assuming it to be a black body. (Take $\sigma = 5 \cdot 7 \times 10^{-8}\ W\,m^{-2}K^{-4}$

Ans. Here, Rate of heat energy radiated, $E = 6 \cdot 3 \times 10^7\ W\,m^{-2}$

Stefan's constant, $\sigma = 5 \cdot 7 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$

Let T be the temperature of the Sun.

Using Stefan's Law, we have

$$E = \sigma T^4$$

$$\therefore \quad 6\cdot 3 \times 01^7\, W m^{-2} = 5\cdot 7 \times 10^{-8}\, W m^{-2} K^{-4} \times T^4$$

or $$T^4 = \frac{6\cdot 3 \times 10^7}{5\cdot 7 \times 10^{-8}} K^4 = 0\cdot 11052 \times 10^{16} K^4$$

or $$T = 5700\, K \text{ Ans.}$$

Ex. TOH-12. A black body is radiating heat at the rate of $7\cdot 8 \times 10^8$ W/m², Calculate the temp. of the black body. Given $\sigma = 5\cdot 7 \times 10^{-8}$ Wm⁻² K⁻⁴.

Sol. Given, $$E = 7\cdot 8 \times 10^8\, W/m^2$$

$$\sigma = 5\cdot 7 \times 10^{-8}\, Wm^{-2} k^{-4}$$

We know that, $$E = \sigma T^4$$

$$\Rightarrow \quad T^4 = \frac{E}{\sigma} \quad \text{or} \quad T = \left(\frac{E}{\sigma}\right)^{\frac{1}{4}}$$

$$\therefore \quad T = \left(\frac{7\cdot 8 \times 10^8}{5\cdot 7 \times 10^{-8}}\right)^{\frac{1}{4}} = \left(\frac{7\cdot 8}{5\cdot 7}\right)^{\frac{1}{4}} \times (10^{16})^{\frac{1}{4}}$$

$$= 10815\cdot 7\, K \text{ Ans.}$$

Ex TOH-13. Calculate the energy radiated per second from the filament of an incandescent lamp at 2000 K if the surface area is $5\cdot 0 \times 10^{-5}$ m² and its relative emittance is 0·85. $\sigma = 5\cdot 7 \times 10^{-8}$ Wm⁻² K⁴

Sol. $$E = Ae\,\sigma T^4$$

Where, $$A = 5\cdot 0 \times 10^{-5}\, m^2, \qquad \sigma = 5\cdot 7 \times 10^{-8}\, Wm^{-2} K^{-4}$$

$$e = 0\cdot 85, \quad T = 2000\, K$$

$$\therefore \quad E = 5\cdot 0 \times 10^{-5} \times 0\cdot 85 \times 5\cdot 7 \times 10^{-8} \times (2000)^4$$

$$E = 38\cdot 76\, W = 38\cdot 76\, J/s \text{ Ans.}$$

Ex TOH-14. Calculate the temperature at which a perfect black body radiate at the rate of 5·67 W cm⁻². Stefan's constant is $5\cdot 67 \times 10^{-8}$ Js⁻¹ m⁻² K⁻⁴

Sol. $$E = 5\cdot 67\, W cm^{-2} = 5\cdot 67 \times 10^4\, W/m^2$$

$$\sigma = 5\cdot 67 \times 10^{-8}\, Js^{-1} m^{-2} K^{-4}$$

Using, $$E = \sigma T^4 \quad \Rightarrow \quad T^4 = \frac{E}{\sigma}$$

or $$T = \left(\frac{E}{\sigma}\right)^{\frac{1}{4}} = \left(\frac{5\cdot 67 \times 10^{-8}}{5\cdot 67 \times 10^4}\right)^{\frac{1}{4}}$$

or $$T = (10^{12})^{\frac{1}{4}} = 10^3 = 1000\ K$$

Hence, $T = 1000\ K$ **Ans.**

Ex TOH-15. Calculate the radiant emittance of a black body at temperature *(i)* 400 K *(ii)* 4000 K. Given $\sigma = 5{\cdot}672 \times 10^{-8}\ Js^{-1}\ m^{-2}K^{-4}$

Sol. According to stefan's law, the radiant emittance of a black body in the amount of heat energy radiated persecond per sq. cm is given by,

$$E = \sigma T^4$$

(i) When $T = 400\ K$

$$E = \sigma T^4 = 5{\cdot}672 \times 10^{-8} \times (400)^4$$

$$E = 5{\cdot}672 \times 10^{-8} \times (4)^4 \times (10)^8$$

$\therefore$ $E = 1452\ J/s$ **Ans.**

(ii) When $T = 4000\ K$

$\therefore$ $$E = \sigma T^4 = 5{\cdot}672 \times 10^{-8} \times (4000)^4$$

$$= 5{\cdot}672 \times 10^{-8} \times (4)^4 \times 10^{12}$$

$\therefore$ $E = 1{\cdot}452 \times 10^7\ J/s$ **Ans.**

TOH—18. WIEN'S LAW

According to Wien's law, *the wavelength corresponding to the maximum heat energy radiated is inversely proportional to the absolute temperature of the body.*

If λ_m is the wavelength corresponding to maximum emission at the absolute temperature T, then

$$\lambda_m \propto \frac{1}{T} \qquad \text{or} \qquad \lambda_m T = b$$

Where b is a constant and is called Wien's constant. Its value in *SI* units for a perfectly black body is $2{\cdot}888 \times 10^{-3}\ m\ K$.

Thus $\lambda_m T = \text{constant}$

This is known as Wien's displacement law. It shows that as the temperature increases the maximum intensity of radiation emitted shifts towards shorter wavelengths hence known as *Wien's displacement law.*

Ex. TOH—16. Calculate the surface temperature of the sun and moon, given that λ_m = 4753 Å and 140000 Å respectively, λ_m being wavelength of maximum intensity of emission.

Sol. λ_m for Sun $= 4753\ A° = 4753 \times 10^{-10}\ m$

Let T_s be the temperature of Sun,

$\therefore$ $$\lambda_m T_s = b$$

or $$4753 \times 10^{-10}\ m \times T_s = 2{\cdot}888 \times 10^{-3}\ m\ K$$

or $$T_{sun} = \frac{2\cdot 888 \times 10^{-3}}{4753 \times 10^{-10}} K = 6076 K$$

For moon, $\lambda_m = 140000\ A° = 140000 \times 10^{-10}\ m$

$$\therefore \quad T_{moon} = \frac{2 \cdot 888 \times 10^{-3}}{140000 \times 10^{-10}} = 206 \cdot 3\ \text{K} \quad \textbf{Ans.}$$

Ex TOH-17. Find the surface temperature of a star for which $\lambda_m = 8000\ A°$. Wein's constant, $b = 2{\cdot}888 \times 10^{-3}\ mK$.

Sol. Given, $\lambda_m = 8000\ A° = 8000 \times 10^{-10}\ m = 8 \times 10^{-7}\ m$

$b = 2{\cdot}888 \times 10^{-3}\ mK.$

We know that, $$\lambda_m = \frac{b}{T} \Rightarrow T = \frac{b}{\lambda_m}$$

or $$T = \frac{2 \cdot 888 \times 10^{-3}}{8 \times 10^{-7}} = \frac{2 \cdot 888}{8} \times 10^4$$

$$\therefore \quad T = 3610\ K \ \textbf{Ans.}$$

Ex TOH-18. Work out the range of temperature corresponding to which a star will appear blue and red respectively. Given,

Wavelength range for blue colour = 4500 A° to 5000 A°

Wavelength range for red colour = 6500 A° to 7000 A°

Sol. According to wein's displacement law,

$$\lambda_m T = b \qquad \text{Also} \quad 1\ A° = 10^{-10}\ m$$

$$\therefore \quad T = \frac{b}{\lambda_m}, \quad b = 2{\cdot}897 \times 10^{-3}\ \text{mK}.$$

For blue Colour

$$T_1 = \frac{b}{\lambda_{m_1}} = \frac{2 \cdot 897 \times 10^{-3}}{4500 \times 10^{-10}} = 6438\ K$$

$$T_2 = \frac{b}{\lambda_{m_2}} = \frac{2 \cdot 897 \times 10^{-3}}{5000 \times 10^{-10}} = 5794\ K$$

$\therefore$ Star will appear blue in the temperature range 5794 K to 6438 K.

For red colour, $$T_1 = \frac{b}{\lambda_{m_1}} = \frac{2 \cdot 897 \times 10^{-3}}{6500 \times 10^{-10}} = 4457\ K$$

$$T_2 = \frac{b}{\lambda_{m_2}} = \frac{2 \cdot 897 \times 10^{-3}}{7000 \times 10^{-10}} = 4139\ K$$

$\therefore$ Star will appear red in the temperature range of 4139 K to 4457 K.

Ex TOH-19. The surface of hot body is 727°C. Find the wave length at which it radiates maximum energy. Wein's constant = 2884×10^{-6} mK.

Sol. Given temp. $= 727°C = 273 + 727 = 1000\ K$

$$\lambda_m = ?, \quad b = 2884 \times 10^{-6}\ mK.$$

We know that, $\lambda_m T = b$

$$\therefore \quad \lambda_m = \frac{b}{T} = \frac{2884 \times 10^{-6}}{1000}$$

$$\lambda_m = 2884 \times 10^{-9}\ m$$

$$\lambda_m = 28840\ A° \text{ Ans.}$$

TOH—19. ENERGY DISTRIBUTION IN SPECTRUM OF A BLACK BODY

The intensity of radiation emitted by a black body is not uniformly distributed over the whole range of wavelength involved. Thus we have to see how the energy of total radiation from black body is distributed among the different wavelengths at various temperature, in other words we have to find how $E\lambda$ for a black body depends on λ and T.

It becomes clear from the fact that when the temperature of a body is steadily raised, the colour of the radiation emitted by it changes. For instance when a blackened platinum wire is raised in temperature by electrical heating, it begins to appear dull-red at about $525°C$, changes to cherry-red at about $900°C$, becomes orange-red at $1100°C$, yellow at $1250°C$ and finally white at about $1600°C$. This shows that the colour emitted becomes richer in waves of shorter wavelengths, in other words, the *maximum intensity of emission shifts towards the shorter wavelength side as the temperature is raised.*

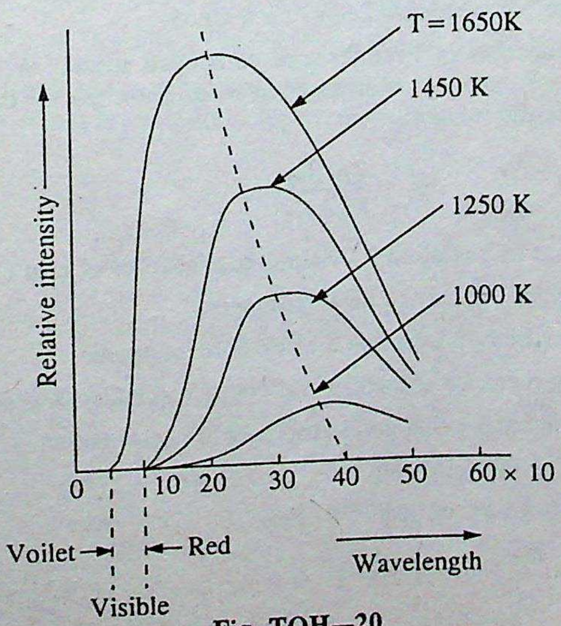


Fig. TOH—20

Graphical representation of black Spectrum. The black body radiation was produced by electrically heating a carbon tube with a small opening. The thermal spectrum was obtained by refraction through a fluorspar prism. A bolometer was used to measure the radiant energy corresponding to small ranges of wavelength along the spectrum.

The experiment was repeated for different temperature of the black body between 1000 K and 1650 K. The results obtained were represented graphically as shown in fig. TOH—20. The amount of energy emitted in a narrow wavelength range was plotted against the mean wavelength of each range. The different curves were obtained for different temperatures at which the black body was maintained. The total energy curve corresponding to that temperature and the horizontal axis. The area is found to increase according to the fourth power of the absolute temperature, which verifies Stefan's law.

The wavelength corresponding to maximum energy represented by the peak of the curve *shifts to shorter wavelength side as the temperature increases,* which confirms Wien's displacement law *i.e.*, $\lambda_m T$ = constant.

QUESTIONS FOR SELF TEST

1. Define conduction of heat and explain it on the basis of molecular theory.
2. Discuss the factors on which heat conducted by a material depends and hence define co-efficient of thermal conductivity.
3. Describe determination of co-eff. of thermal conductivity :

   (i) For good conductor by Searle's method.

   (ii) For poor conductor by Lee's disc method.
4. Discuss conduction of heat through compound media. When two materials of conductivities K_1 and K_2 are arranged in series, show that the equivalent conductivity K is given by the relation,

$$K = \frac{2 K_1 K_2}{K_1 + K_2}$$

5. What is meant by radiation ? Discuss the properties of heat radiation.
6. Discuss Prevost's theory of heat exchanges.
7. What is black body ? Describe a black body in practice.
8. What do you mean by absorbing, reflecting and emissive power of a surface ?
9. Show experimentally that good absorbers are good radiators.
10. Write notes on the following :-

    (i) Kirchhoff's Law of heat radiation.

    (ii) Stefan's law.

    (iii) Wien's Law.
11. Explain with a diagram the energy distribution in a black body spectrum.

PROBLEMS

1. Glass windows of a room have an area of 10 sq m and glass is 4·1 mm thick. Calculate the rate of loss of heat from the room by conduction when the inside temperature is 298 K and outside temperature is 283 K.

   (K for glass = 15·4 W m^{-1} k^{-1}). **(Ans.** 1·786 × 10^3J)

2. Calculate the rate at which ice will melt in a wooden box 2 cm thick, having inside dimensions (100 × 60 × 40 cm). Outside temperature is 20°C and conductivity of wood is 0·04 cal/cm/s/°C. **(Ans.** 120 g/s)

3. A compound plate is made of two different materials placed parallel to each other having conductivities 0·32 and 0·14, their thicknesses are 3·6 and 4·2 cm respectively. If the two faces are maintained at 96°C and 8°C, find the temperature of the interface. **(Ans.** 72°C)

4. A metal pipe having an external diameter of 15 cm carries steam at 200°C. This is covered by a layer of 2·5 cm thick of an insulating material of K = 0·0005. If the outer surface is at 100°C. Calculate the heat lost per meter length per minute. **(Ans.** 6558 cal.)

5. A small hole is made in a hollow sphere whose walls are at 723°C. Find the total energy radiated per second per sq cm. Given Stefan's constant = 5·7 × 10^{-5} erg cm^{-2} s^{-1} K^{-4} **(Ans.** 5·61 × 10^7 ergs)

6. At what temperature will the filament of a 100 W lamp operate if it is supposed to be perfectly black body of area 1 sq cm ?

   Given : $\sigma = 5{\cdot}67 \times 10^{-5}$ erg/cm^2/s/K^4. **(Ans.** 2049 K)

7. Calculate the radiant emittance of a black body at a temperature of 400 K.

   Given : $\sigma = 5{\cdot}672 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}. **(Ans.** 1·452 × 10^7 J s^{-1} m^{-2})

8. Two stars radiate maximum energy at wave lengths 3·6 × 10^{-7} m and 4·8 × 10^{-7} m respectively. What is the ratio of their temperatures. **(Ans.** 4 : 3)

EXPERIMENTAL PHYSICS

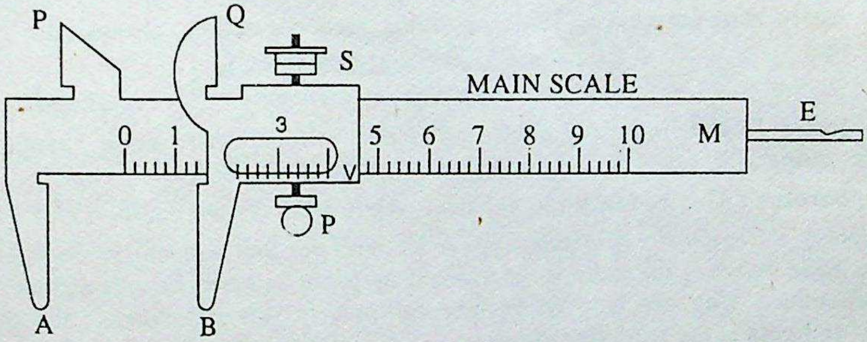
Experiment No—1

L.H.S. *Dated..........*

Object : *To determine the density of a thin cylinder/wire using balance, and vernier callipers.*

Apparatus : Thin rod or wire, balance, weight box, vernier callipers etc.

Diagram :



Vernier Calliper

Observations :

Vernier constant = ...*cm*

Zero error = ...*cm*

Zero correction = – (zero error) = ± ....*cm*

Measurement of mass :

Mass of the cylinder, m = ... gm = ... kg.

S.No.		Reading of the main scale before zero of the vernier scale *(x) cms*	Number of vernier divisions coinciding with main scale divisions *(n)*	Fraction to be added *(y = n V.C)*	Observed value in cm *(x + y)*	Corrected value = observed value + zero corrections	Mean value in cm
1. 2.	*Length*						
3. 4.	*Diameter*						

Calculations : Length of cylinder, l = ...cm = ...m

Diameter of cylinder, d = ...cm = ...m

Volume of the cylinder, $V = \frac{\pi d^2}{4} l$ = ...m^3

$$\text{Density of the cylinder} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V}$$

$$= \frac{m}{\pi \frac{d^2}{4} l} = \frac{4m}{\pi d^2 l} = \text{----------- kg}/m^3$$

Precautions : 1. Do not press the jaws against the rod or wire too hard but make them simply touch the rod gently.

2. Apply zero correction. While applying zero correction, always add it with proper sign.

R.H.S. *Dated..........*

Object : *To determine the density of a thin cylinder wire, using balance and vernier callipers.*

Apparatus : Thin rod or wire, balance, weight box, vernier callipers.

Theory : *The density of a substance is its mass per unit volume.* In the case of a regular solid or wire, the mass is determined by using balance. Length and diameter are determined with the help of vernier calliper. In this experiment, the use of vernier callipers is the main operations.

Use of vernier callipers : The use of varnier calliper consists of the following operations :-

(i) Finding the magnitude of the smallest division on the main scale.

(ii) Finding the vernier constant.

To find the vernier constant, count the total number of divisions on the vernier scale. Slide the movable jaw so that the zero of the vernier scale coincides with the zero (or any division) of the main scale. Now count the number of main scale divisions which coincide with vernier scale divisions. Let us suppose that,

1 Main scale division = 1 mm

and 10 V.D. = 9 S.D.

$$\therefore \quad 1\text{ V.D.} = \frac{9}{10}\text{S.D.} = \frac{9}{10}\text{ mm}$$

Now *vernier constant is the difference between 1 S.D. and 1 V.D.*

$$\therefore \text{Vernier constant (V.C.)} = 1\,mm - \frac{9}{10}\text{ mm}$$

$$\because \quad 1\text{ S.D.} = 1\text{ mm and } \frac{9}{10}\text{S.D.} = \frac{9}{10}\text{ mm}$$

$$= \frac{1}{10}\text{ mm} = 0\cdot1\text{ mm} = 0\cdot01\text{ cm.}$$

Procedure : 1. Find the vernier constant and zero error of the vernier callipers. Take the cylinder lengthwise in between the jaws and note the main scale reading. Find the number of vernier divisions coinciding with the main scale, multiply this by V.C. and get the fraction to be added. The sum of the main scale reading and vernier scale reading gives the observed length of the cylinder. Get the correct length by adding zero correction with proper sign. Repeat the same operation by reversing the rod and take the mean of at least three readings.

2. Similarly, finding the diameter of the cylinder by taking at least three readings at different places of the cylinder with vernier calliper.

3. Find the mass of the rod by using balance.

4. Calculate the volume of the rod from its diameter and length. Divide the mass of the rod by its volume. This gives the density.

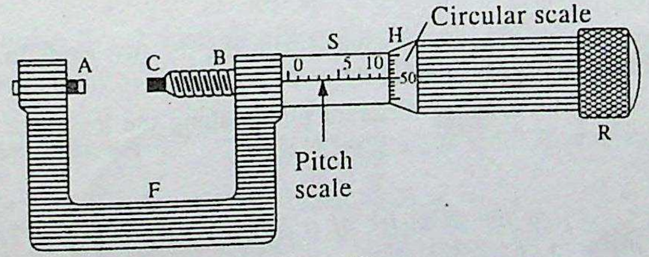
Experiment No—2

L.H.S. *Date..........*

Experiment : *To find the diameter of a cylinder wire using screw gauge and hence determine the area of cross-section.*

Apparatus : Screw gauge, metallic cylinder/wire.

Diagram : Screw Gauge



Screw Gauge

Observations :

(b) **Measurement of diameter with screw gauge :**

Distance travelled by the screw in 4 complete rotations :.......... mm

Pitch of the Screw $= \frac{\text{Distance moved}}{\text{no. of rotations}}$ .... mm

No. of division on the circular scale = ...

Least count $= \frac{\text{Pitch}}{\text{No. of divisions of circular scale}}$ = ... mm.

Zero error (1) ± .... (2) ± .... (3) ± ....

Mean zero correction = ± ....

Table for diameter

S. No.	Linear scale reading (a) mm	No. of C.D. coinciding with index line (n)	Fraction to be added $n \times$ L.C.	Observed diameter $= a + n \times$ L.C. 'd' – do –	Corrected diameter D = 'd' + zero correction
1.					
2.					
3.					
4.					

Mean correct diameter (mm) = ...cm.

or D = ...metre

Calculations :

$$\text{Radius of the cylinder/wire } = r = \frac{D}{2} = \ldots\ldots\ldots\ldots m$$

$$\text{Area of cross-section } = \pi r^2 = \frac{\pi D^2}{4} = \ldots\ldots\ldots\ldots m^2$$

Precautions :

1. Undue pressure should not be applied to bring the two studs in contact. For this, the screw should always be rotated by racket cap.

2. Reading should be taken at different places along the length of wire, taking two at right angles to each other at the same place.

R.H.S.

Date.........

Experiment : *To find the diameter of a cylinder/wire using screw gauge and hence determine the area of cross-section.*

Procedure :

1. Determine the pitch of the screw and the last count as explained before. Bring the two studs of the screw gauge in contact and note the zero error at least three times.

2. To measure the diameter of the rod, turn the screw back and insert the rod between the jaw of the screw gauge. Turn the screw forward till the rod is held gently between the two jaws. Take the reading in this position.

(a) the reading on the pitch scale and *(b)* the reading on the circular scale against the reference line. Multiply the no. of division on the circular scale by the least count and add it to the pitch scale reading or main scale reading *i.e.*, the no. of complete divisions visible on the linear scale. This will give the observed diameter of the wire/rod. Apply zero correction and get the correct diameter. Repeat the observation at least thrice. Now make calculations to get area of cross-section.

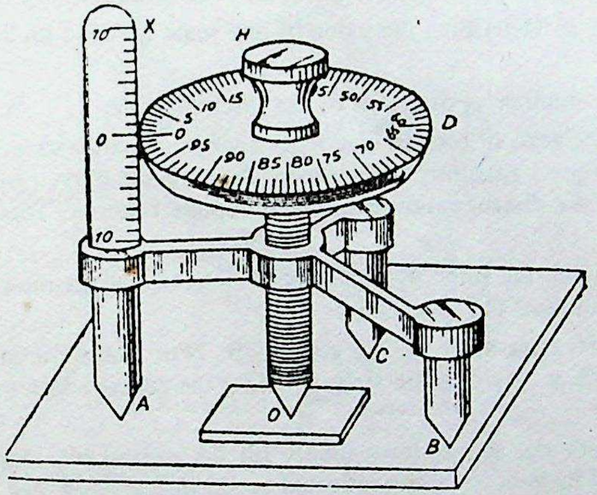
Experiment No—3

L.H.S. Date...........

Experiment : *To determine the thickness of a glass strip by a spherometer.*

Apparatus : Spherometer, glass strip, glass slab and a half metre rod.

Diagram :



Observations : No of rotations given to screw = 4

Distance moved by screw in 4 complete rotations, (d) = ... mm

$$\text{Pitch of the screw} = \frac{d}{\text{no. of rotations}} = \frac{d}{4} = \text{.........}\ \text{mm.}$$

Total number of division on the circular scale, n = ...

$$\text{Least count} = \frac{\text{Pitch}}{n} = \text{.........}\ \text{mm.}$$

Record :

S.No.	Reading on circular scale		Number of complete rotations (n)	Additional circular divisions moved (N)	Thickness $n \times$ pitch + N $\times$ Least Count
	with glass strip	without glass strip			
1.					
2.					
3.					

Mean thickness = ... mm

Precautions : 1. Always rotate the screw in the same direction so as to avoid error due to back-lash of the screw.

2. The screw should be rotated gradually till the central leg just touches the surface of the glass strip. At that time the spherometer just begins to rotate about the central leg.

3. Atleast three readings should be taken at different places of glass strip.

R.H.S.

Experiment : *To determine the thickness of the glass strip by a spherometer.*

Procedure : *(i)* Determine the value of one scale division on the main scale/pitch scale.

(ii) Note the number of divisions on the circular scale.

(iii) Bring the zero of the circular scale against some division on the main scale. Give it four complete rotations and measure the distance moved by it along the main scale. Calculate the distance moved in one complete rotation. This gives the pitch of the screw.

(iv) Now divide the pitch by the number of divisions on the circular head. This gives the least count of the spherometer.

(v) Place the glass strip on the glass slab. Now place the spherometer on the glass slab in such a way that the strip is below the screw and legs do not touch the strip.

(vi) Now turn the screw down gently till the central leg just touches the glass strip, this would happen when the reflected image of the screw just coincides with the tip of the screw and the instrument just begins to rotate about the central leg. Note the division of the circular scale which is opposite to the edge of the main scale. Mark this with pencil and treat it as reference mark.

(vii) Now remove the glass strip and turn the screw downward and go on counting the number of complete rotations till it touches the glass slab. When the stage is reached note the number of circular division coinciding with the egde of the main scale.

(viii) Repeat the expt. three times and make the calculations to get the thickness of the glass strip.

Experiment No—4

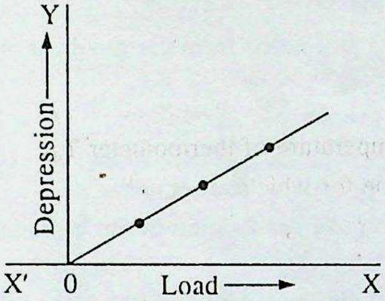
L.H.S.

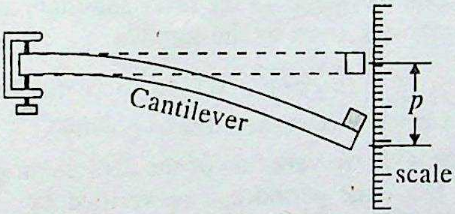
Date.........

Experiment : *Calculation and verification of period of vibrations of a cantilever (use graph).*

Apparatus : Cantilever (*i.e.*, beam fixed horizontally at one end and loaded at the other), stop watch, weight box, metre rod, clamp.

Diagram :





Recording of Time period :

No. of observations	load W in (gm)	Depression P in cm.
1.	20 gm	
2.	40 gm	
3.	60 gm	
4.	80 gm	
5.	120 gm	
6.	140 gm	

No. of observations	Load W	Length of canti lever l = cm.	No. of vibs.	Time taken for 20 vib. '*s*'
1.	100 gm	Length of the cantilever must remain the same	20	
2.	100 gm		20	
3.	100 gm		20	

Mean T = ... S

$\therefore$ Time for one vibration $= \dfrac{T}{20} = \ldots\ldots\ldots\; s$

Verification of time period :

Depression for load (100 gm) noted from the graph, $\rho = \ldots\ldots\ldots$ cm

$$\text{Time period, } t = 2\pi\sqrt{\frac{\rho}{g}} = \ldots\ldots\ldots\; s$$

R.H.S. *Date*..........

Experiment : Calculation and verification of the vibration of a cantilever. (graph method).

Theory : A cantilever is simply a metallic rod of about 1 metre length fixed at one end and free at the other end. If the free end of the cantilever is loaded, a depression is produced upon the length of the lever and the load released, it begins to vibrate. The time of vibration is given by the formula.

$t = 2\pi\sqrt{\rho / g}$ Where, ρ is the depression produced by the suspended load and g is the acceleration due to gravity.

So time period varies with the variation of the depression *i.e.*, with the variation of the load suspended. This time period can be verified by actual measurement by finding the time period for 20 vibrations with the help of stop watch and calculating that for one vibration.

Procedure : 1. Fix firmly one end of a long beam at the of a laboratory table with the help of clamp.

2. Place weights of 20, 40, 60, 80, 120, and 140 gms in turn on the free end of the beam and note the depressions in case with the help of metre rod.

3. Plot a graph between the load W and depression P by selecting a suitable scale.

4. Now place a wt. of, say 100 gm at the end a beam and let it vibrate. Note the time taken for 20 vibrations. Calculate the time period. Repeat the step thrice. Find the mean time period.

5. For verifying the time period use the relation,

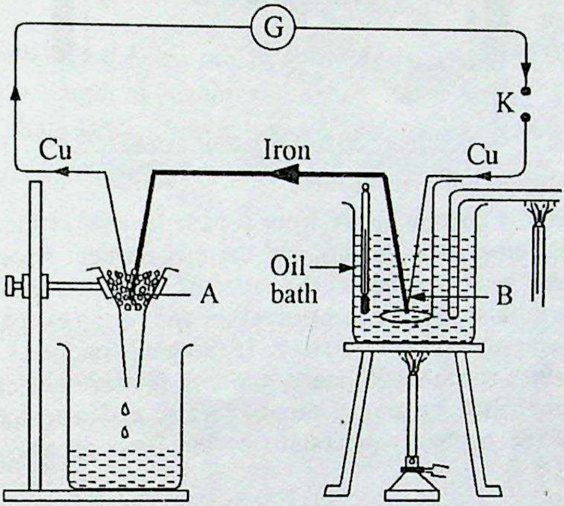
$t = 2\pi\sqrt{\dfrac{\text{depression}}{g}}$ by noting the value of depression from graph for a load of 100 gm.

Experiment No—5

L.H.S.

Experiment : *Callibration of a thermocouple or To plot a graph between temperature of the hot junction and the deflection in an American type galvanometer for a given thermocouple.*

Apparatus : A thermo couple, a clamp stand, an oil bath, a heating arrangement, a funnel stand, a beaker, ice, a thermometer of range 400°C, American type galvanometer, on way key etc.



Observations :

Temperature of cold junction = –°C

S. No.	Temperature of hot Junction	Deflection	S. No.	Temperature of hot Junction	Deflection
1.					
2.					
3.					
4.					
5.					

Precautions : 1. The two wires of the thermocouple should only touch at the junctions and nowhere else.

2. The thermocouple should not touch the bottom or the sides of the vessel.

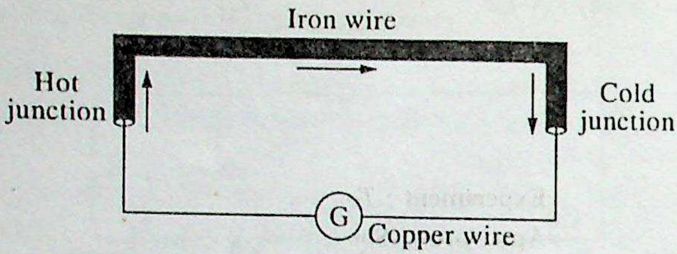
3. The temperature of the hot junction should not be raised beyond the neutral temperature.

R.H.S.

Experiment : *Callibration of a thermocouple or To plot a graph between temperature of the hot junction and the deflection in an American type galvanometer for a given thermocouple.*

Theory :

Thermocouple : Connect two copper wires to a millivoltmeter, a galvanometer and fuse the ends of the copper wires to the ends of a thick iron wire. Heat one of the iron copper junction with a Bunsen burner or spirit lamp keeping the other end at the room temperature, the galvanometer will show deflection showing that there is an e.m.f.



It is found that the current flows from copper to iron at the hot junction. This current is called thermoelectric current and the arrangement is called thermocouple. The e.m.f. generated is very small of the order of a micro volts. If the temperature of the hot junction is raised, the e.m.f. increases and becomes maximum at a certain temperature called the neutral temperature. If the temperature of the hot junction is increased beyond the neutral temperature, the e.m.f. decreases and finally becomes zero at a temperature called the temperature of inversion. The temperature of inversion is as much above the neutral temperature as the latter is above that of the cold junction.

A thermocouple is generally used to measure temperature. For this purpose a graph is plotted between temperature of hot junction and deflection produced in a moving coil galvanometer. The deflection is proportional to the current which in turn is proportional to the e.m.f.

Procedure : *(i)* Draw a diagram showing the scheme of connections as in fig.

(ii) Clean the ends of the wires of the thermocouple. Clamp the thermocouple in a stand. Dip one junction in an oil bath without touching the vessel containing the oil and immerse the other junction in powdered ice placed in a funnel held in a stand with a beaker placed below.

(iii) Connect the two ends of the thermocouple through a one way key to the two terminals of the galvanometer. Suspend a thermometer of range 400°C in the oil hath and also place a stirrer in it.

(iv) Heat the oil hath to a temperature of about 200°C or more till the galvanometer shows full scale maximum deflection.

Maximum deflection can be checked by putting the plug in the key K and noting the deflection then.

When the galvanometer shows maximum deflection a full scale deflection, note the deflection of the galvanometer and also the temperature of the thermometer.

(v) Now remove the burner and allow the temperature to fall. Go on noting the deflection at an interval of about 10°C fall.

(vi) Plot a graph between the temperature of the hot junction and the deflection in the galvanometer.

Note : *(i)* If water bath is used, heat till the water begins to boil.

(ii) If the cold junction instead of being placed in ice is maintained at any other temperature, then the graph is plotted between the deflection and the difference of temperature between the two junctions.

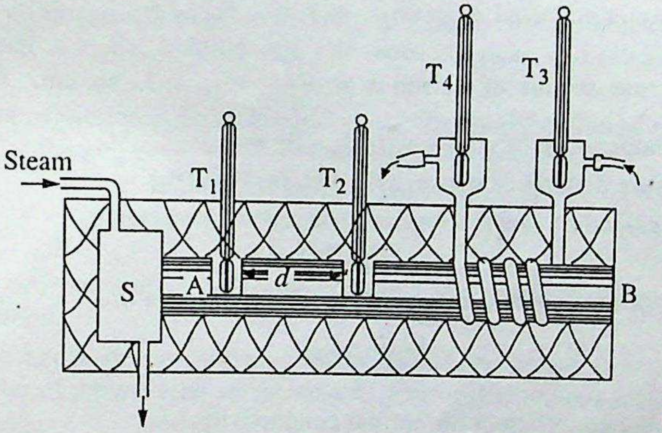
Experiment No—6

L.H.S. Dated..........

Experiment : *To determine the co-efficient of thermal conductivity of a given metal rod by Searle's method.*

Apparatus : *Searle's apparatus, metre rod, steam boiler, two thermometers, stop watch, capillers, constant water flow arrangement, beaker, balance and weight box.*

Diagram :



Observations :

For dia. of the rod,

Vernier constant = ... cm

Zero error = ± .... cm

Zero correction = (– zero error) = ± .... cm

Record : *For diameter :-*

S. No.	Main scale Reading *(i)*	Vernier scale Reading *(ii)*	Observed dia. = *(i)* + *(ii)*	Correct dia. = obs. dia. + zero corrections
1. AB				
CD				
2. AB				
CD				
3. AB				
CD				

Mean correct diameter, D .... cm

Distance between thermometers T_1 and T_2, $d =$ .... cm

Temperature of thermometer T_1, $\theta_1 =$ .... °C

Temperature of thermometer T_2, $\theta_2 =$ .... °C

Temperature of thermometer T_3, $\theta_3 =$ .... °C

Temperature of thermometer T_4, $\theta_4 =$ .... °C

Time for which water collected, $t =$ .... seconds

Mass of empty beaker, $m_1 =$ .... g

Mass of beaker + water collected, $m_2 =$ .... g

Calculations :

Mass of water collected, $m = (m_2 - m_1) =$ .... g

Area of cross-section of the rod $A = \frac{\pi D^2}{4} =$ .... sq. cm.

Rise in temperature of cold water, $\theta_4 - \theta_3 =$ .... °C

Temperature difference between T_1 and T_2, $(\theta_1 - \theta_2) =$ .... °C

Heat gained by water = mass of water × rise in temp.

$= m\,(\theta_4 - \theta_3)$ cals.

Quantity of heat conduced from T_1 to T_2 in t seconds *i.e.*

$$Q = \frac{kA(\theta_1 - \theta_2)t}{d} \text{ cals.}$$

Let k be the co-efficient of thermal conductivity.

Since Heat conducted = Heat gained by water

$$\therefore \quad \frac{kA(\theta_1 - \theta_2)t}{d} = m\,(\theta_4 - \theta_3)$$

or

$$k = \frac{m(\theta_4 - \theta_3)d}{A(\theta_1 - \theta_2)t} = \text{.......... cal/second/sq. cm/°C}$$

Result : Co-efficient of thermal conductivity of the given metal

= .... cal/second/sq. cm/°C

= .... cal/s/m^2/°C

Precautions : 1. Flow of water should be from a constant flow arrangement.

2. The rate of flow of water should be low but continuous.

3. The rod should attain a steady state.

4. Diameter should be measured at a number of places twice at right angles to each other.

R.H.S.

Dated..........

Object : *To determine the co-efficient of thermal conductivity of a given metal rod by Searle's method.*

Theory : For detailed theory, see chapter on transfer of heat.

Procedure : 1. Fill 3/4th of the steam boiler with water and place it on the burner for heating.

2. Take the rod out and measure the distance between T_1 and T_2 with a metre rod. Measure the diameter of the rod at a number of places twice at right angles to each other.

3. Set the apparatus and insert the thermometers T_1 and T_2 in position. Also insert the thermometer T_3 and T_4 in funnels provided to note the temperatures of incoming and outgoing water. Surround the rod with some insulating material and enclose it in a wooden box.

4. Pass steam through the steam chamber. Start circulating water through the copper spiral. Go on noting the temperatures on all the four thermometers till the temperature become constant. Thus at this stage steady state is reached.

5. Weigh a clean and dry beaker. Collect out coming water for a known time in a beaker and weigh it again. Find the mass of water collected and calculate the value of k as on L.H.S.

Applications : Determination of thermal conductivities of different metals.

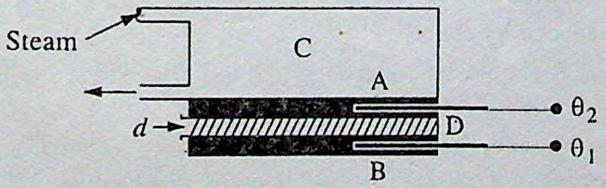
Experiment No—7

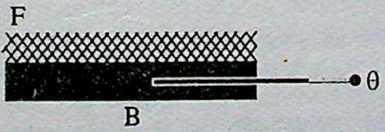
L.H.S. *Dated.........*

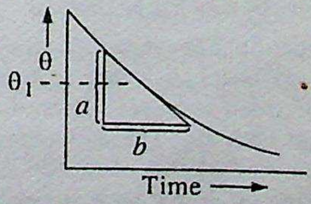
Object : *To determine the co-efficient of thermal conductivity of poor conductors by Lee's disc method.*

Apparatus : Lee's disc apparatus, two thermometers, steam generator, flannel or felt and the disc of the material whose co-efficient of thermal conductivity is to be determined.

Diagram :







Observations :

Area of the disc, D = ... sq. cm

Thickness of the disc, A = ... cm

At steady state

Temperature of disc A, θ_2 = ... °C

Temperature of disc B, θ_1 = ... °C

Rate of fall of temperature $= \left(\frac{a}{b}\right)$ °C = –°C (*from graph*)

Mass of the brass disc B, $m = \ldots\ldots\ldots\; g$

Specific heat capacity of brass disc B, $c = \ldots\ldots\ldots$

Calculations :

Let k be the co-efficient of thermal conductivity of D

∴ Rate of flow of heat through the disc D

$$= \frac{kA(\theta_2 - \theta_1)}{d} \text{ cal./second}$$

Rate of cooling of disc, B $= m \times c \times \frac{a}{b}$ cal./second (*rate of loss of heat*).

$$\therefore \quad \frac{kA(\theta_2 - \theta_1)}{d} = mc\frac{a}{b}$$

$$\text{or} \quad k = \frac{mcad}{A(\theta_2 - \theta_1)b}$$

$$= \ldots\ldots\ldots \text{ cal/g°C/second}$$

$$= \ldots\ldots\ldots\; k \text{ cal/kg/°C}/s.$$

Result : The Co-efficient of thermal conductivity of poor conductor = k cal/kg°C/s.

Precautions : 1. Disc should be of large area and poor thickness.

2. There should be good thermal contact between AD and BD.

3. Disc B should be surrounded by air.

4. While cooling the disc B should be covered at the top with some insulating material.

R.H.S. *Dated.........*

Experiment : *To determine the co-efficient of thermal conductivity of poor conductors by Lee's disc method.*

Theory : The material whose co-efficient of thermal conductivity is to be determined is taken in the form of a disc of large area and small thickness. It is placed between two brass discs of same area, one in contact with steam and the other in contact with air. The temperatures at the two ends of the disc are noted at steady state. The rate of loss of heat at steady state is determined from the lower brass disc by plotting a graph between time and temperature. The rate of loss of heat is compared with the quantity of heat flowing per second through the disc of poor conductor and the value of k calculated, as on L.H.S.

Procedure : 1. Fill some water in the steam generator and start heating.

2. Measure the area and thickness of the disc D of poor conductor (*wood, rubber ; ebonite etc.*). Place it in between two brass discs A and B. Fit the thermometers, θ_1 and θ_2 in the brass discs B and A respectively.

3. Place a steam chamber C on the disc A and start heating. The apparatus

should be arranged in such a way that the lower surface of the brass disc B is not placed in table or anything else, but is surrounded by air. The temperature on thermometers θ_2 and θ_1 goes on rising and when steady state is reached it becomes constant. Note this constant temperature on the thermometers.

4. Remove the disc D of poor conductor and heat the brass disc B to a temperature 10°C above θ_1. Remove the disc B and cover the top with a thick layer of flannel or felt. At intervals of half a minute note the temperature of B, and plot a graph between time and temperature. While cooling, disc B loses heat by radiation and convection. When the disc B passes though the temperature θ_1, it is losing heat at the same rate as in the first part of the experiment, (steady state condition). The rate at which disc B loses heat at the temperature θ_1 is given by,

Heat lost/second = Mass of disc × sp. heat capacity × fall in temperature/second.

$$\therefore \qquad Q = m \times c \times \text{fall in temperature/second}$$

where m and c are respectively the mass and specific heat capacity of disc B

5. To find the rate of fall of temperature at θ_1, draw the tangent to the cooling curve at that pont (θ_1). If the slope of the curve at (θ_1) gives a fall of temperature a°C in b seconds, then the rate of fall of temperature is $\left(\frac{a}{b}\right)$ °C per second.

6. If A is the area of cross-section of D and d its thickness, then rate of flow of heat, Q cal/second $= \frac{kA(\theta_1 - \theta_2)}{d}$ where k is the co-efficient of thermal conductivity of the poor conductor.

Also, Rate of cooling of disc B, Q cal. /second $= m \times c\left(\frac{a}{b}\right)$

But rate of cooling = rate of flow of heat

$$\therefore \qquad \frac{kA(\theta_1 - \theta_2)}{d} = mc\frac{a}{b}$$

or

$$k = \frac{mcad}{A(\theta_1 - \theta_2)b}$$

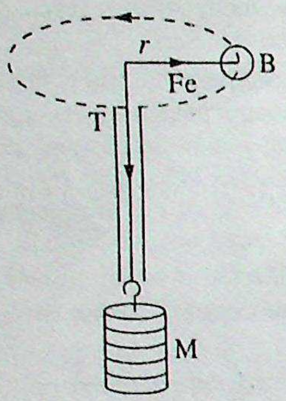
Applications : For the determination of thermal conductivity of poor conductors *i.e.*, insulating materials.

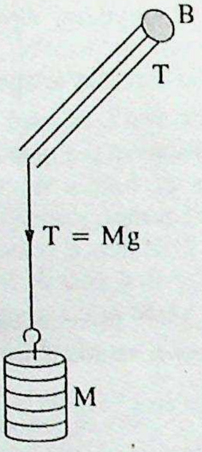
Experiment No—8

L.H.S.

Experiment : *Measurement of centripetal force.*

Apparatus : Centripetal force apparatus, metre rod, slotted weight with a hook, slotted weights, a weight box, a stopwatch vernier callipers and a physical balance.





Observations

Radius of the rubber ball B, y = cm

Mass of the rubber ball, m = gm

S. No.	Time for 10 rotations			Number of rotations per sec (n)	x	$r = x + y$	Centripetal force $= 4\pi^2 n^2 mr$	Mass M	Mg
	1	2	Mean						
1.									
2.									
3.									

Within experimental errors the centripetal force is equal to Mg which supplies it.

Precautions : 1. Counting of rotations should be done only when a steady state has been reached.

2. The speed of rotation should be maintained constant while noting the time for 10 rotations.

R.H.S.

Experiment : *Measurement of centripetal force.*

Theory : Centripetal force. If a body of mass m is rotated in a horizontal circle of radius r with a uniform speed v, it experiences a force which acts along the radius towards the centre of the circle. The force keeps the body to move in a circular path and is called the centripetal force.

It is given by the equation,

$$F = \frac{mv^2}{r} = mr\omega^2$$

Where, w is the angular velocity.

Now angular velocity w in radians per second is given by,

$$w = 2\pi n$$

where n is the frequency *i.e.*, the number of revolutions made per second.

$\therefore$ Centripetal force $= 4\pi^2 n^2 mr$.

Simple centripetal force Appraratus : It consists of a rubber ball B. It is tied to one end of a strong nylon light string. The other end of the string passes through a glass–metal tube T with rounded end and is attached to a known weight with a hook over which different slotted weights can be slipped as shown in the figure.

Procedure : *(i)* Measure radius of the ball with vernier callipers and its mass with physical balance.

(ii) Suspend a hooked mass M of about 100 – 150 gms at the other end of the string passing through the tube T. See that the mass M should be greater than the mass of the rubber ball.

(iii) Hold the tube in a vertical position above your head and twist the ball in a circular path in a horizontal plane. In this position the centripetal force required for the circular motion is supplied through the string by the weight of the hooked mass.

(iv) When the motion is under central (constant) so that the hooked mass is stationary, count the time for 10 circular motions.

(v) Now grasp the string at the bottom of the tube T and mark its position. With the string in this position, measure the distance from the top of the tube T to the surface of the rubber ball B. Let it be x, then radius of the circular path, $r = x +$ radius of the ball B

(vi) Repeat observations for the same hooked mass by changing the radius r of the circular path first to a smaller and then to a larger value and find the time for 10 rotations of the ball.

(vii) Repeat experiment by changing the hooked mass.

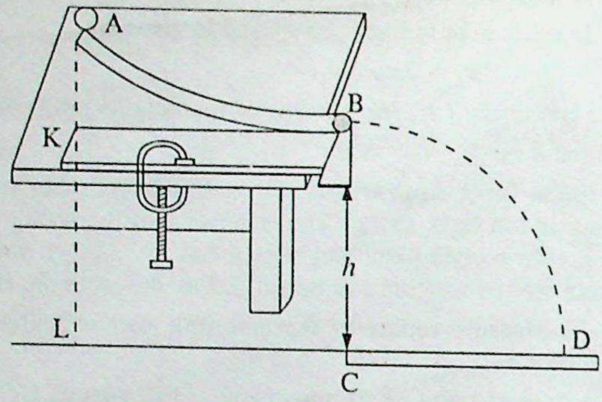
(viii) Record all the observations and make calculations to get the result.

Experiment No—9

L.H.S.

Experiment : *Measurement of kinetic energy gained by a body dropped through a height 'h'.*

Apparatus : A plastic rule with a groove fined on an aluminium strip bent into a parabolic arc and fitted with a stud, a mettalic ball of about 1 cm diameter, a clamp to fit the apparatus on the table, a plumb line, a drawing board, drawing pins, carbon papers, a white sheet of paper, set squares etc.



Record :

Horizontal distance, CD = cm

Mass of the ball M = gm

Height of end A from the board AL = cm

Height of the end B from the board h = cm

Calculations :

Time taken by the ball to strike the drawing board, $t = \sqrt{\frac{2h}{g}}$ sec.

Horizontal distance travelled by the ball in time t sec.

$$CD = \text{cm.}$$

Now horizontal distance, $CD = v \times t$

where v is the uniform velocity at B

$$\therefore \quad v = \frac{CD}{t} \text{ cm/sec.}$$

$$\text{K.E. at B} = \frac{1}{2} MV^2$$

(after the ball has fall on through a height h)

Precautions : 1. The apparatus should be fine, rigid and should not be shaky.

2. The position of the board should be adjusted in such a way that the ball always strike on it.

3. The carbon should be placed with its ink side touching the white paper sheet placed on it.

R.H.S.

Experiment : *Measurement of kinetic energy gained by a body dropped through a height h.*

Theory : Place the spherical ball at the end A of the groove and release it. It will move in the groove and acquire a velocity v in the horizontal direction on reaching the end B. The velocity in the vertically downward direction will be zero. The ball after leaving the groove will move along a parabolic path BD and will strike the ground at D. If C is a point vertically below the lower end of the groove, then the distance travelled by the ball in the horizontal direction will be uniform velocity v during the time t it takes to fall through a vertical high. BC = h is given by CD.

we know that, $$h = \frac{1}{2} g t^2$$

$\therefore$ $$t = \sqrt{\frac{2h}{g}}$$

Also $$\text{CD} = v \times t$$

i.e. $$v = \frac{\text{CD}}{t}$$

Knowing the mass m of the ball and velocity v at B (after it has fallen through a high $h' = \text{AK}$) we can find the kinetic energy gained by the ball after it has fallen through a height h'.

Procedure : 1. Clamp the apparatus rigidly on a table.

2. **Adjustment of the drawing board :** Place metallic ball at the upper end A and allow it to roll down along the groove. Note the position when it strikes the ground. Place the drawing board in such positions that this point where the ball struck the ground and the point vertically below the stud lie on the board. Place a carbon paper on the drawing board with its ink side down on which the white sheet of paper is fixed with the drawing pins.

3. Mark a point vertically below the stud fixed at the lower end of the groove and measure the height BC = h.

4. Place the ball at the upper end A of the groove and allow it to roll down.

5. The ball after leaving the groove at B moves along a parabolic path and strikes the board. The impact of the ball on the paper will produce a fine mark as at D on the side of white paper in contact with the carbon paper. Measure CD.

6. Find the height AL with Plumb line.

7. Find the height $h' = \text{AK}$ by $\text{AL}\cdot\text{BC} = h'$

8. Weight the ball and note its mass M.

9. Now make calculations and get the result.

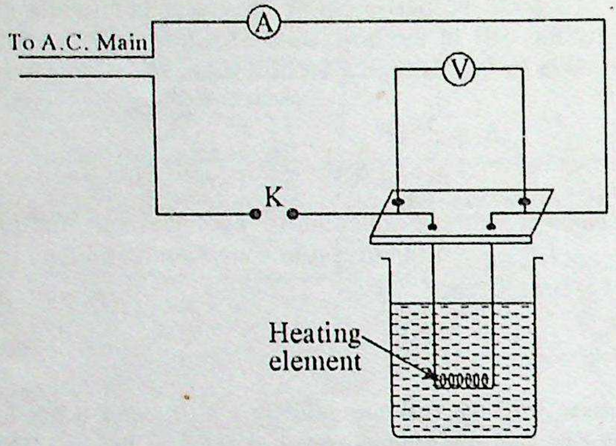
Experiment No—10

L.H.S.

Experiment : *Investigation of conversion of energy i.e., electrical energy to heat.*

Apparatus : Heating element of high resistance or immersion rod, holders/points fitted on with flexible wires. An A.C. ammeter, A.C. voltmeter, flexible connecting wires, a copper calorimeter of 1/2 litre capacity, weight box, physical balance stop watch and a thermometer.

Circuit Diagram :



Observations :

Least count of Ammeter =

Least count of voltmeter =

Mass of empty calorimeter, m_1 =

Mass of empty calorimeter water, m_2 = --------Kg

Initial temp. of water, θ_1 = --------°C

Final temp. of water, θ_2 = --------°C

Time for which current passed, t = --------sec.

Ammeter Reading, I = --------amp.

Voltmeter Reading, V = --------volts.

Calculations :

Mass of water taken $= (m_2 - m_1)$ kg.

Rise of temp. $= \theta_2 - \theta_1$

Let S be the sp. heat of material of calorimeter.

Heat gained by calorimeter + water $= [m_1 s + (m_2 - m_1) \times 1] (\theta_2 - \theta_1)$ ...*(i)*

[Using the formula Heat gained = m.s.t.]

This has been generated by the electrical energy expended.

∴ Electrical energy expended = V.I.t Joules.

Heat generated by electrical energy in kilocalories

$$= \frac{V.I.t}{4200} \text{ kilocalories} \qquad \ldots(ii)$$

Result : Since the heat gained or absorbed and the heat (energy) produced is approximately equal with in experimental error. Therefore conversion of electrical energy into heat energy is investigated.

Precautions : *(i)* Flexible wires should be used for connections.

(ii) Do not touch an uninsulated part of the circuit while working.

(iii) Connections should be neat and light.

(iv) The light means current and stop watch should be started at the same time.

R.H.S.

Experiment : *Investigation of conversion energy i.e.,* electric energy to heat.

Theory : *Thermal effect of electric current* : when an electric flows through conductor, electric energy is expended in overcoming the frictional resistance between the electrons and the molecules of the wire. If V volts potential difference is applied across a conductor and I amperes current flow though it for time t seconds, then the energy expended is given by, V It watt-seconds or joules. Therefore,

$$\text{Energy expended, } W = VIt \text{ joules} \qquad \ldots(I)$$

According to the law of conservation of energy, the electric energy expended is converted into heat energy. The conversion of electrical energy into heat energy is called the thermal effect of electric current. From the expression – (I) ;

Heat generated in joules is given by

$$H = VIt \text{ joules.}$$

This is in accordance with the Joule's law of electrical heating. Using the formula given below, we can find the heat produced/generated in kilocalories.

$$\therefore \qquad H = \frac{V.I.t}{4200} \text{ kilocalories.}$$

Procedure :

1. Draw the neat diagram showing the connections.
2. Arrange the apparatus on the table in order as indicated and make the connections.
3. Now take a copper vessel/calorimeter and weigh it when empty. Note its mass m_1.
4. Then fill 3/4th of the calorimeter with water and weight it. Note the mass m_2.
5. Now place the heating/immersion rod in the calorimeter containing water in such a way that it is completely dipped in water.
6. Place the thermometer in the beeker. Note the room temperature of water before passing the electric current.
7. Now put the plug in the key k and simultaneously start the stop watch. Current will flow through the heating element and heat will be produced and this will heat up the water and it starts rising
8. Note the readings of ammeter and voltmeter.
9. When thermometer shows sufficient rise of temperature, switch off the main and stop the stop watch. Note the maximum temperature reached on the thermometer and note the time for which the current is passed from the stop watch.
10. Now record the observations, make calculations.
11. The experiment can be performed on D.C. Supply (Battery) using a low resistance heating coil D.C. ammeter and voltmeter.

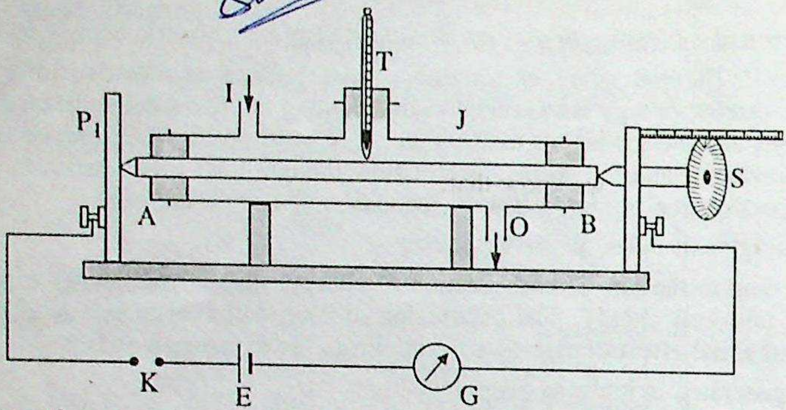
Experiment No—11

Dated.........

L.H.S.

Object : *To determine the co-efficient of linear expansion of a given metal rod, by Gumther's apparatus.*

Diagram :



Observations :

Length of the rod at room temp., L = ... cm

Initial temperature, t_1 = ... °C

Final temperature, t_2 = ... °C

Pitch of the screw of spherometer = ... mm

Least count of spherometer screw = ... mm

Reading of spherometer screw before passing steam, R_1 = ... mm

Reading of spherometer screw after passing steam, R_2 = ... mm

Calculations :

Increase in length = $R_2 - R_1$ = ... mm = ... cm

Rise in temperature = $(t_2 - t_1)$ = ... °C

$$\text{Co-efficient of liner expansion} = \frac{\text{Increase in length}}{\text{Original length} \times \text{Rise in temp.}}$$

$$\therefore \quad \alpha = \frac{R_2 - R_1}{L \times (t_2 - t_1)} \text{ ..../°C}$$

Result : Co-efficient of linear expansion of the given metal rod = /°C

Precautions :

1. While taking initial and final readings, the spherometer screw should be moved very slowly.

2. The reading should be noted when the screw just touches the rod *i.e.*, when the galvanometer just gives deflection.

3. No portion of the rod should be outside the steam jacket.

4. Steam should enter at the upper inlet and leave at the lower outlet,

R.H.S.

Dated.........

Object : *To determine the co-efficient of linear expansion of a given metal rod by Gumther's apparatus.*

Theory : For detailed theory see chapter on expansion of solids.

Procedure : 1. Take a steam boiler and start heating after putting enough water in it.

2. Measure the length of the given metal rod accurately with the help of a metre rod.

3. Insert the rod in the tube and fix it with rubber–corks in position. Place thermometer in the central hole. Make the electrical connections as shown in fig. While making connections the spherometer screw should not touch the rod.

4. Find the pitch and least count of the spherometer screw. Turn the disc of the spherometer so that the screw moves forward. As soon as the screw touches the rod, the galvanometer gives deflection. Remove the plug of the key and note the reading of the spherometer.

6. Move the spherometer screw back to allow space for expansion of the rod. Note the initial reading of the thermometer.

7. Connect the tube from the steam boiler to the inlet I of the jacket and pass steam through it. Place a beaker under the outlet O of the jacket to collect water.

8. When the temperature remains constant for about 5 minutes, insert the key and turn the spherometer disc so that the screw moves forward and just comes in contact with rod again. when the galvanometer gives deflection, note the reading of the spherometer. Note the final temperature from the thermometer and calculate the value of α as on L.H.S.

Application : The co–efficients of linear expansion of different metals can be determined.

Experiment No—12

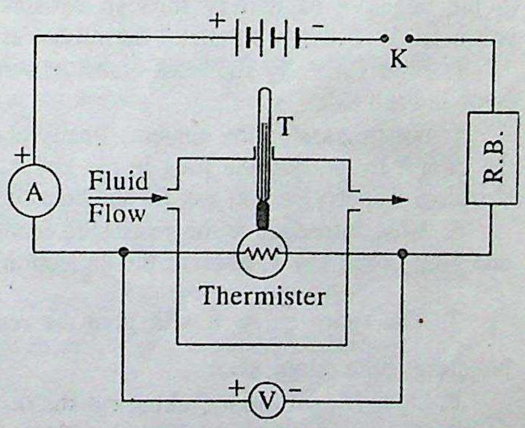
Experiment : *Study of resistance variation of*

(a) Thermistor and

(b) Copper wire 40 S.W.G. with temperature

Apparatus : Thermistor, copper wire 40 S.W.G. Ammeter, Voltmeter, One way key, A battery, Thermometer, Steam generator and Connecting wires.

Circuit Diagram :



Observations :

Least count of Ammeter = —Amp.

Least count of voltmeter = —volts.

S. No.		Temperature in °C	voltmeter Reading in Volts (V)	Ammeter Reading in amperes (I)	V/I
1.	Thermistor				
2.					
3.					
4.	Copper				
5.					
6.					

Record :

Result : From the above, it clear that as the temperature is raised, the resistance of the thermistor decreases while that of copper wire increases and vice versa. When the temperature is lowered.

R.H.S.

Experiment : *Study of resistance variation of*

(a) Thermistor and

(b) Copper wire 40 S.W.G. with temperature.

Theory : Thermistors are semiconductors which have a high negative temperature coefficient in the order of – 4% per degree centigrade. Thermistor may be used for temperature measurement and control.

Thermistors or thermal resistors consists of metallic oxides such as manganese, cobalt, copper, iron and uranium.

Temperature coefficient of resistance *(α)* is defined as the change in resistance per unit resistance at 0°C per degree centigrade rise in temperature.

Procedure :

1. Draw a neat diagram of the circuit as shown in figure.

2. Arrange the apparatus on the table in the order in accordance with the circuit diagram and make the connections.

3. Connect positive terminal of the battery to the positive terminal of ammeter. Connect one end of thermistor to the negative terminal of ammeter and the other end to the negative of battery through resistance box (R.B.) and key *k*. connect the voltmeter V across two ends of thermistor as shown in fig.

4. Find carefully the least count of Ammeter and voltmeter and note the zero error in each case.

5. Before passing the current, finally check the connections once again and close the circuit by putting the plug in the key *k*. See that the ammeter and voltmeter are working properly or not, ask for the help of teacher.

6. Now introduce some resistance from the R.B. (for safe flow of the current) and plug in the key *k*, observe the deflection of the ammeter and record it.

7. The ratio $\frac{V}{I} = R$ will give the resistance of the thermistor at room temp. Note the room temp. also.

8. Open the circuit by taking out the key *k*.

9. Now raise the temperature of the thermistor by surrounding the thermistor with hot water or steam.

10. Keeping the value of R.B. resistance same, plug in the key *k* and take the reading of the voltmeter and ammeter at different temperatures and record them. Find $\frac{V}{I} = R$ in each case. It will be found that resistance decreases with the rise of temp.

11. Now repeat the whole experiment with copper wire (S.W.G. 40) and find the result, the result will be opposite to that of the thermistor.

Precautions : 1. Correspondings both for thermistors and copper should be taken for the same temperatures.

Chapter

Viva-Voce

MECHANICS

MEASUREMENTS

Q. 1. *What are fundamental quantities ?*

Ans. Fundamental quantities are the quantities which cannot be expressed in terms of any other physical quantities.

Mass, length, time, electric current, luminous intensity, thermodynamic temperature, amount of substance, plane angle and solid angle are the fundamental quantities of the *S.I.* units.

Q. 2. *What are fundamental units ?*

Ans. The units selected for measuring the fundamental quantities (such as mass, length, time etc.) are called fundamental units.

Q. 3. *What is a unit ?*

Ans. A unit is a convenient magnitude (fixed upon by mutual agreement or by some law of nature) in terms of which other quantities are expressed.

Or

The quantity used as the standard for measurement is called the unit.

For Example : kilogram is the unit of mass
metre is the unit of length
ampere is the unit of electric current.

Q. 4. *What are derived units ?*

Ans. The units of physical quantities which can be derived from fundamental units are called derived units. For example the units of force, density, velocity are derived units.

Q. 5. *What are S.I. units ?*

Ans. *S.I.* unit is abbreviated form for **System International units.**

S.I. units are based on seven fundamental units or base units. The seven base units are *metre* (*m*) for length ; *kilogram* (*kg*) for mass, *second* (*s*) for time ; *ampere* (*A*) for electric current ; *cendela* (*cd*) for luminous intensity ; kelvin (*K*) for temperature ; *mole* (*mol.*) for amount of substance.

In addition to seven *S.I.* base units there are two supplementry *S.I.* units as :- *radian* (*rad.*) for plane angle ; *steradian* (*sr*) for solid angle.

Q. 6. *Define one second.*

Ans. One second is defined to be equal to 9192631770 periods of the Cesium-clock.

Q. 7. *What is the number of wavelengths of* K_r^{86} *contained in 1 meter ?*

Ans. 1650763·73 wavelengths.

Q. 8. *What is an error ?*

Ans. The difference between actual value and the observed value is called error.

Q. 9. *What is percentage error ?*

Ans. $\% \text{ age error} = \dfrac{\text{Actual value} \sim \text{observed value}}{\text{Actual value}} \times 100$

Q. 10. *Which of the following measurement of length is most accurate ?*

(i) 3·00 cm ; (ii) 0·003 mm (iii) 30·00 mm

Ans. 30·00 *mm.*

Q. 11. *Which of the following readings is more accurate and why ?*

(i) $2{\cdot}5 \times 10^3$ *kg* *(ii)* $2{\cdot}50 \times 10^3$ *kg* *(iii)* $2{\cdot}500 \times 10^3$ *kg.*

Ans. The possible error in $2{\cdot}5 \times 10^3$ kg can be $0{\cdot}1 \times 10^3$ kg.

$\therefore$ Percentage error in $2{\cdot}5 \times 10^3 = \dfrac{0 \cdot 1 \times 10^3}{2 \cdot 5 \times 10^3} \times 100 = 4\%$

Possible error in $2{\cdot}50 \times 10^3$ kg can be $0{\cdot}01 \times 10^3$ kg.

$\therefore$ Percentage error in $2{\cdot}50 \times 10^3 = \dfrac{0 \cdot 01 \times 10^3}{2 \cdot 5 \times 10^3} \times 100 = 0 \cdot 4\%$

Possible error in $2{\cdot}500 \times 10^3$ kg can be $0{\cdot}001 \times 10^3$ kg.

$\therefore$ Percentage error in $2{\cdot}500 \times 10^2 = \dfrac{0 \cdot 001 \times 10^3}{2 \cdot 5 \times 10^3} \times 100 = 0 \cdot 04\%$

Hence, $2{\cdot}500 \times 10^2$ kg is more accurate.

Q. 12. *In measuring the side of a cube, error caused is 5%. What is the error in the volume of the cube ?*

Ans. volume, $V = l^3$

$$\therefore \quad \left|\frac{\Delta V}{v}\right| . \times 100 = 3\left|\frac{\Delta l}{l}\right| \times 100 = 3 \times 5 = 15\%.$$

VERNIER CALLIPERS

Q. 13. *What is vernier scale ?*

Ans. A small subsidiary scale, which slides along the main scale of a measuring instrument its called vernier scale. It was named after its inventor Pierre Vernier.

Q. 14. *What is the relation between the main scale divisions and vernier scale divisions ?*

Ans. $(n - 1)$ divisions on the main scale coincide with n divisions on the vernier scale.

Q. 15. *What is vernier constant or least count of a vernier calliper ?*

Ans. Vernier constant or least count is the minimum distance which can be measured with the instrument. It is equal to the difference between one main scale division and one vernier scale division.

Q. 16. *How do you find the vernier constant of an instrument ?*

Ans. Let $S.D.$ be the value of one main scale division and $V.D.$ be the value of one vernier scale division.

Again let $(n - 1)$ divisions of the main scale coincide with n divisions of the vernier scale.

$$\therefore \quad (n - 1)\, S.D. = n\, V.D.$$

$$\text{or} \quad n\, S.D. - n\, V.D. = 1\, S.D.$$

$$\text{Vernier constant} = 1\, S.D. - 1\, V.D. = \frac{1\, S.D.}{n}$$

$$\text{Thus, vernier constant} = \frac{1\, S.D.}{n}$$

$$= \frac{\text{value of one main scale division}}{\text{total number of divisions on the vernier scale}}$$

Q. 17. *What is meant by "zero error" of a vernier calliper ?*

Ans. When the two jaws of the vernier calliper are brought into contact, the following points are observed :-

(i) If the zero of vernier scale coincides with the zero of main scale, then the instrument is correct *i.e.*, there is no zero error

(ii) If the zero of vernier scale does not coincide with the zero of main scale, then the instrument is incorrect *i.e.*, there is zero error in the instrument.

Q. 18. *When zero error is +ve and when it is –ve ?*

Ans. *(i)* If the zero of vernier scale remains behind(towards right) of the zero of main scale when the jaws are in contact, the error is +ve (*positive*)

(ii) If the zero of vernier scale passes beyond (towards left) of the zero of main scale when the jaws are in contact, the error is –ve (*negative*).

Q. 19. *How zero error in the vernier callipers is determin`ed ?*

Ans. Bring the two jaws in contact. If zero of *V*. Scale does not coincide with the zero of *M*. Scale, determine the error as given below.

(i) When zero of *V*. scale is towards right of the zero of Main Scale. Error is +ve. Start from the zero of *V*. scale, go on counting and find the number of divisions on the vernier scale which coincides with any of the main scale divisions. Let it be *N*. Multiply *N* by vernier constant. This is zero error (+ve)

∴ Zero Error = N × Vernier constant = +.....

(ii) When zero of *V*. scale is towards left of the zero of Main scale. Error is −ve. Start from zero of *V*. scale, go on counting and find the number of divisions on the *V* scale which coincides with any of the main scale divisions. Let it be n. Now subtract total number of divisions (p) on vernier scale from n and multiply $(n - p)$ by vernier constant. This is zero error (−ve).

∴ Zero Error = $(n - p)$ × Vernier constant = − .....

Q. 20. *How do you apply zero error to get the corrected reading of vernier callipers.*

Ans. Zero error is always subtracted with proper sign from the observed reading.

Q. 21. *What is zero correction and how is it applied ?*

Ans. Zero correction = − (Zero error). Zero correction is always added with proper sign to the observed reading.

SCREW GUAGE

Q. 22. *What is a screw gauge ?*

Ans. A screw gauge is a measuring instrument made on the principle of the screw.

Q. 23. *Sometimes screw guage is also called micrometer, screw gauge, why ?*

Ans. A screw gauge which can make measurements upto a micrometer is called a micrometer screw gauge.

1 micrometer (μm) = 10^{-6} metre.

Q. 24. *What is principle of a screw ?*

Ans. When rotatory motion is given to the head of the screw, a motion of translation along the length of the screw also takes place.

Principle of Screw– *The linear motion of the screw is directly proportional to the amount of rotation given to the head of screw.*

Q. 25. *What is pitch of a screw ?*

Ans. Pitch of a screw is the distance travelled by the screw in one complete rotation.

Or

Pitch is the thickness of the thread of the screw.

Or

Pitch is the distance between two consective threads of the screw.

Q. 26. *How pitch of a screw gauge can be determined ?*

Ans. Make the zero of circular scale to coincide with reference line. Note the reading on the main scale. Give some (say 4) complete rotations to the head of screw gauge and find the distance travelled along the linear scale. Divide the total distance travelled by the number of rotations. This gives the pitch of the screw.

$$\text{Pitch} = \frac{\text{Distance travelled in } n \text{ complete rotations}}{\text{No. of rotations } (n)}$$

Q. 27. *What is least count of the screw gauge ?*

Ans. Least count is the minimum distance which can be measured with the screw gauge.

Q. 28. *How least count of a screw gauge is determined ?*

Ans. $$\text{Least count} = \frac{\text{Pitch of the screw}}{\text{No. of divisions on the circular scale}}$$

Q. 29. *Distance moved by screw in 4 complete rotations is 2 mm. What is the least count if the total number of divisions on the circular scale is 100 ?*

Ans. $$\text{Pitch of screw} = \frac{2\ mm}{4} = 0 \cdot 5\ mm$$

$$\therefore \quad \text{Least count} = \frac{\text{Pitch}}{\text{No. of divisions on circular scale}} = \frac{0 \cdot 5\ mm}{100} = 0 \cdot 005\ mm$$

Q. 30. *What is zero error of a screw gauge ?*

Ans. When the screw just touches the fixed stud opposite to it, then if zero of circular scale does not coincide with the reference line, the screw gauge is said to have zero error.

Q. 31. *When there is no zero error in the screw gauge ?*

Ans. If on bringing the screw just in contact with the opposite fixed stud, the zero of circular scale falls on the zero mark of the main scale and coincides with reference line, the screw gauge is said to have no zero error.

Q. 32. *When zero error is positive and when is it negative ?*

Ans. When the screw is touching the fixed stud opposite to it and the zero of circular scale does not coincide with reference line (or base line) of the linear scale, then +ve or −ve zero error is determined as :–

(i) If the zero of circular scale lies below the reference line (or base line), zero error is positive.

(ii) If the zero of circular scale lies above the reference line, zero error is negative.

Q. 33. *How zero error of a screw gauge is determined ?*

Ans. As explained above find the number of divisions on the circular scale coinciding with reference line, then

zero error (±) = No of divisions coinciding × least count

Q. 34. *How do you apply zero error ?*

Ans. Zero error is always subtracted with proper sign from the observed reading.

Q. 35. *What is back-lash error ? How does it arise ?*

Ans. When the head of screw is given rotation in one direction, the screw moves forward but if we start rotating the screw in the opposite direction, then it is generally observed that the screw does not move backwards immediately, this defect is called the back-lash error.

Back-lash error arises due to the constant use and imperfect construction of the instrument, which causes looseness of the screw in the stud or nut.

Q. 36. *How back-lash error is avoided ?*

Ans. Back-lash error can be avoided by rotating the screw in one direction only while taking the observation.

Q. 37. *For what purposes do you make use of the screw gauge ?*

Ans. Screw gauge is used either to measure the diameter of thin wires or the thickness of thin plates.

Q. 38. *Why readings at different places in case of thin sheets and mutually perpendicular directions in case of wires are taken ?*

Ans. The objects may not be of uniform, thickness or diameter. Therefore, mean of several readings gives us the correct measurement ?

Spherometer

Q. 39. *Why a spherometer is called so ?*

Ans. A spherometer is called so because it is used for measuring the radius of curvature of spherical surfaces.

Q. 40. *What is the principle of a spherometer ?*

Ans. A spherometer works on the principle that linear motion of a screw is directly proportional to the rotatory motion.

Q. 41. *What is pitch of spherometer screw ?*

Ans. The distance travelled by the screw when disc is given one complete rotation is called pitch.

Q. 42. *What is least count of a spherometer ?*

Ans. Least count of an instrument is the minimum measurement that can be made with it. In case of spherometer,

$$\text{Least count} = \frac{\text{Pitch of the Screw}}{\text{No. of div. on the disc. of spherometer}}$$

Q. 43. *Is it essential to determine the zero error in case of spherometer ?*

Ans. We usually require the difference between two readings, therefore, it is not necessary to find the zero error in case of spherometer. The zero error if any gets cancelled while taking difference.

Q. 44. *How to avoid back–lash error ?*

Ans. While taking measurements, the screw should be rotated in one direction only.

Q. 45. *What is radius of curvature of a spherical surface ?*

Ans. Radius of curvature of a spherical surface is the radius of the sphere of which the surface forms a part.

Q. 46. *What is the formula for finding the radius of curvature of a spherical surface ?*

Ans. The formula is, $R = \frac{l^2}{6h} + \frac{h}{2}$

Where, R = radius of curvature

l = mean distance between any two fixed legs.

h = height or depth of the curved surface from the plane of the legs.

Q. 47. *Why should there be three legs of spherometer ?*

Ans. The tips of the legs should be in the same plane so that the instrument is balanced when put on the spherical surfaces. Thus three legs keep the instrument balanced because maximum number of points which can lie in the same plane is *three*.

USE OF BALANCE

Q. 48. *What is the principle of a balance ?*

Ans. Balance is a lever and works on the principle of *moments.*

Q. 49. *What is mass ? How is it measured ?*

Ans. Mass is the quantity of matter contained in a body. It is measured with a common balance.

Q. 50. *What is meant by the moment of a force ?*

Ans. Moment of a force is equal to the product of the force and perpendicular distance of the line of action of the force from the axis of rotation.

Q. 51. *What is principle of moments ?*

Ans. The algebraic sum of moments of all the forces acting on a body, in equilibrium, is zero.

Or

Sum of clockwise moments = Sum of anti clock–wise moments.

Q. 52. *What do you measure with a physical balance-mass or weight ?*

Ans. We measure mass with a common balance because 'g' is same on both the pans.

Q. 53. *How weight of a body is measured ?*

Ans. Weight of a body is measured with the help of a spring balance.

Q. 54. *What is density ? What are its units ?*

Ans. Density of a body is its mass per unit volume. The units are $kg\ m^{-2}$.

ACCELERATION DUE TO GRAVITY

Q. 55. *What is relation between mass and weight ?*

Ans. Weight = mass × acceleration due to gravity

i.e. $W = mg$

Q. 56. *Define acceleration due to gravity (g).*

Ans. Earth attracts everybody towards its centre. The acceleration produced in a body due to this attraction is called acceleration due to gravity. It is denoted by '*g*' and is equal to the force exerted by the earth on a unit mass.

Q. 57. *What are the units of 'g' on the S.I. units ?*

Ans. The units of '*g*' are meter/second^2 or $m\ s^{-2}$.

Q. 58. *Is 'g' a vector or a scalar quantity ?*

Ans. '*g*' is a vector quantity, since it acts vertically downwards through C.G. of the body.

Q. 59. *How does 'g' vary at different places on earth ?*

Ans. *(i)* '*g*' is maximum at the poles.

(ii) '*g*' is minimum at the equator.

Q. 60. *Why does 'g' vary from place to place ?*

Ans. '*g*' varies from place to place due to,

(i) Variation of distance of the body from the C.G. of the earth.

(ii) Rotation of earth, causing centrifugal force.

Q. 61. *What is the type of motion of a simple pendulum ?*

Ans. Simple Harmonic Motion (S.H.M.)

Q. 62. *What is S.H.M. ?*

Ans. When a body moves to and fro about its mean position in a straight line such that its acceleration is always directed towards the mean position and is proportional to its displacement from the mean position, then the body is said to execute S.H.M.

Q. 63. *What is an oscillation ?*

Ans. When a vibrating body passes through its mean position once towards right and again after completing one vibration, towards right, through the same position in the same direction, we have one *oscillation*.

Q. 64. *What is time period ? What is the formula for the time period of a simple pendulum.*

Ans. The time taken by a pendulum (or vibrating body) to complete one vibration is called time period.

The formula for the time period of a simple pendulum is,

$$t = 2\pi\sqrt{\frac{l}{g}}$$

Where, t = time period

l = length of pendulum

g = acceleration due to gravity.

Q. 65. *Give the positions where the velocity and acceleration of the bob of pendulum are maximum ?*

Ans. Velocity is maximum at mean position. Acceleration is maximum at the extreme positions.

Q. 66. *What is second's Pendulum ?*

Ans. A pendulum whose time period is 2 seconds is called a second's pendulum.

Q. 67. *What is the length of second's pendulum ?*

Ans. Length of second's pendulum = 99·4 cm.

Q. 68. *Is the value of 'g' on earth and moon same ?*

Ans. No. Value of 'g' on moon is 1/6th of the value of 'g' on the earth.

SURFACE TENSION

Q. 69. *What is surface tension ?*

Ans. Surface tension is the property of a liquid due to which its free surface at rest behaves like a stretched membrane or an elastic skin and tends to contract.

Q. 70. *How surface tension is measured ?*

Ans. Surface tension is measured as the force acting per unit length of an imaginary line drawn on the free surface of liquid perpendicular to the line and tangential to the surface.

Q. 71. *What are the units of surface tension ?*

Ans. Since $T = \frac{F}{l}$, $\therefore$ units are newton per metre $N\,m^{-1}$.

Or $T = \frac{W}{\Delta A} = \frac{\text{work done}}{\text{increase in surface area}}$, units are joules/$m^2$

Q. 72. *Why liquid drops are spherical ?*

Ans. Free surface of a liquid tends to contract *i.e.*, tends to have the minimum surface area. We know that for a given volume, sphere has the minimum surface area. Hence liquid drops aquire spherical shape.

Q. 73. *What is angle of contact.*

Ans. Angle of contact is the angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid for that pair of liquid and solid.

Q. 74. *What is the angle of contact for water and glass ; for mercury and glass ?*

Ans. For water and glass, angle of contact is about 8° and for mercury and glass angle of contact is about 138°.

Q. 75. *What is force of cohesion ?*

Ans. Force of cohesion is the force of attraction between the molecules of the same substance. For example force of attraction between the molecules of water is the force of cohesion.

Q. 76. *What is the force of adhesion ?*

Ans. The force of attraction existing between the molecules of different substances is called the force of adhesion. For example force of attraction between the ink molecules and paper molecules is the force of adhesion.

Q. 77. *What is a capillary tube ?*

Ans. A tube of very fine bore is called a capillary tube.

Q. 78. *What is capillarity or capillary action ?*

Ans. The rise or fall of a liquid in a capillary tube is called capillarity or capillary action.

Q. 79. *Why does oil rises in a cottonwick of a kerosine lamp ?*

Ans. Oil rises in the cottonwick due to capillary action.

Q. 80. *Which liquids rise in a capillary tube and which liquids fall in a capillary tube.*

Ans. *(i)* The liquids which wet the surfaces, rise in a capillary tube. For example, water rises in a capillary tube.

(ii) The liquids which do not wet the surface fall in a capillary tube. For example mercury falls in a capillary tube.

Q. 81. *What is the shape of meniscus of water and mercury in contact with a solid ?*

Ans. The shape of water surface in contact with a solid is *concave*. The shape of mercury surface in contact with a solid is *convex*.

Q. 82. *What is surface film ?*

Ans. The top-most layer of liquid having thickness equal to the molecular range is called surface film.

Q. 83. *What is surface energy ?*

Ans. The work done to increase the surface area of a surface film through unity is called surface energy.

Q. 84. *Why does a piece of camphor dance when placed on the surface of water ?*

Ans. Surface tension of a solution of camphor in water is less than the surface tension for water.

Q. 85. *What is the cleansing action of soap ?*

Ans. Soap decreases the surface tension of water.

HEAT

Q. 1. *What is the difference between heat and temperature ?*

Ans. Heat is a form of energy which causes the rise in temperature of the body while temperature is the degree of hotness of a body. It represents the thermal state of the body.

Q. 2. *What is the unit of heat ? Define a calorie.*

Ans. Unit of heat is calorie in C.G.S. system and kilocalorie in *M.K.S.* system. The amount of heat which raises the temperature of 1 gm. of water by 1°C is called one calorie.

Q. 3. *What is the relation between calorie and kilocalorie ?*

1 kilo cal. $= 10^3$ cal.

Q. 4. *What is a theromometer ?*

Ans. A thermometer is a device used to measure the temperature.

Thermometer → thermo (heat) + meter (scale) *i.e.*, scale of measuring heat (temperature).

Q. 5. *What are the scales for measuring temperature ?*

Ans. The scales are (*a*) Celcius, (*b*) Fahrenheit and (*c*) kelvin.

Q. 6. *How are these three scales related among themselves ?*

Ans. $$\frac{C}{100} = \frac{F-32}{180} = \frac{K-273}{100}$$

Q. 7. *Can you tell any temperature which will have the same reading on centigrade and Fahrenheit thermometers ?*

Ans. Yes, – 40°

Q. 8. *Which liquid is filled in the thermometers ?*

Ans. Pure and dry mercury.

Q. 9. *Why is mercury used in thermometers ?*

Ans. Mercury is filled in thermometers because :—

(*i*) It is a good conductor of heat and therefore quickly transmits heat throughout its mass hence the temperature remains uniform throughout.

(*ii*) It is an opaque and shining substance, hence is easily readable in capillary tubes.

(*iii*) Its range is fairly wide, B.P. 357°C, and F.P. – 39°C.

(*iv*) Its coefficient of expansion is also large. For a small temperature rise it shows a fairly measurable expansion.

(*v*) Its thermal capacity is low.

(*vi*) It does not wet the glass. It is available in pure and dry state.

Q. 10. *Define specific heat of a substance.*

Ans. Specific heat of a substance is the amount of heat required to raise the temperature of 1 kg of the substance by 1°C.

Q. 11. *What are the units of specific heat ?*

Ans. Kilo cal kg^{-1} $°C^{-1}$ or Joules/kg/°C

e.g. sp. heat of water = 1 kilo cal/kg°C = $4 \cdot 18 \times 10^3$ (joules/kg °C)

Q. 12. *Define thermal capacity.*

Ans. Thermal capacity of a body is the amount of heat required to raise the temperature of the body through 1°C

Numerically, thermal capacity = mass × sp. heat.

Q. 13. *How does sp. heat of water vary with temperature ?*

Ans. From 0°C to 40°C sp. heat decreases slightly with value 1 at 15°C. From 40°C it increases and is equal to 1 at 70°C again

Q. 14. *Define "water equivalent" of a calorimeter ?*

Ans. It is the equivalent mass of water which requires the same amount of heat as the calorimeter to rise through 1°C.

Numerically, water-equivalent = mass of cal. × sp. heat.

Q. 15. *What is J and give its value ?*

Ans. J is a conversion factor with the help of which heat can be converted into work and vice-versa.

Q. 16. *What precautions do you take to ensure that no heat is lost by conduction, convection and radiation ?*

Ans. To avoid loss of heat due to conduction, the calorimeter is enclosed and surrounded by some non-conducting material such as wood, felt, cork etc.

To avoid loss of heat due to radiation, the calorimeter is brightly polished.

Q. 17. *State Newton's Law of cooling.*

Ans. The rate of cooling of a body by radiation is proportional to the mean difference of temperature between the body and its surroundings provided this difference is not large.

Q. 18. *Define the law of radiations or Stefan's Law.*

Ans. It states that the amount of radiation per second per unit surface area, from a body, is proportional to the 4th power of its absolute temperature, *i.e.*,

$E = \sigma T^4$ where σ is a constant called Stefan's constant.

Q. 19. *What are limitations of Stefan's Law.*

Ans. It holds exactly for perfectly black bodies and for large differences of temperature (upto 1500°C).

Q. 20. *What are the factors on which radiation depends ?*

Ans. *(i)* The difference of the temperature between the body and its surroundings.

(ii) Area of the surface exposed.

(iii) Time

(iv) Emissivity of the surface.

Q. 21. *Distinguish between the rate of cooling and the rate of loss of heat of a body.*

Ans. The rate of loss of heat of a body depends upon the temperature of the body, the area of its surface exposed and the nature of the surface exposed.

But the rate of cooling is the rate of loss of heat divided by the thermal capacity of the body, *i.e.*, rate of fall of temperature.

Q. 22. *What is a perfectly black body ?*

Ans. A perfectly black body is one which absorbs all the radiations falling on it irrespective of their wavelengths.

Q. 23. *Define latent heat of steam and latent heat of fusion of ice ?*

Ans. Latent heat of steam is the amount of heat required to convert 1 kg. of water at its boiling point into steam at the same temperature. Latent heat of fusion of ice is the amount of heat required to melt 1 kg of ice at 0°C.

Q. 24. *Why is it called latent heat ?*

Ans. The word 'latent' heat means hidden or that which cannot be detected. Hence it is latent for the thermometer which does not record any rise of temperature during a change of state.

Q. 25. *Why is the temperature of ice and salt mixture much lower than ice ?*

Ans. As salt is put in ice, a part of it dissolves in the water of the ice and the latent heat required is taken from the mixture whose temperature is thereby lowered.

Q. 26. *If we drop ether or alcohol on the palm, why do we feel cold ?*

Ans. Ether or alcohol are volatile liquids. For vaporisation they need latent heat and that is taken from our palm, so we feel cold.

Q. 27. *How does water cool in an earthen pot ?*

Ans. Water leaks out of the pot through the pores. On coming out, it evaporates taking the required heat from the pot, thereby decreasing the temperature of the liquid.

Q. 28. *Why do you feel more heat in steam than in water at 100°C ?*

Ans. Because steam at 100°C has latent heat as 536 kilo cal/kg. more than in water at 100°C.

Q. 29. *Why it is difficult to cook rice at hills.*

Ans. Because on hills the atmospheric pressure is low and water will boil below 100°C and rice will not get sufficient heat for being cooked.

Q. 30. *Define coefficient of linear expansion ?*

Ans. It is the ratio of the increase in length of the original length for unit rise in temperature. Mathematically,

$$a = \frac{L_2 - L_1}{L_1(t_2 - t_1)}$$

i.e., increase in length per unit original length per degree centigrade rise in temperature.

Q. 31. *What is unit of co-efficient of linear expansion.*

Ans. Per degree Celcius ...../°C.

Q. 32. *What are the other co-efficients of expansion ?*

Ans. Co-eff. of superficial and co-eff. of cubical expansions.

Q. 33. *What is the relation between them ?*

Ans. The co-efficients of superficial and cubical expansions are twice and thrice respectively of the value of co-efficient of linear expansions, *i.e.*,

$$\beta = 2\alpha \text{ and } \gamma = 3\alpha$$

Q. 34. *Why is it easy to fuse platinum wire into glass ?*

Ans. Because their expansions are equal.

Q. 35. *What is meant by "Negative co-efficient of expansion' ?*

Ans. It is the co-efficient of expansion for the substance which contracts on heating.

Q. 36. *What is the substance which has low expansion co-efficient ?*

Ans. Invar, an alloy of 64% Fe and 36% Ni.

Q. 37. *What are the practical uses of expansion ?*

Ans. *(i)* In the construction of pendulum and balance wheels.

(ii) Shoeing of wooden wheels with iron-tyres.

(iii) Thermostats.

Q. 38. *What is dew point ?*

Ans. Dew point is the temperature at which a given volume of air becomes just saturated with the water vapour actually present in it.

Q. 39. *What do you mean by relative humidity ?*

Ans. It is the ratio of mass of water vapour actually present in a given volume of air at ordinary temperature to the mass of water vapour required to saturate it at the same temperature.

Q. 40. *If R.H. is 100% what does it mean ?*

Ans. At that time any volume of air will be saturated by the vapours present.

Q. 41. *Why the wet clothes dry slowly in rainy season than in summer, the temperature being the same ?*

Ans. The Relative humidity in the rainy season is greater than the summer, so the rate of evaporation is slow.

Q. 42. *What is the use of the knowledge of R.H. ?*

Ans. Its knowledge is of very great value for meteorological stations and public health deptt. Forecast of weather is based on R.H. Its large value tells, that it may rain. This information is of great value for the pilots, textile mills, cold storages and agriculture.

Q. 43. *What factors affect the dew formation ?*

Ans. *(i)* Presence of clouds in the sky.

(ii) Presence of good radiaters on earth.

(iii) Wind.

Q. 44. *The most suitable relative humidity for health is—*

Ans. 50%

Q. 45. *What are different modes of transfer of heat ?*

Ans. *(i)* Conduction – for solids and mercury.

(ii) Convection – for liquids and gases.

(iii) Radiation – This does not require any medium.

Q. 46. *What is thermal conductivity ?*

Ans. The property of a substance by virtue of which heat is transmitted through the substance. Heat being handed over from molecule to molecule.

Q. 47. *In solids, what factors give the flow of heat ?*

Ans. After reaching the steady state, the amount of heat flowing through the solids in t sec. depends on area of the Surfaces.

i.e., Q $\propto$ Area of two surfaces

$\propto$ Temperature gradient between the two surfaces.

$\propto$ Time, t.

Q. 48. *What is temperature gradient ?*

Ans. Rate of fall of temperature with distance between two isothermal surfaces

i.e. $\left[-\dfrac{\Delta\,\theta}{\Delta\,x}\right]$, –ve sign indicates that θ decreases when x increases.

Q. 49. *What is meant by steady state ?*

Ans. When the temperature of different points of the conductor, becomes steady. At this state heat is not at all absorbed by any part of the rod and the rate of conduction depends only on the conductivity of the material. This is steady state.

Q. 50. *What is coefficient of thermal conductivity ?*

Ans. From the formula,

$Q = \dfrac{kA(\theta_2 - \theta_1)t}{d}$ we can define K. It is the amount of heat which in steady state flows between the opposite surfaces of cube of 1 meter side per second normal to the surfaces when they are kept at a temperature difference of 1 °C.

Q. 51. *Why mica is used in heaters ?*

Ans. Mica is conductor of heat but insulator for electricity.

Q. 52. *Why in metals transmission of heat is very quick ?*

Ans. In metals, (Hg also) the transmission of heat is due to free electrons in the metal and not by the molecules.

Q. 53. *Which has more conductivity, Cu or Ag ?*

Ans. Ag.

Q. 54. *Two metallic plates of thickness d_1 and d_2 and thermal conductivities K_1 and K_2 are placed in contact. What is their equivalent thermal conductivity ?*

Ans. $$K = \frac{d_1 + d_2}{\left[\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}\right]}$$

Q. 55. *Two metallic plates of equal thickness and same area of cross-section and conductivities K_1 and K_2 are placed in contact. What is their equivalent thermal conductivity ?*

Ans. $$K = \frac{2K_1K_2}{K_1 + K_2}$$

Q. 56. *A cloudy night is hotter than a clear sky-night. Why ?*

Ans. In the cloudy night the heat radiated out from the earth is reflected by the cloud back to the earth. Therefore the temperature of the earth does not fall.

Q. 57. *Define co-efficient of superficial expansion. How is it related to α.*

Ans. Co-efficient of superficial expansion is defined as the increase in area per unit area per degree centigrade rise in temperature. Its value is twice that of co-efficient of linear expansion *i.e.*, $\beta = 2\alpha$.

Q. 58. *Define co-efficient of cubical expansion. How is it related to α.*

Ans. It is defined as increase in volume per unit volume per degree centigrade rise in temperature. Its value is thrice the value of co-efficient of linear expansion *i.e.*, $\gamma = 3\alpha$.

Q. 59. *Why is some gap left between two rails ?*

Ans. In order to make allowance for the expansion in summer.

Q. 60. *Why are the telegraph wires left loose between two poles ?*

Ans. In order to make allowance for contraction in winter.

G.M. College of Education
Raipur, Bantalab
Jammu.
Acc No....2.624 (D)
Dated.....1/11/2015